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Lagrangian Mechanical Systems with Four Almost Complex Structures on Symplectic Geometry

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Abstract: *In this paper we presented an analysis of Lagrange formulas. with four Almost Complex Structures we have reached important results in differential geometry that can be applied in theoretical physics*
Keywords: *Symplectic geometry, four almost complex structure, Lagrangian Mechanical Systems.*

1. Introduction

The geometric study of dynamical systems is an important chapter of contemporary mathematics due to its applications in Mechanics, Theoretical Physics**.** the most important papers on the topic entitled Mechanical Equations.

R.Ye developed a general framework for embedded (immersed) J-holomorphic curves and a systematic treatment of the theory of filling by holomorphic curves in 4-dimensional symplectic manifolds [1]**. Audin** and Lafontaine introductioned to symplectic geometry and relevant techniques of Riemannian geometry, proofs of Gormov.s compactness theorem, an investigation of local properties of holomorphic curves, including positivity of intersections, and applications to Lagrangian embeddings problems [2]. **Tekkoyun** submitted paracomplex analogue of the Euler-Lagrange equations was obtained in the framework of para-Kählerian manifold and the geometric results on a paracomplex mechanical systems were found [3]. **Lisi** considered three applications of pseudoholomorphic curves to problems in Hamiltonian dynamics [4]. **Tekkoyun and Yayli** shown that generalized-quaternionic Kählerian analogue of Lagrangian and Hamiltonian mechanical systems. Eventually, the geometric-physical results related to generalized-quaternionic Kählerian mechanical systems are provided [5]. **Kasap** submitted Weyl-Euler-Lagrange equations of motion on .at manifold [6]. **Kasap and Tekkoyun** obtained Lagrangian and Hamiltonian formalism for mechanical systems using para/pseudo-Kahler manifolds, representing an interesting multidisciplinary. field of research. Also, the geometrical, relativistical, mechanical and physical results related to para/pseudo-Kahler mechanical systems were given, too [7].

Kasap examined Weyl. Euler. Lagrange and Weyl. Hamilton equations on R_n^{2n} which is a model of tangent manifolds of constant W-Sectional curvature [8]. **Oguzhan and Zeki Kasap** submitted Mechanical Equations with Two Almost Complex Structures on Symplectic Geometry, using two complex structures, examined mechanical systems on symplectic geometry. [9].

In this paper, we study dynamical systems with four Almost Complex Structures. After Introduction in Section 1, we consider Historical Background paper basic. Section 2 (Preliminaries) deals with the study Almost Complex Structures. Section 3 is devoted to study Lagrangian Dynamics.

2. Preliminaries

In this preliminary chapter, we recall basic definitions, results and formulas which we shall use in the subsequent chapters of the paper. Most of material included in this chapter occurs in standard literatures namely

2.1. Symplectic Manifolds

Definition 2.1.1 A symplectic manifold is a pair $(\mathcal{M},\sigma)$such that $\mathcal{M}$ is a smooth manifold and σ is a closed non-degenerate differential 2-form on $\mathcal{M}$. This means that in each tangent space $T_p\mathcal{M}$, σ gives a non degenerate, bilinear, skew symmetric form $\sigma_p : T_p\mathcal{M} \times T_p\mathcal{M} \longrightarrow \mathbb{R}$ such that σ_p varies smoothly in p.

2.2. Complex Manifolds

Let $\mathcal{M}$ be configuration manifold of real dimension m. A tensor field J on $T\mathcal{M}$ is called an almost complex structure on $T\mathcal{M}$ if at every point p of $T\mathcal{M}$, J is endomorphism of the tangent space $T_p(T\mathcal{M})$ such that $J^2 = -I$. A manifold $T\mathcal{M}$ with fixed almost complex structure J is called almost complex manifold. Assume that (x_i) be coordinates of $\mathcal{M}$ and (x_i, y_i) be a real coordinate system on a neighborhood U of any point p of $T\mathcal{M}$. Also, let us to be $\left\{\left(\frac{\partial}{\partial x_i}\right)_p, \left(\frac{\partial}{\partial y_i}\right)_p\right\}$ and $\{(dx_i)_p, (dy_i)_p\}$ to natural bases over R of tangent space $T_p(T\mathcal{M})$ and cotangent space $T_p^*(T\mathcal{M})$ of $T\mathcal{M}$, respectively.

Let $T\mathcal{M}$ be an almost complex manifold with fixed almost complex structure J. The manifold $T\mathcal{M}$ is called complex manifold if there exists an open covering {U}

of $T\mathcal{M}$ satisfying the following condition: There is a local coordinate system (x_i, y_i)on each U, such that

$$J\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial y_i} \quad , \quad J\left(\frac{\partial}{\partial y_i}\right) = -\frac{\partial}{\partial x_i} \tag{1}$$

2.3. Integrable Almost Complex Structures

Definition 2.3.1[6]

Every complex manifold is itself an almost complex manifold. In local holomorphic coordinates $Z = x_k + iy_k$ one can define the maps

$$J\left(\frac{\partial}{\partial x_k}\right) = \frac{\partial}{\partial y_k} \quad , \quad J\left(\frac{\partial}{\partial y_k}\right) = -\frac{\partial}{\partial x_k}$$

Proposition 2.3.2

Suppose that $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, be a real coordinate system on $(\mathcal{M}, J)$. Then we denote by

$$\left\{\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_6}, \frac{\partial}{\partial x_7}, \frac{\partial}{\partial x_8}\right\}$$

$$\{dx_1, dx_2, dx_3, dx_4, dx_5, dx_6\}$$

$$J\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2} \quad , \quad J\left(\frac{\partial}{\partial x_2}\right) = -\frac{\partial}{\partial x_1} \quad ,$$

$$J\left(\frac{\partial}{\partial x_3}\right) = \frac{\partial}{\partial x_4} \quad , \quad J\left(\frac{\partial}{\partial x_4}\right) = -\frac{\partial}{\partial x_3}$$

$$J\left(\frac{\partial}{\partial x_5}\right) = \frac{\partial}{\partial x_6} \quad , \quad J\left(\frac{\partial}{\partial x_6}\right) = -\frac{\partial}{\partial x_5}$$

$$J\left(\frac{\partial}{\partial x_7}\right) = \frac{\partial}{\partial x_8} \quad , \quad J\left(\frac{\partial}{\partial x_8}\right) = -\frac{\partial}{\partial x_7} \tag{2}$$

Let

$$z_1 = x_1 + ix_2 \ , \ z_2 = x_3 + ix_4 \quad , \ z_3 = x_5 + ix_6 \ , \quad z_4 = x_7 + ix_8$$

$$J^2\left(\frac{\partial}{\partial x_1}\right) = \frac{\partial}{\partial x_2} = J\left(\frac{\partial}{\partial x_2}\right) = -\frac{\partial}{\partial x_1}$$

$$J^2\left(\frac{\partial}{\partial x_2}\right) = J\left(-\frac{\partial}{\partial x_1}\right) = -\frac{\partial}{\partial x_2}$$

$$J^2\left(\frac{\partial}{\partial x_3}\right) = J\left(\frac{\partial}{\partial x_4}\right) = -\frac{\partial}{\partial x_3}$$

$$J^2\left(\frac{\partial}{\partial x_4}\right) = J\left(-\frac{\partial}{\partial x_3}\right) = -\frac{\partial}{\partial x_4}$$

$$J^2\left(\frac{\partial}{\partial x_5}\right) = J\left(\frac{\partial}{\partial x_6}\right) = -\frac{\partial}{\partial x_5}$$
$$J^2\left(\frac{\partial}{\partial x_6}\right) = J\left(-\frac{\partial}{\partial x_5}\right) = -\frac{\partial}{\partial x_6}$$
$$J^2\left(\frac{\partial}{\partial x_7}\right) = J\left(-\frac{\partial}{\partial x_8}\right) = -\frac{\partial}{\partial x_7}$$
$$J^2\left(\frac{\partial}{\partial x_8}\right) = J\left(-\frac{\partial}{\partial x_7}\right) = -\frac{\partial}{\partial x_8}$$

Theorem 2.3.3 [3] Let $\mathcal{M}$ be m-real dimensional configuration manifold .A tensor field J on $T^*\mathcal{M}$ is called an almost complex structure on $T^*\mathcal{M}$ if at every point p of $T^*\mathcal{M}$, J is endomorphism of the tangent space $T_p{}^*(\mathcal{M})$ such that $J^2 = -1$ are complex is $J^{*2} = J^* \circ J^* = -1$ is called structures are complex manifold

3. Lagrangian Dynamical Systems

In this section, we shall obtain the version Euler-Lagrange equations for classical mechanics structured with Four Almost Complex Structures on Symplectic Geometry introduced in

Definition 3.1. A Lagrangian function for a Hamiltonian vector field X on $\mathcal{M}$ is a smooth function $L : T\mathcal{M} \rightarrow R$ such that

$$i_X \Phi_L = dE_L \quad (3)$$

Let ξ be the vector field by

$$\xi = X_1\frac{\partial}{\partial x_1} + X_2\frac{\partial}{\partial x_2} + X_3\frac{\partial}{\partial x_3} + X_4\frac{\partial}{\partial x_4} + X_5\frac{\partial}{\partial x_5} + X_6\frac{\partial}{\partial x_6} + X_7\frac{\partial}{\partial x_7} + X_8\frac{\partial}{\partial x_8} \quad (4)$$

And

$X_1 = \dot{x}_1$, $X_2 = \dot{x}_2$, $X_3 = \dot{x}_3$, $X_4 = \dot{x}_4$, $X_5 = \dot{x}_5$, $X_6 = \dot{x}_6, X_7 = \dot{x}_7$, $X_8 = \dot{x}_8$

$$U = J(\xi) = X_1\frac{\partial}{\partial x_1} - X_2\frac{\partial}{\partial x_2} + X_3\frac{\partial}{\partial x_3} - X_4\frac{\partial}{\partial x_4} + X_5\frac{\partial}{\partial x_5} - X_6\frac{\partial}{\partial x_6} + X_7\frac{\partial}{\partial x_7} - X_8\frac{\partial}{\partial x_8} \quad (5)$$

Let that Liouville Vector field on complex manifold $(\mathcal{M}, U)$

Kinetic energy given $T: T\mathcal{M} \rightarrow \mathcal{M}$

$$T = \frac{1}{2}m_i(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{x}_4^2 + \dot{x}_5^2 + \dot{x}_6^2 + \dot{x}_7^2 + \dot{x}_8^2)$$

Potential energy $P: T\mathcal{M} \rightarrow \mathcal{M}$

$$P = m_i gh$$

Here m_i, g and h stand for mass of a mechanical system having m particles, the gravity acceleration and distance to the origin of mechanical system .

Then $L : \mathcal{M} \rightarrow R$ is a map that satisfies the conditions;

$i) L = T - P$ is a Lagrangian function,

ii) the function determined by $E_L = V(L) - L$, is energy function.

The function i_{F^0} induced by F^0 and denoted by

$$i_J\omega(X_1, X_2, \dots, X_r) = \sum_{i=1}^{r} \omega(X_1, \dots, JX_i, \dots, X_r) \quad (6)$$

is said to be vertical derivation, where $\omega \in \wedge^r \mathcal{M}, X_i \in \mathcal{X}(\mathcal{M})$. The vertical differentiation d_J is defined by

$$d_J = [i_J, d] = i_J d - d i_J$$

where d is the usual exterior derivation.

$\phi_L = dd_J L$ such that

$$d_J = \frac{\partial}{\partial x_2}dx_1 - \frac{\partial}{\partial x_1}dx_2 + \frac{\partial}{\partial x_4}dx_3 - \frac{\partial}{\partial x_3}dx_4 + \frac{\partial}{\partial x_6}dx_5 - \frac{\partial}{\partial x_5}dx_6 + \frac{\partial}{\partial x_8}dx_7 - \frac{\partial}{\partial x_7}dx_8 \quad (7)$$

Defined by operator $d_J : A(\mathcal{M}) \to \wedge^1 \mathcal{M}$

$$d_J L = \left(\frac{\partial}{\partial x_2}dx_1 - \frac{\partial}{\partial x_1}dx_2 + \frac{\partial}{\partial x_4}dx_3 - \frac{\partial}{\partial x_3}dx_4 + \frac{\partial}{\partial x_6}dx_5 - \frac{\partial}{\partial x_5}dx_6 + \frac{\partial}{\partial x_8}dx_7 - \frac{\partial}{\partial x_7}dx_8\right)L$$

$$d_J L = \frac{\partial L}{\partial x_2}dx_1 - \frac{\partial L}{\partial x_1}dx_2 + \frac{\partial L}{\partial x_4}dx_3 - \frac{\partial L}{\partial x_3}dx_4 + \frac{\partial L}{\partial x_6}dx_5 - \frac{\partial L}{\partial x_5}dx_6 + \frac{\partial L}{\partial x_8}dx_7 - \frac{\partial L}{\partial x_7}dx_8 \quad (8)$$

That

$$\phi_L = -d\left(d_{G_1}\right) = -d\left(\frac{\partial}{\partial x_2}dx_1 - \frac{\partial}{\partial x_1}dx_2 + \frac{\partial}{\partial x_4}dx_3 - \frac{\partial}{\partial x_3}dx_4 + \frac{\partial}{\partial x_6}dx_5 - \frac{\partial}{\partial x_5}dx_6 + \frac{\partial}{\partial x_8}dx_7 dx_8\right)$$

$$\phi_L = -\frac{\partial^2 L}{\partial x_1 \partial x_2}dx_1 \wedge dx_1 + \frac{\partial^2 L}{\partial x_1 \partial x_1}dx_1 \wedge dx_2 - \frac{\partial^2 L}{\partial x_1 \partial x_4}dx_1 \wedge dx_3 + \frac{\partial^2 L}{\partial x_1 \partial x_3}dx_1 \wedge dx_4 - \frac{\partial^2 L}{\partial x_1 \partial x_6}dx_1 \wedge dx_5 + \frac{\partial^2 L}{\partial x_1 \partial x_5}dx_1 \wedge dx_6 - \frac{\partial^2 L}{\partial x_1 \partial x_8}dx_1 \wedge dx_7 + \frac{\partial^2 L}{\partial x_1 \partial x_7}dx_1 \wedge dx_8 - \frac{\partial^2 L}{\partial x_2 \partial x_2}dx_2 \wedge dx_1 + \frac{\partial^2 L}{\partial x_2 \partial x_1}dx_2 \wedge dx_2 - \frac{\partial^2 L}{\partial x_2 \partial x_4}dx_2 \wedge dx_3 + \frac{\partial^2 L}{\partial x_2 \partial x_3}dx_2 \wedge dx_4 - \frac{\partial^2 L}{\partial x_2 \partial x_6}dx_2 \wedge dx_5 + \frac{\partial^2 L}{\partial x_2 \partial x_5}dx_2 \wedge dx_6 - \frac{\partial^2 L}{\partial x_1 \partial x_8}dx_2 \wedge dx_7 + \frac{\partial^2 L}{\partial x_2 \partial x_7}dx_2 \wedge dx_8 - \frac{\partial^2 L}{\partial x_3 \partial x_2}dx_3 \wedge dx_1 + \frac{\partial^2 L}{\partial x_3 \partial x_1}dx_3 \wedge dx_2 - \frac{\partial^2 L}{\partial x_3 \partial x_4}dx_3 \wedge dx_3 + \frac{\partial^2 L}{\partial x_3 \partial x_3}dx_3 \wedge dx_4 - \frac{\partial^2 L}{\partial x_3 \partial x_6}dx_3 \wedge dx_5 + \frac{\partial^2 L}{\partial x_3 \partial x_5}dx_3 \wedge dx_6 - \frac{\partial^2 L}{\partial x_3 \partial x_8}dx_3 \wedge dx_7 + \frac{\partial^2 L}{\partial x_3 \partial x_7}dx_3 \wedge dx_8 - \frac{\partial^2 L}{\partial x_4 \partial x_2}dx_4 \wedge dx_1 + \frac{\partial^2 L}{\partial x_4 \partial x_1}dx_4 \wedge dx_2 - \frac{\partial^2 L}{\partial x_4 \partial x_4}dx_4 \wedge dx_3 + \frac{\partial^2 L}{\partial x_4 \partial x_3}dx_4 \wedge dx_4 - \frac{\partial^2 L}{\partial x_4 \partial x_6}dx_4 \wedge dx_5 + \frac{\partial^2 L}{\partial x_4 \partial x_5}dx_4 \wedge dx_6 - \frac{\partial^2 L}{\partial x_4 \partial x_8}dx_4 \wedge dx_7 + \frac{\partial^2 L}{\partial x_4 \partial x_7}dx_4 \wedge dx_8 - \frac{\partial^2 L}{\partial x_5 \partial x_2}dx_5 \wedge dx_1 + \frac{\partial^2 L}{\partial x_5 \partial x_1}dx_5 \wedge dx_2 - \frac{\partial^2 L}{\partial x_5 \partial x_4}dx_5 \wedge dx_3 + \frac{\partial^2 L}{\partial x_5 \partial x_3}dx_5 \wedge dx_4 - \frac{\partial^2 L}{\partial x_5 \partial x_6}dx_5 \wedge dx_5 + \frac{\partial^2 L}{\partial x_5 \partial x_5}dx_5 \wedge dx_6 - \frac{\partial^2 L}{\partial x_5 \partial x_8}dx_5 \wedge dx_7 + \frac{\partial^2 L}{\partial x_5 \partial x_7}dx_5 \wedge dx_8 - \frac{\partial^2 L}{\partial x_6 \partial x_2}dx_6 \wedge dx_1 + \frac{\partial^2 L}{\partial x_6 \partial x_1}dx_6 \wedge dx_2 - \frac{\partial^2 L}{\partial x_6 \partial x_4}dx_6 \wedge dx_3 + \frac{\partial^2 L}{\partial x_6 \partial x_3}dx_6 \wedge dx_4 - \frac{\partial^2 L}{\partial x_6 \partial x_6}dx_6 \wedge dx_5 + \frac{\partial^2 L}{\partial x_6 \partial x_5}dx_6 \wedge dx_6 - \frac{\partial^2 L}{\partial x_6 \partial x_8}dx_6 \wedge dx_7 + \frac{\partial^2 L}{\partial x_6 \partial x_7}dx_6 \wedge dx_8 - -\frac{\partial^2 L}{\partial x_7 \partial x_2}dx_7 \wedge dx_1 + \frac{\partial^2 L}{\partial x_7 \partial x_1}dx_7 \wedge dx_2 - \frac{\partial^2 L}{\partial x_7 \partial x_4}dx_7 \wedge dx_3 + \frac{\partial^2 L}{\partial x_7 \partial x_3}dx_7 \wedge dx_4 - \frac{\partial^2 L}{\partial x_7 \partial x_6}dx_7 \wedge dx_5 + \frac{\partial^2 L}{\partial x_7 \partial x_5}dx_7 \wedge dx_6 - \frac{\partial^2 L}{\partial x_7 \partial x_8}dx_7 \wedge dx_7 + \frac{\partial^2 L}{\partial x_7 \partial x_7}dx_7 \wedge dx_8 - \frac{\partial^2 L}{\partial x_8 \partial x_2}dx_8 \wedge dx_1 + \frac{\partial^2 L}{\partial x_8 \partial x_1}dx_8 \wedge dx_2 - \frac{\partial^2 L}{\partial x_8 \partial x_4}dx_8 \wedge dx_3 + \frac{\partial^2 L}{\partial x_8 \partial x_3}dx_8 \wedge dx_4 - \frac{\partial^2 L}{\partial x_8 \partial x_6}dx_8 \wedge dx_5 + \frac{\partial^2 L}{\partial x_8 \partial x_5}dx_8 \wedge dx_6 - \frac{\partial^2 L}{\partial x_8 \partial x_8}dx_8 \wedge dx_7 + \frac{\partial^2 L}{\partial x_8 \partial x_7}dx_8 \wedge dx_8$$

Calculate $\phi_L(\xi)$

$$i_X \phi_L = \phi_L(\xi) = (-\frac{\partial^2 L}{\partial x_1 \partial x_2}dx_1 \wedge dx_1 + \frac{\partial^2 L}{\partial x_1 \partial x_1}dx_1 \wedge dx_2 - \frac{\partial^2 L}{\partial x_1 \partial x_4}dx_1 \wedge dx_3 + \frac{\partial^2 L}{\partial x_1 \partial x_3}dx_1 \wedge dx_4 - \frac{\partial^2 L}{\partial x_1 \partial x_6}dx_1 \wedge dx_5 + \frac{\partial^2 L}{\partial x_1 \partial x_5}dx_1 \wedge dx_6 - \frac{\partial^2 L}{\partial x_1 \partial x_8}dx_1 \wedge dx_7 + \frac{\partial^2 L}{\partial x_1 \partial x_7}dx_1 \wedge dx_8 - \frac{\partial^2 L}{\partial x_2 \partial x_2}dx_2 \wedge dx_1 + \frac{\partial^2 L}{\partial x_2 \partial x_1}dx_2 \wedge dx_2 - \frac{\partial^2 L}{\partial x_2 \partial x_4}dx_2 \wedge dx_3 + \frac{\partial^2 L}{\partial x_2 \partial x_3}dx_2 \wedge dx_4 - \frac{\partial^2 L}{\partial x_2 \partial x_6}dx_2 \wedge dx_5 + \frac{\partial^2 L}{\partial x_2 \partial x_5}dx_2 \wedge dx_6 - \frac{\partial^2 L}{\partial x_1 \partial x_8}dx_2 \wedge dx_7 + \frac{\partial^2 L}{\partial x_2 \partial x_7}dx_2 \wedge dx_8 - \frac{\partial^2 L}{\partial x_3 \partial x_2}dx_3 \wedge dx_1 + \frac{\partial^2 L}{\partial x_3 \partial x_1}dx_3 \wedge dx_2 - \frac{\partial^2 L}{\partial x_3 \partial x_4}dx_3 \wedge dx_3 + \frac{\partial^2 L}{\partial x_3 \partial x_3}dx_3 \wedge dx_4 - \frac{\partial^2 L}{\partial x_3 \partial x_6}dx_3 \wedge dx_5 + \frac{\partial^2 L}{\partial x_3 \partial x_5}dx_3 \wedge dx_6 - \frac{\partial^2 L}{\partial x_3 \partial x_8}dx_3 \wedge dx_7 +$$

$\frac{\partial^2 L}{\partial x_3 \partial x_7} dx_3 \wedge dx_8 \quad - \frac{\partial^2 L}{\partial x_4 \partial x_2} dx_4 \wedge dx_1 + \frac{\partial^2 L}{\partial x_4 \partial x_1} dx_4 \wedge dx_2 - \frac{\partial^2 L}{\partial x_4 \partial x_4} dx_4 \wedge dx_3 + \frac{\partial^2 L}{\partial x_4 \partial x_3} dx_4 \wedge dx_4 - \frac{\partial^2 L}{\partial x_4 \partial x_6} dx_4 \wedge dx_5 + \frac{\partial^2 L}{\partial x_4 \partial x_5} dx_4 \wedge dx_6 - \frac{\partial^2 L}{\partial x_4 \partial x_8} dx_4 \wedge dx_7 + \frac{\partial^2 L}{\partial x_4 \partial x_7} dx_4 \wedge dx_8 - \frac{\partial^2 L}{\partial x_5 \partial x_2} dx_5 \wedge dx_1 + \frac{\partial^2 L}{\partial x_5 \partial x_1} dx_5 \wedge dx_2 - \frac{\partial^2 L}{\partial x_5 \partial x_4} dx_5 \wedge dx_3 + \frac{\partial^2 L}{\partial x_5 \partial x_3} dx_5 \wedge dx_4 - \frac{\partial^2 L}{\partial x_5 \partial x_6} dx_5 \wedge dx_5 + \frac{\partial^2 L}{\partial x_5 \partial x_5} dx_5 \wedge dx_6 - \frac{\partial^2 L}{\partial x_5 \partial x_8} dx_5 \wedge dx_7 + \frac{\partial^2 L}{\partial x_5 \partial x_7} dx_5 \wedge dx_8 \quad - \frac{\partial^2 L}{\partial x_6 \partial x_2} dx_6 \wedge dx_1 + \frac{\partial^2 L}{\partial x_6 \partial x_1} dx_6 \wedge dx_2 - \frac{\partial^2 L}{\partial x_6 \partial x_4} dx_6 \wedge dx_3 + \frac{\partial^2 L}{\partial x_6 \partial x_3} dx_6 \wedge dx_4 - \frac{\partial^2 L}{\partial x_6 \partial x_6} dx_6 \wedge dx_5 + \frac{\partial^2 L}{\partial x_6 \partial x_5} dx_6 \wedge dx_6 - \frac{\partial^2 L}{\partial x_6 \partial x_8} dx_6 \wedge dx_7 + \frac{\partial^2 L}{\partial x_6 \partial x_7} dx_6 \wedge dx_8 - - \frac{\partial^2 L}{\partial x_7 \partial x_2} dx_7 \wedge dx_1 + \frac{\partial^2 L}{\partial x_7 \partial x_1} dx_7 \wedge dx_2 - \frac{\partial^2 L}{\partial x_7 \partial x_4} dx_7 \wedge dx_3 + \frac{\partial^2 L}{\partial x_7 \partial x_3} dx_7 \wedge dx_4 - \frac{\partial^2 L}{\partial x_7 \partial x_6} dx_7 \wedge dx_5 + \frac{\partial^2 L}{\partial x_7 \partial x_5} dx_7 \wedge dx_6 - \frac{\partial^2 L}{\partial x_7 \partial x_8} dx_7 \wedge dx_7 + \frac{\partial^2 L}{\partial x_7 \partial x_7} dx_7 \wedge dx_8 - \frac{\partial^2 L}{\partial x_8 \partial x_2} dx_8 \wedge dx_1 + \frac{\partial^2 L}{\partial x_8 \partial x_1} dx_8 \wedge dx_2 - \frac{\partial^2 L}{\partial x_8 \partial x_4} dx_8 \wedge dx_3 + \frac{\partial^2 L}{\partial x_8 \partial x_3} dx_8 \wedge dx_4 - \frac{\partial^2 L}{\partial x_8 \partial x_6} dx_8 \wedge dx_5 + \frac{\partial^2 L}{\partial x_8 \partial x_5} dx_8 \wedge dx_6 - \frac{\partial^2 L}{\partial x_8 \partial x_8} dx_8 \wedge dx_7 + \frac{\partial^2 L}{\partial x_8 \partial x_7} dx_8 \wedge dx_8)$

$$\left(X^1 \frac{\partial}{\partial x_1} + X^2 \frac{\partial}{\partial x_2} + X^3 \frac{\partial}{\partial x_3} + X^4 \frac{\partial}{\partial x_4} + X^5 \frac{\partial}{\partial x_5} + X^6 \frac{\partial}{\partial x_6} + X^7 \frac{\partial}{\partial x_7} + X^8 \frac{\partial}{\partial 8}\right) \tag{9}$$

From the energy equation we get

$$E_L = V(L) - L = X^1 \frac{\partial L}{\partial x_2} - X^2 \frac{\partial L}{\partial x_1} + X^3 \frac{\partial L}{\partial x_4} - X^4 \frac{\partial L}{\partial x_3} + X^5 \frac{\partial L}{\partial x_6} - X^6 \frac{\partial L}{\partial x_5} + X^7 \frac{\partial L}{\partial x_7} - X^8 \frac{\partial L}{\partial x_7} - L \qquad (10\,)$$

In the equation of the energy equation we obtain

$$dE_L = \left(\frac{\partial}{\partial x_2} dx_1 - \frac{\partial}{\partial x_1} dx_2 + \frac{\partial}{\partial x_4} dx_3 - \frac{\partial}{\partial x_3} dx_4 + \frac{\partial}{\partial x_6} dx_5 - \frac{\partial}{\partial x_5} dx_6 + \frac{\partial}{\partial x_8} dx_7 - \frac{\partial}{\partial x_7} dx_8\right)\Big(X^1 \frac{\partial L}{\partial x_2} - X^2 \frac{\partial L}{\partial x_1} + X^3 \frac{\partial L}{\partial x_4} - X^4 \frac{\partial L}{\partial x_3} + X^5 \frac{\partial L}{\partial x_6} - X^6 \frac{\partial L}{\partial x_5} + X^7 \frac{\partial L}{\partial x_8} - X^8 \frac{\partial L}{\partial x_7} - L\Big)$$

$$dE_L = X^1 \frac{\partial^2 L}{\partial x_1 \partial x_2} dx_1 + X^1 \frac{\partial^2 L}{\partial x_2 \partial x_2} dx_2 + X^1 \frac{\partial^2 L}{\partial x_3 \partial x_2} dx_3 + X^1 \frac{\partial^2 L}{\partial x_4 \partial x_2} dx_4 + X^1 \frac{\partial^2 L}{\partial x_5 \partial x_2} dx_5 + X^1 \frac{\partial^2 L}{\partial x_6 \partial x_2} dx_6 + X^1 \frac{\partial^2 L}{\partial x_7 \partial x_2} dx_7 + X^1 \frac{\partial^2 L}{\partial x_8 \partial x_2} dx_8$$

$$-X^2 \frac{\partial^2 L}{\partial x_1 \partial x_1} dx_1 - X^2 \frac{\partial^2 L}{\partial x_2 \partial x_1} dx_2 - X^2 \frac{\partial^2 L}{\partial x_3 \partial x_1} dx_3 - X^2 \frac{\partial^2 L}{\partial x_4 \partial x_1} dx_4 - X^2 \frac{\partial^2 L}{\partial x_5 \partial x_1} dx_5 - X^2 \frac{\partial^2 L}{\partial x_6 \partial x_1} dx_6 - X^2 \frac{\partial^2 L}{\partial x_7 \partial x_1} dx_7 - X^2 \frac{\partial^2 L}{\partial x_8 \partial x_1} dx_8$$

$$+X^3 \frac{\partial^2 L}{\partial x_1 \partial x_4} dx_1 + X^3 \frac{\partial^2 L}{\partial x_2 \partial x_4} dx_2 + X^3 \frac{\partial^2 L}{\partial x_3 \partial x_4} dx_3 + X^3 \frac{\partial^2 L}{\partial x_4 \partial x_4} dx_4 + X^3 \frac{\partial^2 L}{\partial x_5 \partial x_4} dx_5 + X^3 \frac{\partial^2 L}{\partial x_6 \partial x_4} dx_6 + X^3 \frac{\partial^2 L}{\partial x_7 \partial x_3} dx_7 + X^3 \frac{\partial^2 L}{\partial x_8 \partial x_3} dx_8$$

$$-X^4 \frac{\partial^2 L}{\partial x_1 \partial x_3} dx_1 - X^4 \frac{\partial^2 L}{\partial x_2 \partial x_3} dx_2 - X^4 \frac{\partial^2 L}{\partial x_3 \partial x_3} dx_3 - X^4 \frac{\partial^2 L}{\partial x_4 \partial x_3} dx_4 - X^4 \frac{\partial^2 L}{\partial x_5 \partial x_3} dx_5 - X^4 \frac{\partial^2 L}{\partial x_6 \partial x_3} dx_6 - X^4 \frac{\partial^2 L}{\partial x_7 \partial x_4} dx_7 - X^4 \frac{\partial^2 L}{\partial x_8 \partial x_4} dx_8$$

$$+X^5 \frac{\partial^2 L}{\partial x_1 \partial x_6} dx_1 + X^5 \frac{\partial^2 L}{\partial x_2 \partial x_6} dx_2 + X^5 \frac{\partial^2 L}{\partial x_3 \partial x_6} dx_3 + X^5 \frac{\partial^2 L}{\partial x_4 \partial x_6} dx_4 + X^5 \frac{\partial^2 L}{\partial x_5 \partial x_6} dx_5 + X^5 \frac{\partial^2 L}{\partial x_6 \partial x_6} dx_6 + X^5 \frac{\partial^2 L}{\partial x_7 \partial x_5} dx_7 + X^5 \frac{\partial^2 L}{\partial x_8 \partial x_5} dx_8$$

$$-X^6 \frac{\partial^2 L}{\partial x_1 \partial x_5} dx_1 - X^6 \frac{\partial^2 L}{\partial x_2 \partial x_5} dx_2 - X^6 \frac{\partial^2 L}{\partial x_3 \partial x_5} dx_3 - X^6 \frac{\partial^2 L}{\partial x_4 \partial x_5} dx_4 - X^6 \frac{\partial^2 L}{\partial x_5 \partial x_5} dx_5 - X^6 \frac{\partial^2 L}{\partial x_6 \partial x_5} dx_6 - X^6 \frac{\partial^2 L}{\partial x_7 \partial x_6} dx_7 - X^6 \frac{\partial^2 L}{\partial x_8 \partial x_6} dx_8$$

$$+X^7\frac{\partial^2 L}{\partial x_1\partial x_8}dx_1+X^7\frac{\partial^2 L}{\partial x_2\partial x_8}dx_2+X^7\frac{\partial^2 L}{\partial x_3\partial x_8}dx_3+X^7\frac{\partial^2 L}{\partial x_4\partial x_8}dx_4+X^7\frac{\partial^2 L}{\partial x_5\partial x_8}dx_7+X^7\frac{\partial^2 L}{\partial x_6\partial x_8}dx_6+X^7\frac{\partial^2 L}{\partial x_7\partial x_8}dx_7+X^7\frac{\partial^2 L}{\partial x_8\partial x_8}dx_8$$

$$-X^8\frac{\partial^2 L}{\partial x_1\partial x_7}dx_1-X^8\frac{\partial^2 L}{\partial x_2\partial x_7}dx_2-X^8\frac{\partial^2 L}{\partial x_3\partial x_7}dx_3-X^8\frac{\partial^2 L}{\partial x_4\partial x_7}dx_4-X^8\frac{\partial^2 L}{\partial x_5\partial x_7}dx_5-X^8\frac{\partial^2 L}{\partial x_6\partial x_7}dx_6-X^8\frac{\partial^2 L}{\partial x_7\partial x_7}dx_7-X^8\frac{\partial^2 L}{\partial x_8\partial x_7}dx_8$$

$$-\frac{\partial L}{\partial x_1}dx_1-\frac{\partial L}{\partial x_2}dx_2-\frac{\partial L}{\partial x_3}dx_3-\frac{\partial L}{\partial x_4}dx_4-\frac{\partial L}{\partial x_5}dx_5-\frac{\partial L}{\partial x_6}dx_6-\frac{\partial L}{\partial x_7}dx_7-\frac{\partial L}{\partial x_8}dx_8 \quad (11)$$

Equation of Equation (10) with Equation (11) we obtain

$i_X\Phi_L = dE_L$

$$-\left(X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}dx_5+X^8\frac{\partial}{\partial x_8}\right)\left(\frac{\partial L}{\partial x_2}\right)dx_1+\frac{\partial L}{\partial x_1}dx_1$$

$$\left(X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}dx_5+X^8\frac{\partial}{\partial x_8}\right)\left(\frac{\partial L}{\partial x_1}\right)dx_2+\frac{\partial L}{\partial x_2}dx_2$$

$$-\left(X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}dx_5+X^8\frac{\partial}{\partial x_8}\right)\left(\frac{\partial L}{\partial x_4}\right)dx_3+\frac{\partial L}{\partial x_3}dx_3$$

$$\left(X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}dx_5+X^8\frac{\partial}{\partial x_8}\right)\left(\frac{\partial L}{\partial x_3}\right)dx_4+\frac{\partial L}{\partial x_4}dx_4$$

$$-\left(X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}dx_5+X^8\frac{\partial}{\partial x_8}\right)\left(\frac{\partial L}{\partial x_6}\right)dx_5+\frac{\partial L}{\partial x_5}dx_5$$

$$\left(X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}dx_5+X^8\frac{\partial}{\partial x_8}\right)\left(\frac{\partial L}{\partial x_5}\right)dx_6+\frac{\partial L}{\partial x_6}dx_6$$

$$-\left(X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}dx_5+X^8\frac{\partial}{\partial x_8}\right)\left(\frac{\partial L}{\partial x_8}\right)dx_7+\frac{\partial L}{\partial x_7}dx_7$$

$$\left(X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}dx_5+X^8\frac{\partial}{\partial x_8}\right)\left(\frac{\partial L}{\partial x_7}\right)dx_8+\frac{\partial L}{\partial x_8}dx_8=0 \quad (12)$$

Be an integral curve .in local coordinates it is obtained that

Suppose that a curve

$\alpha: I \subset R \rightarrow T^*\mathcal{M} = R^{2n}$

is an integral curve of the Lagrangian vector field X_H, i.e.,

$$X_L(\alpha(t)) = \frac{d\alpha(t)}{dt} \quad , \quad t \in I.$$

In the local coordinates, if it is considered to be

$\alpha(t) = (x_1(t), x_2(t), x_2(t), x_4(t), x_5(t), x_6(t), x_7(t), x_8(t))$

we obtain

$$\frac{d\alpha(t)}{dt} = \frac{dx_1}{dt}\frac{\partial}{\partial x_1}+\frac{dx_2}{dt}\frac{\partial}{\partial x_2}+\frac{dx_3}{dt}\frac{\partial}{\partial x_3}+\frac{dx_4}{dt}\frac{\partial}{\partial x_4}+\frac{dx_5}{dt}\frac{\partial}{\partial x_5}+\frac{dx_6}{dt}\frac{\partial}{\partial x_6}+\frac{dx_7}{dt}\frac{\partial}{\partial x_7}+\frac{dx_8}{dt}\frac{\partial}{\partial x_8}$$

$$X^1\frac{\partial}{\partial x_1}+X^2\frac{\partial}{\partial x_2}+X^3\frac{\partial}{\partial x_3}+X^4\frac{\partial}{\partial x_4}+X^5\frac{\partial}{\partial x_5}dx_5+X^6\frac{\partial}{\partial x_6}+X^7\frac{\partial}{\partial x_7}+X^8\frac{\partial}{\partial x_8}=\frac{\partial}{\partial \mathrm{t}} \tag{13}$$

Taking the equation (12) = the equation (13)

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_2}\right)dx_1+\frac{\partial L}{\partial x_1}dx_1=0\rightarrow\quad -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_2}\right)+\frac{\partial L}{\partial x_1}=0$$

$$\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_1}\right)dx_2+\frac{\partial L}{\partial x_2}dx_2=0\quad\rightarrow\quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_1}\right)+\frac{\partial L}{\partial x_2}=0$$

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_4}\right)dx_3+\frac{\partial L}{\partial x_3}dx_3=0\rightarrow\quad -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_4}\right)+\frac{\partial L}{\partial x_3}=0$$

$$\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_3}\right)dx_4+\frac{\partial L}{\partial x_4}dx_4=0\quad\rightarrow\quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_3}\right)+\frac{\partial L}{\partial x_4}=0$$

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_6}\right)dx_5+\frac{\partial L}{\partial x_5}dx_5=0\rightarrow -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_6}\right)+\frac{\partial L}{\partial x_5}=0$$

$$\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_5}\right)dx_6+\frac{\partial L}{\partial x_6}dx_6=0\quad\rightarrow\quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_5}\right)+\frac{\partial L}{\partial x_6}=0$$

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_8}\right)dx_7+\frac{\partial L}{\partial x_7}dx_7=0\rightarrow -\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_8}\right)+\frac{\partial L}{\partial x_7}=0$$

$$\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_7}\right)dx_8+\frac{\partial L}{\partial x_8}dx_8=0\quad\rightarrow\quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_7}\right)+\frac{\partial L}{\partial x_8}=0$$

And

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_2}\right)+\frac{\partial L}{\partial x_1}=0\quad,\quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_1}\right)+\frac{\partial L}{\partial x_2}=0\ ,$$

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_4}\right)+\frac{\partial L}{\partial x_3}=0\quad,\quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_3}\right)+\frac{\partial L}{\partial x_4}=0$$

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_6}\right)+\frac{\partial L}{\partial x_5}=0\quad,\quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_5}\right)+\frac{\partial L}{\partial x_6}=0$$

$$-\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_8}\right)+\frac{\partial L}{\partial x_7}=0\quad,\quad \frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial L}{\partial x_7}\right)+\frac{\partial L}{\partial x_8}=0 \tag{14}$$

Hence the triple $(\mathcal{M}, \phi_L, \xi)$ is shown to be a Lagrangian mechanical system which are deduced by means of an almost real structure J and using of basis $\left\{\frac{\partial}{\partial x_i} : i = 1,2,3,4,5,6,7,8\right\}$ on the distributions $\mathcal{M}$

4. Conclusions

Thus, equations Lagrangian of equations (14). with four Almost Complex Structures.

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