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Bisection Method by using Fuzzy Concept

Nasr Al Din IDE*

Professor, Aleppo University, Aleppo, Syria

***Corresponding Author:** ***Nasr Al Din IDE,** *Professor, Aleppo University, Aleppo, Syria*

Abstract: *Finding the roots of nonlinear algebraic equations is an important problem in science and engineering, later many methods have been developed by using Taylor's series, decomposition, homotopy, for solving nonlinear equations [1-25]. Fuzzy numbers are foundation of fuzzy sets and fuzzy mathematics that extends the domain of numbers from real numbers to fuzzy numbers. Several researchers investigated a number of methods of numerical analysis by using Fuzzy theory. Recently, various methods have been developed for solving linear programming problems with using fuzzy number. Many research works were done on fuzzy numbers and on its applications in various field [26-34], but very few developments have been seen in the area of numerical methods using fuzzy triangular numbers. Toralima Bora and G.C. Hazarika [26], Treat the fuzzification of Newton Raphson method to find the solution of cubic equation in the form of triangular numbers along with the membership function. In this work the fuzzification of Bisection method to give the solution of nonlinear equations. Results have been obtained in the form of triangular numbers with the membership function also. These methods based on the method given by Toralima Bora and G.C. Hazarika [26], 2017. We verified the work by an example and numerical results obtained to clarify the present method.*

Keywords: *Bisection Method, Newton Raphson Method, Fuzzy membership function, Triangular fuzzy number, α-cut.*

1. Introduction

Solving nonlinear equations f (x)=0, is one of the most important problem in scientific and engineering applications. There are several well-known methods for solving nonlinear algebraic equations of the form:

$$F(x)=0 \tag{1}$$

Where F denote a continuously differentiable function on [a, b]$\subset \mathscr{R}$, and has at least one root r, in [a, b] Such as Newton's Method, Bisection method, Regula Falsi method, Nonlinear Regression Method and several another methods see for example [1-25]. Here we describe a new method by using Fuzzy Concept, then we find that, this procedure lead us to the root r of equation (1) between two fuzzy sets A and B.

2. The Present Method -Fuzzification of BISECTION METHOD

Fuzzification of the Bisection method will be done as it treated in [26] by using triangular fuzzy number.

Let us consider equation (1): F(X) =0, Let the function F(X) changes its sign over an interval X_0 and X_1. Let X_0=[X_0', X_0'', X_0'''] and X_1=[X_1', X_1'', X_1''']. Then there is a root of F(X)=0 lying between X_0 and X_1. Fuzzy membership function of X_0 and $X_1$are respectively,

$$\mu_{X_0} = \begin{cases} \dfrac{X - X_0'}{X_0'' - X_0'}; & X_0' \le X \le X_0'' \\ \dfrac{X - X_0'''}{X_0'' - X_0'''}; & X_0'' \le X \le X_0''' \\ 0 & otherwise \end{cases} \quad ; \quad \mu_{X_1} = \begin{cases} \dfrac{X - X_1'}{X_1'' - X_1'}; & X_1' \le X \le X_1'' \\ \dfrac{X - X_1'''}{X_1'' - X_1'''}; & X_1'' \le X \le X_1''' \\ 0 & otherwise \end{cases}$$

with α-cuts:

$$[X_0]^{\alpha} = [X_0' \;, X_0'' \;, X_0'''] = [X_0' + \alpha (X''_0 - X_0') \;, X_0''' + \alpha (X_0'' - X_0''')]$$

$[X_1]^\alpha = [X_1', X''_1, X_1'''] = [X_1' + \alpha(X''_1 - X_1'), X_1''' + \alpha(X_1'' - X_1''')]$

According to Bisection method, if $F(X_0).F(X_1)<0$ then, the first approximation of the root of $F(X)=0$ is X_2 which it is given as following:

$$X_2=[\frac{X_0+X_1}{2}]=[\frac{X_0'+X_1'}{2}, \frac{X_0''+X_1''}{2}, \frac{X_0'''+X_1'''}{2}]=[X_2', X_2'', X_2''']$$

With the membership function:

$$\mu_{X_2}(X)=\begin{cases} \frac{X-X_2'}{X_2''-X_2'}; & X_2' \le X \le X_2'' \\ \frac{X-X_2'''}{X_2''-X_2'''}; & X_2'' \le X \le X_2''' \\ 0 & otherwise \end{cases}$$

And $[X_2]^\alpha = [X_2', X''_2, X_2'''] = [X_2' + \alpha(X''_2 - X_2'), X_2''' + \alpha(X_2'' - X_2''')]$

After that, we study the sign of $f(X_2)$, if $f(X_0).f(X_2)<0$, then the solution is between X_0 and X_2 else the solution is between X_1 and X_2. Suppose that $f(X_0).f(X_2)<0$, then the solution is between X_0 and X_2, so, Similarly X3 is given by:

$$X_3=[\frac{X_0+X_2}{2}]=[\frac{X_0'+X_2'}{2}, \frac{X_0''+X_2''}{2}, \frac{X_0'''+X_2'''}{2}]=[X_3', X_3'', X_3''']$$

with membership function:

$$\mu_{X_3}(X)=\begin{cases} \frac{X-X_3'}{X_2''-X_3'}; & X_3' \le X \le X_3'' \\ \frac{X-X_3'''}{X_3''-X_3'''}; & X_3'' \le X \le X_3''' \\ 0 & otherwise \end{cases}$$

And $[X_3]^\alpha = [X_3', X''_3, X_3'''] = [X_3' + \alpha(X''_3 - X_3'), X_3''' + \alpha(X_3'' - X_3''')]$

And so, we can find X_4,

$X_4=[\frac{X_0+X_3}{2}]=[\frac{X_0'+X_3'}{2}, \frac{X_0''+X_3''}{2}, \frac{X_0'''+X_3'''}{2}]=[X_4', X_4'', X_4''']$ so on, we can find X_n.

3. The Error of Present Method

For the error, we can study the error as it given by [35], we select a tolerance $\varepsilon>0$ and generate $\boldsymbol{X_2}$, $\boldsymbol{X_3}$, $\boldsymbol{X_4}$, …..,$\boldsymbol{X_n}$ until one of the following conditions is met:

a) $|\boldsymbol{X_n} - \boldsymbol{X_{n-1}}| < \varepsilon$

b) $\frac{|\boldsymbol{X_n} - \boldsymbol{X_{n-1}}|}{|\boldsymbol{X_n}|} < \varepsilon$; $|\boldsymbol{X_n}| \neq \boldsymbol{0}$.

c) $|\boldsymbol{F}(\boldsymbol{X_n})| < \varepsilon$

4. Numerical Example

Let us consider the algebraic equation ***[26],*** $\boldsymbol{F(X)=X^3-6X+4}$

Let $X_0=[-0.01, 0, 0.01]$ and $X_1=[0.99, 1, 1.01]$ since,

$F(X_0)=F[-0.01, 0, 0.01]=\boldsymbol{X_0^3}-6X_0+4=[3.93, 4, 4{,}07]$,

$F(X_1)= F=[0.99, 1, 1.01]=\boldsymbol{X_1^3}-6X_1+4=[-1.09, -1, -0.89]$

Since $F(X_0)$ and $F(X_1)$ are of opposite sign, a root r lies between

$X_0=[-0.01, 0, 0.01]$ and $X_1=[0.99, 1, 1.01]$, if $F(X_0).F(X_1)<0$, then the solution is between X_0 and X_1

Hence, $X_2=[\frac{\boldsymbol{X_0}+\boldsymbol{X_1}}{\boldsymbol{2}}]=[0.49, 0.05, 0.51]$,

The membership function of X_2 is

$$\mu_{X_2}(X)=\begin{cases}\frac{X-0.49}{0.05-0.49}; & 0.49\le X\le 0.05\\ \frac{X-0.51}{0.05-0.51}; & 0.05\le X\le 0.51\\ 0 & otherwise\end{cases}$$

And $[X_2]^{\alpha}=[X_2', X''_2, X_2''']=[0.49+\alpha(0.05-0.49), 0.51+\alpha(0.05-0.51)]$

$$=[0.49-0.44\,\alpha, 0.51-0.46\alpha)]$$

we stydy the sign of F(X_2), we see that

$F(X_2)=[1.177649, 1.125, 1.072651]>0$, so, the solution is between X_0 and X_2

So, $X_3=[\frac{X_0+X_2}{2}]=[0.24, 0.25, 0.26]$. The membership function of X_3 is

$$\mu_{X_3}(X)=\begin{cases}\frac{X-0.24}{0.25-0.24}; & 0.24\le X\le 0.25\\ \frac{X-0.26}{0.25-0.26}; & 0.25\le X\le 0.26\\ 0 & otherwise\end{cases}$$

And $[X_3]^{\alpha}=[X_3', X''_3, X_3''']=[0.24+\alpha(0.25-0.24), 0.26+\alpha(0.25-0.26)]$

$$=[0.24+0.01\,\alpha, 0.26-\mathbf{0.01\alpha}]$$

we study the sign *of* $F(X_3)$, we see that

$F(X_3)=[2.573824, 2.515625., 2.457576]>0$, so the solution is between X_0 and X_3 and so**.**

5. Conclusion

In this work we use the fuzzy concept to find the root of nonlinear algebraic equation by the Bisection method, using the triangular number, we treat one example to clarify this work.

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