



A New Way of Understanding why the Harmonic Series is Divergent

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Abstract: *The harmonic series is one of the most important series in mathematics. Here in this short paper we will establish a new lemma to prove that the harmonic series is divergent. The main purpose of establishing this lemma is to demonstrate a new way of understanding why the harmonic series must be divergent.*

1. INTRODUCTION

We know the harmonic series is divergent. There are several well-known proofs of the divergence of the harmonic series. But here we will show a different kind of proof which will help us to easily understand why the harmonic series is divergent. Here we will establish a new lemma which will be helpful to understand for some infinite series like harmonic series that the series is divergent or not.

2. PROPOSING THE LEMMA

Lemma 1: Suppose a series $S_1 = a_1 + a_2 + a_3 + \dots$ and another convergent series $S_2 = b_1 + b_2 + b_3 + \dots$ where $S_2 \neq 0$. After adding the series S_2 with the series S_1 or subtracting the series S_2 from the series S_1 if all the nonzero terms of $S_1 + S_2$ or $S_1 - S_2$ become the same as the nonzero terms of S_1 , then S_1 must be a divergent series.

Proof: If all the nonzero terms of $S_1 + S_2$ or $S_1 - S_2$ become the same as the nonzero terms of S_1 , then it will be true if and only if $S_1 = a_1 + a_2 + a_3 + \dots = +\infty$ or $S_1 = a_1 + a_2 + a_3 + \dots = -\infty$ because if we add anything or subtract anything from infinity ($+\infty$ or $-\infty$), the result will be infinity ($+\infty$ or $-\infty$). If S_1 is a convergent series then the nonzero terms of $S_1 + S_2$ or $S_1 - S_2$ will not become the same as the nonzero terms of S_1 .

This lemma is not helpful for every divergent series. It would be applicable for some infinite series to check that the series is divergent or convergent. Now, let us apply this lemma for harmonic series.

$$S_1 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$$

$$S_2 = -\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots\right) = -\ln 2$$

$$1 \quad + \frac{1}{2} \quad + \frac{1}{3} + \dots$$

Here, after subtracting the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$ from the harmonic series, the nonzero terms of $S_1 - S_2$ become the same as the nonzero terms of the harmonic series. So, the harmonic series is divergent.

3. CONCLUSION

The proof of the divergence of the harmonic series showed in the paper will help us to understand why the Harmonic series is divergent from a new perspective.

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