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On Disjoint Stationary Sets in Topological Spaces

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Abstract: *In this paper we study some new techniques from set theory to general topology and its applications , here we study some the partition relations ,cardinal functions and its principals and some more theorems . Further we develop the combinatory of stationary sets we have aimed at person with some knowledge of topology and a little knowledge of set theory.*

Keywords: *Stationary sets ,homeomorphic, topologicalspaces, Bairespaces.*

1. Introduction

Here we will discuss some interesting subsets of ω_1 – sets, Borel subsets, stationary and bi-stationary subsets of ω_1 – and some theorems . In addition, we will mention some topological applications of stationary sets . Let us know that ω_1 is an uncountable well-ordered set with the special property that if $\alpha < \omega_1$, then the initial segment $[0, \alpha)$ of ω_1 is countable, and that the countable union of countable sets is countable. Like any linearly ordered set, ω_1 has an open interval topology. Le us consider that α is a limit point of the space ω_1.

2. Main Results

Theorem 1.1 : A Borel subset of B(k) which is not in $\sigma - LW(<k)$ is a homeomrphic to B(k).

Definition : B(k) is the Baire zero dimensional space of weight k, is a function from $\chi \rightarrow k$ with the given $d(f,g) = 2^{-n}$,where n is the least of $f/n \neq g/n$.

Definition: A space ‘x’ is a Baire space iff intersection of countable many dense open subsets of X is a dense in X.

Definition: Any set that intersects every club-set in ω_1 is called a stationary subset of ω_1.

Definition: Any set $S \subseteq \omega_1$ with the property that both S and $\omega_1 - S$ intersect every club-set is called a bi-stationary set.

Definition: A topological space X is said to be metrizable if there exists a metric d such that $\{Br(x) : x \in X \text{ and } r > 0\}$ forms a basis generating the topology on X.

Proposition: Any subspace of a metrizable space is metrizable.

Lemma : Let X is a metrizable space with weight k, where k is an uncountable regular cardinal , let $(d_\alpha : \alpha < k)$ be a dense subset of X. For $\delta < k$.

Theorem 1.2: There is a subset “s” of B(w) such that both s and B(k)-S meet every subset of B(w) is homeomorphic.

Proof : Let us consider the note that there are 2^w *of* G_δ subsets of B(w) and that each uncountable G_δ has 2^ω points. Here we consider the well ordered uncountable G_δ subsets and inductively choose any two points from each one be in S, the other to be in B(w)-S. The induction continuous because at each step there are less than 2^ω points which are chosen so that there are plenty of possible new points.

We would like to construction with B(k) in the place of B(w) , However ,when $k = 2^\omega$ Then there are 2^k open sets, but only k points in the space.

Theorem 1.3: Let k be an uncountable regular cardinal, then their S of B(k) such that both S and B(k)-S meet every subset of B(k) is homeomorphic to B(k).

Proof : Let introduce the new function with B(k) ,for $\alpha < k$, Let us consider $\sum_{\alpha} \alpha = \{\alpha : \alpha \text{ is a function}, dom\ \alpha \in \omega, ran\ \sigma \subset \alpha\}$ For $\sigma \in \sum_{k} \alpha$ define for $\alpha = \{f \in B(k) : \sigma \subset f\}$. Such that for each $[\alpha]$ is closed and open subset of B(k),and $\{[\sigma] : \sigma \in \sum_{i}\}$is a base for B(k),for B(k) define $f^* = Sup\ ran\ f\ \ for\ \sigma \in \sum_{k}$ define $\sigma^* = Sup\ ran \sigma$. Note that for each $\alpha < k, \{\sigma : \sigma^* < \alpha\} < k$. By using the above Lemma it is clear that $A \subset CF\ \omega \cap k$ such that both A and CF of $CF\ \ \omega \bigcap k$ are stationary sets in k, Thus z satisfies the conclusion of Theorem 1.3 .

Theorem 1.4: There are two Baire spaces whose product is not a Baire.

Proof : Let A and B are any disjoint stationary sub sets of ω_1. Further consider $X = \{f \in B(\omega_1) : f^* \in A\} : Y = \{g \in B(\omega_1) : g^* \in B\}$, for every dense of G_δ of B(k) is homeomorphic to B(k), so by Definition , X any Y are Baire spaces.

For $n \in \omega$, let $U_n = \{(f, g) \in X \times Y : f(n) < g^*\ and\ g(n) < f^*\}$.It is to verify that U_n is dense and open subset of XxY. Thus the Contradiction.

Theorem 1.5: Suppose there is a continuous injective mapping h: S → T where S and T are stationary sets. Then S ∩ T is also stationary.

Proof: Because there is a continuous injective mapping from S to T, then S − T cannot be stationary. But S = (S ∩ T) ∪ (S − T) is stationary so that the set S ∩ T must be stationary.

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