International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 6, Issue 6, 2018, PP 15-22
ISSN No. (Print) 2347-307X & ISSN No. (Online) 2347-3142
DOI: http://dx.doi.org/10.20431/2347-3142.0606002
www.arcjournals.org

On the Norm of Basic Elementary Operator in a Tensor Product

Peter G. Muiruri[1], Denis N. King'ang'i[2], Sammy W. Musundi[3]

[1]*Department of Physical Sciences, Chuka University, Nyahururu, Kenya*

[2]*Department of Mathematics and Computer Science, University of Eldoret, Eldoret, Kenya*

[3]*Department of Physical Sciences, Chuka University, Chuka, Kenya*

Corresponding Author:** ***Peter G. Muiruri, *Department of Physical Sciences, Chuka University, Nyahururu, Kenya*

Abstract: *In this paper, we determine the norm of an elementary operator in a tensor product. More precisely, we investigate the bounds of the norm of a basic elementary operator in a tensor product. We employ the techniques of tensor products and finite rank operators to express the norm of an elementary operator in terms of its coefficient operators. We also show that the norm of a basic elementary operator on* $\mathcal{B}(H\otimes K)$ *is expressible in terms of the norms of basic elementary operators on* $\mathcal{B}(H)$ *and* $\mathcal{B}(K)$*.*

Keywords: *Basic Elementary Operator, Finite rank Operator and Tensor product.*

1. Introduction

Many researchers have studied the properties of elementary operators, including numerical ranges, spectrum, compactness and rank in great depth and many results obtained. The norm property has also been considered in large number of studies but remains an interesting area of research for many researchers. More specifically, deriving a formula to express the norm of an arbitrary elementary operator in terms of its coefficient operators remains a topic for research in operator theory. In this paper the properties of tensor products and finite rank operators are applied in determining the norm of a basic elementary operator in a tensor product.

Let H be a complex Hilbert space and $\mathcal{B}(H)$ be the set of bounded linear operators on H. We define the *elementary operator* $T_n: \mathcal{B}(H) \longrightarrow \mathcal{B}(H)$ by;

$T_n(X) = \sum_{i=1}^{n} A_i X B_i$, for all $X \in \mathcal{B}(H)$ where A_i, B_i are fixed elements of $\mathcal{B}(H)$.

When $n = 1$, then we obtain a *basic elementary operator* $T: \mathcal{B}(H) \longrightarrow \mathcal{B}(H)$, defined by;

$T(X) = AXB$, for all $X \in \mathcal{B}(H)$, where A, B are fixed elements of $\mathcal{B}(H)$.

We denote the basic elementary operator by $M_{A,B}$.

When $n = 2$, we obtain the elementary operator of length two, whereby;

$T_2(X) = A_1 X B_1 + A_2 X B_2$ for all $X \in \mathcal{B}(H)$, where A_i, B_i are fixed elements of $\mathcal{B}(H)$ for $i = 1,2$.

The Jordan Elementary operator $U_{A,B}: \mathcal{B}(H) \longrightarrow \mathcal{B}(H)$, defined by;

$U_{A,B}(X) = AXB + BXA$, for all $X \in \mathcal{B}(H)$ where A, B are fixed elements of $\mathcal{B}(H)$ is an example of an elementary operator.

A functional, f, is a mapping that maps vectors to real value, that is, $f: x \rightarrow \mathbb{R}$. A linear functional on a linear space L satisfies the following two properties:

i) Additive: $f(x + y) = f(x) + f(y)$

ii) Homogeneous: $f(\alpha x) = \alpha f(x)$

Now let $u: H \rightarrow \mathbb{R}$ be functional, for a complex Hilbert space H, with dual H^*. We define a *finite rank operator* $u \otimes x: H \longrightarrow H$ by $(u \otimes x)y = u(y)x, \forall\, y \in H$, where $u \in H^*$ and $x \in H$ is a unit vector, with the norm determined as follows:

$$\| u \otimes x \| = sup\{\| (u \otimes x)y \| : y \in H, \| y \| = 1\}$$

$$= sup\{\| u(y)x \| : y \in H, \| y \| = 1\}$$

$$= sup\{ |u(y)| \ \| x \| : y \in H, \| y \| = 1\}$$

$$= sup\{ |u(y)| : y \in H, \| y \| = 1\} = |u(y)|$$

In this paper, we use the above finite rank operator and properties of tensor products to determine the norm of a basic elementary operator in a tensor product. The approaches used by Okelo and Agure (2011) [11] and King'ang'i *et al* (2014)[12] are employed in obtaining our results.

2. The Norm of Elementary Operator

In this section, we highlight some results obtained in this area.

Let H be a complex Hilbert space, $\mathcal{B}(H)$ be the algebra of bounded linear operators on H, and $A, B \in \mathcal{B}(H)$ be fixed. For a Jordan elementary operator $U_{A,B}$, Mathieu [1], in 1990 proved that in the case of a prime $C^* - algebra$, the lower bound of the norm of $U_{A,B}$ can be estimated by:

$$\| U_{A,B} \| \geq \frac{2}{3} \| A \| \| B \|.$$

Also, Cabrera and Rodriguez [2] in 1994 proved that:

$$\| U_{A,B} \| \geq \frac{1}{20412} \| A \| \| B \|.$$

On their part, Stacho and Zalar [3] in 1996, worked on the standard operator algebra (which is sub-algebra of $\mathcal{B}(H)$ that contains all finite rank operators). They first showed that $U_{A,B}$ actually represents a Jordan triple structure of a $C^* - algebra$. They also showed that if T a standard Operator algebra is acting on Hilbert space H and $A, B \in T$ then:

$$\| U_{A,B} \| = 2(\sqrt{2} - 1) \| A \| \| B \|.$$

They later in 1998, [4] proved that: $\| U_{A,B} \| \geq \| A \| \| B \|$ for the algebra of symmetric operators acting on Hilbert space. They generally attached a family of Hilbert spaces to standard operator algebra and used inner product in them to obtain their results.

In the year 2001, Barraa and Boumazgour [5] used the concept of numerical range of A relative to B, denoted by $W_B(A^*B)$, to obtain their results. They employed the idea of finite rank operators to prove the following theorem;

Theorem 2.1

Let H be a complex Hilbert space, $B(H)$ be the algebra of bounded linear operators on H. If $A, B \in \mathcal{B}(H)$ with $B \neq 0$ then:

$$\| U_{A,B} \| \geq sup_{\lambda \in W_B(A^*B)} \left\{ \| \| B \| A + \frac{\lambda}{\|B\|} B \| \right\}.$$

The proof of the above Theorem can be obtained from Barraa and Boumazgour [5] Theorem 2.1.

As a consequence of this they proved the following Corollary and Proposition;

Corollary 2.2

Let H be a complex Hilbert space, $\mathcal{B}(H)$ be the algebra of bounded linear operators on H. If $A, B \in \mathcal{B}(H)$ and $0 \in W_B(A^*B) \cup W_A(B^*A)$, then:

$$\| U_{A,B} \| \geq \| A \| \| B \|.$$

Proposition 2.3

Let H be a complex Hilbert space, $B(H)$ be the algebra of bounded linear operators on H . If

$A, B \in \mathcal{B}(H)$ and $\| A \| \| B \| \in W_B(A^*B) \cap W_A(B^*A)$, then:

$\| U_{A,B} \| = 2 \| A \| \| B \|$.

The proofs of the above corollary and proposition can also be found in Barraa and Boumazgour [5] Corollary 2.1 and Proposition 2.3.

Further, in 2003, Timoney [6] [7] and in 2004, Blanco et al [8] arrived to the result that:

$\| U_{A,B} \| = \| A \| \| B \|$.

On his part, Boumazgour [9] in 2008 obtained the norm inequality for sum of two basic elementary operators. He proved the following Theorem:

Theorem 2.4

If A, B, C and D are operators on $\mathcal{B}(H)$, then:

$$\| M_{A,B} + M_{C,D} \| \leq [(Max \| B \|^2, \| D \|^2 + \| BD^* \|)(Max \| A \|^2, \| C \|^2 + \| AC^* \|)]^{\frac{1}{2}}.$$

Later, in 2011 Okelo [10] used the numerical range and finite rank operators for the case of norm-attained operators on $\mathcal{B}(H)$ and showed that:

$\| M_{A,B} \| = \| A \| \| B \|$.

In 2011, Okelo and Agure [11] also used the concept of finite rank operators to determine the norm of elementary operator and arrived at a following result;

Lemma 2.5

Let H be a complex Hilbert space, $\mathcal{B}(H)$ be the algebra of bounded linear operators on H. If

$M_{A,B}: \mathcal{B}(H) \longrightarrow \mathcal{B}(H)$is defined by $M_{A,B}(X) = AXB$ where $A, B \in \mathcal{B}(H)$ then:

$\| M_{A,B} \| = \| A \| \| B \| \ \forall X \in \mathcal{B}(H)$, with $\| X \| = 1$, and $X(x) = x$ for all $x \in H$.

The proof to this Lemma can be obtained from Okelo and Agure [11] Lemma 4.2.

In 2014 King'ang'i *et al* [12] used finite rank operator to determine the norm of the elementary operatorT_2. Below is the result that they proved:

Theorem 2.6

Let H be a complex Hilbert space, $\mathcal{B}(H)$ be the algebra of bounded linear operators on H. Let T_2 be the elementary operator on $\mathcal{B}(H)$ of length two. If for an operator $X \in \mathcal{B}(H)$ with $\| X \| = 1$, we have $X(x) = x$ for all $x \in H$ then $\| T_2 \| = \sum_{i=1}^{2} \| A_i \| \| B_i \| \forall A_i, B_i$ fixed in ,$\mathcal{B}(H)$ and $i = 1,2$.

For the proof see King'ang'i *et al* [12] theorem 2.5

This work and that of Okelo and Agure [11] forms basis of the results in this paper.

King'ang'i [13] in 2017 also employed the concept of the maximal numerical range of A^*B relative to B to determine the lower norm of an elementary operator length two. He proved the following Theorem:

Theorem 2.7

Let T_2 be an elementary operator of length two on $\mathcal{B}(H)$. Then:

$$\| T_2 \| \geq sup_{\lambda \in W_{B_1}(B_2^* B_1)} \left\{ \| \| B_1 \| A_1 + \frac{\bar{\lambda}}{\| B_1 \|} A_2 \| \right\}$$

Where $\forall A_i, B_i$ fixed in $\mathcal{B}(H)$ and $i = 1,2$.

He also determined the conditions on which the norm of T_2 is expressible in terms of the norm of its coefficient operators by proving the following Corollary and theorem:

Corollary 2.8

Let H be a complex Hilbert space and A_i, B_i be bounded linear operators on $H$for $i = 1,2$. Let $0 \in W_{B_1}(B_2{}^*B_1) \cup W_{B_1}(B_2{}^*B_1)$ then $\| T_2 \| \geq \| A_1 \| \| B_1 \|$ where T_2 is as defined earlier.

Theorem 2.9

Let H be a complex Hilbert space and A_i, B_i be bounded linear operators on H for $i = 1,2$. If

$\| A_1 \| \| A_2 \| \in W_{A_1^*}(A_2 A_1^*)and\| B_1 \| \| B_2 \| \in W_{B_2}(B_2 B_1^*)$ then:

$\| T_2 \| = \sum_{i=1}^{2} \| A_i \| \| B_i \|$.

3. Elementary Operators in a Tensor Product

Before embarking on our main result, we introduce tensor products and define elementary operators in a tensor product.

Let $H = \{x_1, x_2, \ldots\}$ and $K = \{y_1, y_2, \ldots\}$ be Hilbert spaces with inner/scalar products $\langle x_1, x_2 \rangle$ and $\langle y_1, y_2 \rangle$ respectively. A ***tensor product*** of H and K is a Hilbert space $H \otimes K$ where $\otimes : H \times K \longrightarrow H \otimes K, (x, y) \mapsto x \otimes y$ is a bilinear mapping such that;

i) The vectors $x \otimes y$ form a total subset of $H \otimes K$, that is, the closed linear span of the set of all the vectors $x \otimes y$ is $H \otimes K$. *NB*: A subset T of a topological space V is a ***total/fundamental*** set if the linear span of T is dense in V.

ii) $\langle x_1 \otimes y_1, x_2 \otimes y_2 \rangle = \langle x_1, x_2 \rangle \langle y_1, y_2 \rangle, \forall x_1, x_2 \in H, y_1, y_2 \in K$. This implies that

$\|x \otimes y\| = \|x\| \|y\|$for all $x \in H,\ y \in K$.

If $A \in \mathcal{B}(H), B \in \mathcal{B}(K)$, then $\mathcal{B}(H \otimes K)$ is a Hilbert space and for $A \otimes B \in \mathcal{B}(H \otimes K)$, we have

$A \otimes B(x \otimes y) = Ax \otimes By$ for all $x \in H, y \in K$.

The following are properties of members of $\mathcal{B}(H \otimes K)$:

i) $(A \otimes B)(C \otimes D) = AC \otimes BD$.

ii) $\| A \otimes B \| = \| A \| \| B \|$ for all $A, C \in \mathcal{B}(H), B, D \in \mathcal{B}(K)$.

Let$H$ and Kbe complex Hilbert spaces, $B(H \otimes K)$be the set of bounded linear operators on $H \otimes K$ and $A \otimes B$, $C \otimes D$ being fixed elements of, $\mathcal{B}(H \otimes K)$, where $A, C \in \mathcal{B}(H)$, the set of bounded linear operators on H, and $B, D \in \mathcal{B}(K)$, the set of bounded linear operators on K;

Then the *elementary operator* $T_n : \mathcal{B}(H \otimes K) \longrightarrow \mathcal{B}(H \otimes K)$is defined as:

$T_n(X \otimes Y) = \sum_{i=1}^{n} (A_i \otimes B_i)(X \otimes Y)(C_i \otimes D_i)$, for all $X \otimes Y \in \mathcal{B}(H \otimes K)$and $A_i \otimes B_i, C_i \otimes D_i$ being fixed elements of $\mathcal{B}(H \otimes K)$.

When $n = 1$, then we obtain a *basic elementary operator* $T : \mathcal{B}(H \otimes K) \longrightarrow \mathcal{B}(H \otimes K)$,

$T(X \otimes Y) = (A \otimes B)(X \otimes Y)(C \otimes D)$, for all $X \otimes Y \in \mathcal{B}(H \otimes K)$ and $A \otimes B, C \otimes D$ are fixed elements of $\mathcal{B}(H \otimes K)$.

We denote the basic elementary operator by $M_{A \otimes B, C \otimes D}$, where $A, C \in \mathcal{B}(H)$and $B, D \in \mathcal{B}(K)$.

When$n = 2$, we obtain the *elementary operator of length two*, where by:

$T_2(X \otimes Y) = (A_1 \otimes B_1)(X \otimes Y)(C_1 \otimes D_1) + (A_2 \otimes B_2)(X \otimes Y)(C_2 \otimes D_2)$ for all $X \otimes Y \in \mathcal{B}(H \otimes K)$, where $A_i \otimes B_i, C_i \otimes D_i$ are fixed elements of $\mathcal{B}(H \otimes K)$ for $i = 1,2$.

The *Jordan elementary operator* $U_{A\otimes B, C\otimes D}: \mathcal{B}(H \otimes K) \longrightarrow \mathcal{B}(H \otimes K)$, is defined as;

$U_{A\otimes B, C\otimes D}(X \otimes Y) = (A \otimes B)(X \otimes Y)(C \otimes D) + (C \otimes D)(X \otimes Y)(A \otimes B)$ for all $X \in \mathcal{B}(H)$ and $A \otimes B, C \otimes D$ are fixed elements of $\mathcal{B}(H \otimes K)$, where $A, C \in \mathcal{B}(H)$ and $B, D \in \mathcal{B}(K)$.

4. Norm of Elementary Operator in Tensor Product

As our main result, we investigate the bounds of the norm of basic elementary operator in tensor product defined in section three. We express the norm of this operator in terms of the operators on H and K. Finally, we show that the norm of the basic elementary operator on $\mathcal{B}(H \otimes K)$ can be expressed in terms of norms of the basic elementary operators on $\mathcal{B}(H)$ and $\mathcal{B}(K)$.

Theorem 4.1

Let H and K be complex Hilbert spaces and $\mathcal{B}(H \otimes K)$ be the set of bounded linear operators on $H \otimes K$. Then for all, $X \otimes Y \in \mathcal{B}(H \otimes K)$ with $\| X \otimes Y \| = 1$, we have

$\| M_{A\otimes B, C\otimes D} \| = \| A \| \| B \| \| C \| \| D \|$, where A, B and C, D are fixed elements in $\mathcal{B}(H)$ and $\mathcal{B}(K)$ respectively.

Proof

By Definition

$\| M_{A\otimes B, C\otimes D} \backslash \mathcal{B}(H \otimes K) \| = \sup \{\| M_{A\otimes B, C\otimes D}(X \otimes Y) \| : X \otimes Y \in \mathcal{B}(H \otimes K), \| X \otimes Y \| = 1\}$

$\| M_{A\otimes B, C\otimes D} \backslash \mathcal{B}(H \otimes K) \| \geq \| M_{A\otimes B, C\otimes D}(X \otimes Y) \| : X \otimes Y \in \mathcal{B}(H \otimes K), \| X \otimes Y \| = 1$

Then we have, $\forall \varepsilon \geq 0$

$\| M_{A\otimes B, C\otimes D} \backslash \mathcal{B}(H \otimes K) \| - \varepsilon < \| M_{A\otimes B, C\otimes D}(X \otimes Y) \| \; X \otimes Y \in \mathcal{B}(H \otimes K), \| X \otimes Y \| = 1$

That is;

$\| M_{A\otimes B, C\otimes D} \backslash \mathcal{B}(H \otimes K) \| - \varepsilon < \| (A \otimes B)(X \otimes Y)(C \otimes D) \| = \| (A \otimes B)(XC \otimes YD) \| = \| AXC \otimes BYD \| = \| AXC \| \| BYD \|$

Since $\varepsilon \geq 0$ was arbitrarily taken then $\| M_{A\otimes B, C\otimes D} \backslash \mathcal{B}(H \otimes K) \| \leq \| AXC \| \| BYD \|$

Since $\| X \| = 1$, then; $\| AXC \| \leq \| A \| \| X \| \| C \| = \| A \| \| C \|$

Thus; $\| AXC \| \leq \| A \| \| C \|$ (1)

Also, since $\| Y \| = 1$, then we have; $\| BYD \| \leq \| B \| \| Y \| \| D \| = \| B \| \| D \|$ (2)

Thus from (1) and (2) then:

$\| M_{A\otimes B, C\otimes D} \backslash \mathcal{B}(H \otimes K) \| \leq \| A \| \| B \| \| C \| \| D \|$ (3)

Conversely, let there exist a sequence $\{e_n \otimes f_n\}$ of unit vectors in $H \otimes K$ for each $e_n \in H$ and $f_n \in K$ then for each $n \geq 1$, we have;

$\| M_{A\otimes B, C\otimes D}(X \otimes Y)(e_n \otimes f_n) \| \leq \| M_{A\otimes B, C\otimes D}(X \otimes Y) \| \| (e_n \otimes f_n) \| \leq \| M_{A\otimes B, C\otimes D} \| \| (X \otimes Y) \| \| (e_n \otimes f_n) \|$

But; $\| M_{A\otimes B, C\otimes D} \| \| (X \otimes Y) \| \| (e_n \otimes f_n) \| = \| M_{A\otimes B, C\otimes D} \| \| X \| \| Y \| \| e_n \| \| f_n \|$

Since $\| X \| = 1, \| Y \| = 1, \| e_n \| = 1$ and $\| f_n \| = 1$ then:

$\| M_{A\otimes B, C\otimes D} \| \geq \| M_{A\otimes B, C\otimes D}(X \otimes Y)(e_n \otimes f_n) \| = \| \{A \otimes B(X \otimes Y) C \otimes D\}(e_n \otimes f_n) \|$

But: $\| \{A\otimes B(X \otimes Y)C\otimes D\}(e_n\otimes f_n) \| = \| A\otimes B(X \otimes Y)Ce_n\otimes Df_n \| = \| A\otimes B(XCe_n\otimes YDf_n) \|$

Also: $\| A\otimes B(XCe_n\otimes YDf_n) \| = \| AXCe_n\otimes BYDf_n \| = \| AXCe_n \| \| BYDf_n \|$

We obtain that:

$$\| M_{A\otimes B,C\otimes D} \| \geq \| AXCe_n \| \| BYDf_n \| \quad (4)$$

Now let, $u, v: H \longrightarrow \mathbb{R}^+$ be functionals

From (4), choosing unit vectors x_1, x_2 and define the finite rank operators $A = u\otimes x_1$, $x_1 \in H \, \| x_1 \| = 1$ by $Ae_n = (u\otimes x_1)e_n = u(e_n)x_1$and $C = v\otimes x_2$, $x_2 \in H \, \| x_2 \| = 1$ by $Ce_n = (v\otimes x_2)e_n = v(e_n)x_2$.

We observe that the norm of A is:

$\| A \| = \sup \{\| (u\otimes x_1)e_n \| : e_n \in H, \| e_n \| = 1\}$

$= \sup \{\| u(e_n)x_1 \| : e_n \in H, \| e_n \| = 1\}$

$= \sup \{|u(e_n)| \| x_1 \| : e_n \in H, \| e_n \| = 1\}$

$= \sup\{|u(e_n)| : e_n \in H, \| e_n \| = 1\} = |u(e_n)|$

This is $\| A \| = |u(e_n)|$ for any unit vectors $e_n \in H$

Likewise, the norm of C is $\| C \| = |v(e_n)|$ for any unit vectors $e_n \in H$

Therefore from (1) we have $\| AXCe_n \| = \| (u\otimes x_1)X(v\otimes x_2)e_n \|$

By the definition of Finite rank operator $\| AXCe_n \| = \| (u\otimes x_1)X(v\otimes x_2)e_n \| = \| (u\otimes x_1)Xv(e_n)x_2 \|$

$= \| (u\otimes x_1)v(e_n)Xx_2 \| = |v(e_n)| \, \| (u\otimes x_1)X(x_2) \|$

$= |v(e_n)| \, \| (u\otimes x_1)X(x_2) \| = |v(e_n)| \, \| u(X(x_2))x_1 \|$

$= |v(e_n)| \, |u(X(x_2))| \, \| x_1 \| = \| A \| \| C \|$

Thus; $\| AXCe_n \| = \| A \| \| C \|$

Following the same steps then: $\| BYDf_n \| = \| B \| \| D \|$

Thus, it is clear that:

$\| M_{A\otimes B,C\otimes D} \| \geq \| AXCe_n \| \| BYDf_n \| = \| A \| \| C \| \| B \| \| D \|$

This shows that:

$$\| M_{A\otimes B,C\otimes D} \| \geq \| A \| \| B \| \| C \| \| D \| \quad (5)$$

From (3) and (5) we obtain that:

$\| M_{A\otimes B,C\otimes D} \| = \| A \| \| B \| \| C \| \| D \|$ ∎

Corollary 4.2

Let H and K be complex Hilbert spaces and $\mathcal{B}(H \otimes K)$ be the set of bounded linear operators on $H \otimes K$. If for all, $X \otimes Y \in \mathcal{B}(H \otimes K)$ with $\| X \otimes Y \| = 1$, then we have $\| M_{A\otimes B,C\otimes D} \| = \| M_{A,C} \| \| M_{B,D} \|$, where $M_{A,C}$ and $M_{B,D}$are basic elementary operators on $\mathcal{B}(H)$ and $\mathcal{B}(K)$ respectively (As defined in section 1).

Proof

The proof this corollary follows directly from theorem 4.1.

Recall that from Okelo and Agure (2011) lemma 4.2; $\|M_{A,C}\| = \| A \| \| C \|$, while $\|M_{B,D}\| = \| B \| \| D \|$.

Now, from theorem 4.1, we have: $\| M_{A\otimes B,C\otimes D} \| = \| A \| \| B \| \| C \| \| D \|$.

We can rearrange this as: $\| M_{A\otimes B,C\otimes D} \| = \| A \| \| C \| \| B \| \| D \|$.

Notice that $A, C \in \mathcal{B}(H)$, while $B, D \in \mathcal{B}(K)$.

Then substituting, we obtain;

$\| M_{A\otimes B,C\otimes D} \| = \| M_{A,C} \| \| M_{B,D} \|$ ■

5. Conclusion

In this paper, we have investigated in theorem 4.1 the bounds of the norm of a basic elementary operator in a tensor product defined in section 3 and expressed the norm of this operator in terms of the operators on H and K .Finally, in Corollary 4.2, we have showed that the norm of a basic elementary operator on $\mathcal{B}(H\otimes K)$ can be expressed in terms of norms of the basic elementary operators on $\mathcal{B}(H)$ and $\mathcal{B}(K)$.

Acknowledgements

Special thanks to Chuka University for providing the required environment to carry out this study and all people who have given us valuable ideas and guidance towards coming up with this paper.

References

[1] Mathieu, M., More Properties of the Product of two Derivations of a C*-Algebra. Bull. Austral. Math. Soc.42, 115-120(1990).

[2] Cabrera, M., and Rodriquez, A., Non-degenerately ultra-prime Jordan Banach Algebras. Proc. London Math.Soc . 69, 576-604 (1994).

[3] Stacho, L., andZalar, B., On the Norm of Jordan Elementary Operator in Standard Operator Algebras. Publ. Math. Debrecen. 49, 127-134. (1996).

[4] Stacho, L., andZalar, B., Uniform primeness of the Jordan Algebra of Symmetric Operators. Proc. Amer.Math.Soc.126, 2241-2247(1998).

[5] Barraa, M., and Boumazgour, M., A Lower bound of the Norm of Operator $\rightarrow \mathrm{AXB}+\mathrm{BXA}$. Extracta Math.16, 223-227 (2001).

[6] Timoney, R., Norm and CB norms of Jordan Elementary Operators. School of math. Trimity college, Dublin2, Ireland, 597-603(2003).

[7] Timoney, R., Computing the Norm of Elementary Operators. Illinois, J.Math .47, 1207-1226 (2003).

[8] Blanco, A., Boumazgour, M., and Ransford, T., On Norm of Elementary Operator. J.London Math,Soc. 70,479-498 (2004).

[9] Boumazgour, M., Norm inequalities for sum of two Basic Elementary operators. j.math.appl.342, 386-393 (2008).

[10] Okelo, N., On Dvoretsky's theorem and Norm of Elementary Operators. Int. J. Pure Appl.Sci.Technol. 2(2), 46-53(2011)

[11] Okelo, N., and Agure , J., (2011). A two sided Multiplication Operator Norm. General mathematics Notes. 2, 18-23 (2011).

[12] Kingangi, D., Agure, J., and Nyamwala, F., On the Norm of Elementary Operator. Advance in pure mathematics.4,309-316(2014)

[13] Kingangi. D., On the norm of elementary operator of length two. International Journal of Science and innovation Mathematical Research. 5, 34-38 (2017)

AUTHORS' BIOGRAPHY

Mr. Peter Guchu Muiruria, is an MSc student pursuing pure mathematics at Chuka University. He is currently a tutor in Mathematics and Chemistry at Simbara Secondary School, Nyahururu-kenya

Dr. Denis Njue King'ang'I, holds a PhD in pure mathematics. He is currently lecturing at the University of Eldoret, Kenya. His research interests are in Operator Theory and Graph Theory

Citation: *Muiruri, P. (2018). On the Norm of Basic Elementary Operator in a Tensor Product. International Journal of Scientific and Innovative Mathematical Research (IJSIMR), 6(6), pp.15-22. http://dx.doi.org/10.20431/2347-3142.0606002*