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Effect of Thermal Radiation, Joule Heating, Heat Sources on Hydromagnetic Flow of Micropolar Fluid past a Stretching Surface with Convective Boundary Conditions

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Abstract: *In this paper, an attempt has been made to investigate the influence of thermal radiation, dissipation, heat sources and convective boundary condition on the flow of a micropolar fluid past a stretching sheet. The non-linear coupled equations have been solved numerically. The effect of various parameters on the flow characteristics are depicted through graphs and tables.*

Keywords: *Thermal radiation, Heat Sources, Hydromagnetic, Stretching Surface, Convective Boundary conditions*

1. INTRODUCTION

Coupled heat and mass transfer by mixed convection in a micropolar fluid saturated porous medium due to a stretching surface has received great attention during the last decades because of numerous applications in geophysics and energy related engineering problems that includes both metal and polymer sheets. Sakiadis[25]first investigated flow in the boundary layer . on moving solid surface ,who found that the boundary layer is in the direction of motion of the continuous solid surface and deviates from the classical Blassius flow past a flat plate. Later, Grubka and Bobba [11] have investigated the stretching sheet problems. Mixing and recirculation of local fluid moves through paths in packed beds. This hydrodynamic mixing of fluid at pore level influences thermal and solutal dispersion in the porous medium. More discussion and applications of convective transport in porous media can be found in the book by Nield and Bejan [22] and Ingham and Pop[15] have reported similarity solutions for mixed convection flow over horizontal and inclined plated embeddede in fluid saturated porous media in the presence of surface mass flux. On the other hand, Mikowycz et al[21] have discuss the effect of surface mass transfer on buoyancy-induced Darcian flow adjacent to a horizontal surface using non-similarity solutions. Hopper et al[13] have considered the problem of non-similar mixed convection flow along an isothermal vertical plate in porous media with uniform surface suction or injection and a introduced a single parameter for the entire regime of free –forced –mixed convection. Wang and Chen[32]presented a numerical set and modelling results of three dimensional Electro osmotic flows[EOPs]in homogeneously charged micro and nano scale random porous media. Wang et al[33] analysed numerical modelling results of the EOP through charger anisotropic porous media using the lattice Poisson-Boltzmann method. The above mentioned applications may be treated well with micropolar fluid under can be described through new material constants in additions to those of a classical Newtonian fluid. Erigen's micropolar model includes the classical Navier -Stokes equations for a viscous and incompressible fluid as special case. Extensive reviews of the theory and applications of micropolar fluids were provided by Ariman et al [4].These micropolar fluids are suitable in modelling the body fluids and cerebro-spinal fluid. Hoyt and Fabulla [12] have shown experimentally that fluids containing minute polymeric additives exhibits a considerable reduction in the skin friction and hence this concept can be well explained by the theory of micropolar fluids. Thus study of heat and mass transfer considering micropolar fluid is of special interest because of the cooling of the fibre in the formation processes[9]. The effect of radiation on heat and mass transfer flow have been discussed by several authors [7,8,10]

In industrial and chemical engineering processes in which heat and mass transfer is a consequence of buoyancy effects caused by diffusion and concentrations vary from point to point resulting in mass transfer. The Dufour effect was found to be of order of considerable magnitude such that it cannot be neglected[12].Soret and Dufour effects have been found to appreciably influence the flow field in mixed convection boundary layer over a vertical surface embedded in a porous medium. Bourich et al[5] studied analytically and numerically the Soret effect on the onset of convection in a vertical porous layer subject to uniform heat flux.Alam and Rahman[1] investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Postelnicu [23] analyse the influence of chemical reaction on heat and mass transfer by natural convection from vertical surface in porous media in the presence of Soret band Dufour effects. Tsai and Huang [30] focussed mainly on the heat and mass transfer under a chemical reaction. Heat source4, radiation, stretching surface and variable viscosity coupled with the Soret and Dufour effects which occur in a porous medium for the Hiemenz flow. El-Arabawy [10] investigated the heat and mass transfer by natural convection from vertical surface embedded in a fluid saturated porous media considering Sore tans Dufour effects with variable surface temperature and constant concentration. Reddy and Reddy[24]examined the Soret and Dufour effects on steady MHD free convection boundary layer flow past a semi-infinite moving vertical plate embedded in a porous medium by taking viscous dissipation into account. Dulal Pal and Sewli Chatterjee [7] have discussed mixed convection magnetohydrodynamic heat and mass transfer past a stretching surface in a micropolar fluid saturated porous medium under the influence of Ohmic heating ,Soret and Dufour effects.,

In all the studies cited above, the flow is driven either by prescribed surface temperature or by a prescribed surface heat flux. Here, a relatively different driving mechanism for unsteady free convection along a vertical surface is considered where it is assumed that the flow is also start up by Newtonian heating from the surface. Keeping this in view several authors [2,3,6,14,16-20,26,28,29] have been investigated the effect Newtonian heating on convection flow in different configurations. Ramzan et al [27]have discussed radiative and Joule heating effects in the MHD flow of a micropolar fluid with partial slip and convective boundary condition.

In this paper we investigate the effect of non-uniform heat source on convective heat and mass transfer flow of a micro polar fluid past stretching sheet with convective boundary condition. The governing equation have been solved by employing finite element technique.

2. Formulation of the Problem

We analyse an incompressible two-dimensional MHD micropolar's fluid flow over a permeable stretching surface with slip velocity. The entire fluid system is under the influence of a uniform magnetic field. Thermal radiation and Joule heating effects due to magnetic and electric fields are also encountered (see fig.1).The equations governing the flow, heat and mass transfer under Boussinesque approximation and Roseland approximation are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(\frac{\mu+k}{\rho})\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial \omega}{\partial y}+\frac{\sigma}{\rho}(E_0B_0-B_0^2u)+G(\theta+N\phi) \quad (2)$$

$$u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=(\frac{\gamma}{\rho j})\frac{\partial^2 \omega}{\partial y^2}-\frac{k}{\rho j}(2N+\frac{\partial u}{\partial y}) \quad (3)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=k_f\frac{\partial^2 T}{\partial y^2}+\left(\frac{ku_s}{x\nu}\right)\left[A1(T_1-T_1)f'(\eta)+B1(T-T_2)\right]+\frac{16\sigma^* T_\infty^3}{3\beta_R}\frac{\partial^2 T}{\partial y^2} \quad (4)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=(D_B)\frac{\partial^2 C}{\partial y^2}-k_c(C-C_\infty) \quad (5)$$

The boundary conditions are

$$\left.\begin{array}{l} u = ax + \alpha((\mu+k)\dfrac{\partial u}{\partial y} + k\omega), v = -v_0, \omega = -s\dfrac{\partial u}{\partial y}, \\ -k_f\dfrac{\partial T}{\partial y} = h_f(T_f - T), -D_B\dfrac{\partial C}{\partial y} = h_c(C_f - C) \qquad on\ y = 0 \end{array}\right| \tag{6}$$

$$u \to 0, \omega \to 0, T \to T_\infty, C \to C_\infty \qquad as\ y \to \infty \tag{7}$$

Where u and v are the velocity components along x and y directions,

$\mu, \omega, k, \rho, j, \gamma, \sigma, Bo, Eo, k_f, C_p, D_B, v_w, \sigma^{\bullet}, \beta_R, q_R, h_f, h_c, q''', T, T_f,$

C, C_f, C_w, C_∞ and T_∞ are the viscosity, micro-rotation or angular velocity, vortex viscosity, density, microinertia density, spin gradient viscosity, electrical conductivity, applied magnetic field strength, applied electric field, thermal conductivity, specific heat at constant pressure, molecular diffusivity, slip coefficient, suction /injection velocity, Stefan-Boltzmann constant, mean absorption coefficient, radiative heat transfer, heat transfer coefficient, mass transfer coefficient strength of the non-uniform heat source, fluid and ambient temperature, fluid and ambient concentration respectively, wall concentration, fluid and ambient concentration respectively.

A linear relationship between the nicro-rotation function ω and the surface stress ($\dfrac{\partial u}{\partial y}$) is chosen for investigating the effect to different surface conditions for the microrotation. Here S is the boundary parameter and varies from 0 to 1.The first boundary condition (S=0) is a generalisation of the no slip condition, which requires that the fluid particles close to a solid boundary stick to it neither translating nor rotating. The second boundary condition i.e., micro-rotation is equal to the fluid vortices at the boundary(S#0) means that in the neighbourhood of the boundary, the only rotation is due to fluid shear and therefore, the gyration vector must be equal to fluid vortices.

The coefficient q''' is the rate of internal heat generation (>0) or absorption(<0). The internal heat generation /absorption q''' is modelled as

$$q''' = \left(\frac{ku_s}{x\nu}\right)\left[A1\left(T_1 - T_1\right) f'(\eta) + B1\left(T - T_2\right)\right] \tag{8}$$

Where A1 and B1 are coefficients of space dependent and temperature dependent internal heat generation or absorption respectively. It is noted that the case A1>0 and B1>0, corresponds to internal heat generation and that A1<0 and B1<0, the case corresponds to internal heat absorption case.

On setting

$$\begin{array}{l} \eta = y\sqrt{\dfrac{a}{\nu}}, \psi = \sqrt{a\nu x}\, f(\eta), \omega = ax\sqrt{\dfrac{a}{\nu}}h(\eta), \\ \theta = \dfrac{T - T_\infty}{T_w - T_\infty}, \phi = \dfrac{C - C_\infty}{C_w - C_\infty} \end{array} \tag{9}$$

Equation (1) is automatically satisfied and equations (2)-(4) take the form

$$\begin{array}{l} (1+\Delta) f''' + (ff'' - (f')^2) + G(\theta + N\phi) + \\ \qquad + \Delta h' + M^2 E - M^2 f' = 0 \end{array} \tag{10}$$

$$(f\dot{f} - fh') = (1+\Delta)h'' - \Delta(2h + f'') \tag{11}$$

$$(1 + \frac{4Rd}{3})\theta'' + 0.5\Pr f\theta' + (A1 f' + B1\theta) + M^2 Ec\Pr((f')^2 + E^2 - 2Ef') = 0 \tag{12}$$

$$\phi'' + 0.5Scf\phi' - \gamma Sc\phi + ScSo\theta'' = 0 \qquad (13)$$

$$f = f_w\ ,\ f'(0) = 1 + A(1+\Delta)f''(0), h(0) = -sf''(0),$$
$$\theta'(0) = -Bi(1-\theta(0)), \phi'(0) = -Bc(1-\phi(0)) \qquad (14)$$

$$f'(\infty) = 0,\ h(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \qquad (15)$$

Where $G = \dfrac{\beta g(T_w - T_\infty)}{a^2 x}, M^2 = \dfrac{\sigma B_o^2}{\rho a}, N = \dfrac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)}, \Delta = \dfrac{k}{\mu}$

$$\Delta = \frac{k_1}{\mu}, E^2 = \frac{E_0^2}{u_w^2 B_o^2}, \Pr = \frac{\mu C_p}{k_f}, Bi = \frac{h_f}{k_f}(\sqrt{\frac{\nu}{a}}), Bc = \frac{h_c}{D_m}(\sqrt{\frac{\nu}{a}})$$

$$Rd = \frac{4\sigma^\bullet T_\infty^3}{\beta_R k_f}, Ec = \frac{\rho u_w^2}{C_p(T_w - T_\infty)}, Sc = \frac{\nu}{D_B}, \gamma = \frac{k_r'}{a}, A = \alpha'\mu\sqrt{\frac{a}{\nu}},$$

$$f_w = -(a\nu)^{1/2} v_w, So = \frac{D_m K_T m_w k_f}{T_\infty q_w D_m}$$

are the Grashof number, Magnetic parameter, buoyancy ratio, micropolar parameter, Micropolar parameter, dimensionless spin gradient viscosity parameter, Electric parameter, Prandtl number, convective heat transfer constant, radiation parameter, heat source parameter, Eckert parameter, Schmidt number, chemical reaction parameter, slip parameter, suction/injection velocity, Soret parameter

The local skin friction, Nusselt number and Sherwood number on the wall are defined by

$$C_{fx} = \frac{2\tau_w}{\rho(ax)^2}\ ,\quad Nux = \frac{xq_w}{k_f(T_w - T_\infty)}\quad ,\quad Shx = \frac{xm_w}{D_B(C_w - C_\infty)}$$

where τ_w (wall shear stress), q_w(heat flux), m_w(mass flux) are given by

$$\tau_w = ((\mu + k)\frac{\partial u}{\partial y} + k\omega)_{y=0}, q_w = -k_f(\frac{\partial T}{\partial y})_{y=0},\ \ m_w = -D_B(\frac{\partial C}{\partial y})_{y=0,}$$

which in the non-dimensional form reduces

$$0.5C_{fx}\operatorname{Re}_x^{1/2} = (1 + (1-n)\Delta)f''(0), Nu_x \operatorname{Re}_x^{1/2} = -\theta'(0), Sh_x \operatorname{Re}_x^{1/2} = -\phi'(0)$$

Where the local Reynolds number is given by $\operatorname{Re}_x = \dfrac{u_w x}{\nu}$

3. Method of Solution

The Galerkin finite element method has been implemented to obtain numerical solutions of coupled non-linear equations (9) to (10) of third-order in f and second order in h,θ,φ under boundary conditions (11&12) .This technique is extremely efficient and allows robust solutions of complex coupled, nonlinear multiple degree differential equation systems. The ultimate coupled global matrices are solved to determine the unknown global values of velocity, temperature and concentration in the fluid region. In solving these matrices an iteration procedure has been adopted.

Table1. *Comparison: In the absence of convection (G=0) the results are in good agreement with Ramzan et al[27].*

Parameters					Ramzan et al [27] Nu(0)	Present Results Nu(0)
Δ	E	α	γ	s		
0.1	0.2	0.1	0.1	0.1	0.08501	0.08502
0.3	0.2	0.1	0.1	0.1	0.08540	0.08541
0.6	0.2	0.1	0.1	0.1	0.08440	0.08442
0.1	0.3	0.1	0.1	0.1	0.08580	0.08579
0.1	0.6	0.1	0.1	0.1	0.08605	0.08602
0.1	0.2	0.3	0.1	0.1	0.08545	0.08544
0.1	0.2	0.6	0.1	0.1	0.08440	0.08441
0.1	0.2	0.1	0.3	0.1	0.20211	0.20212
0.1	0.2	0.1	0.6	0.1	0.34401	0.34400
0.1	0.2	0.1	0.1	0.3	0.08745	0.08746
0.1	0.2	0.1	0.1	0.6	0.08951	0.08952

4. Discussion of the Numerical Results

In this analysis we analyse the effect of partial slip, thermal radiation, dissipation, on convective heat and mass transfer flow of a viscous, electrically conducting fluid past a stretching sheet in the presence of non-uniform heat source with convective boundary conditions. The results are presented graphically in figures.2a-17d for different parametric variations. Comparison of the present results with previously works are performed and excellent agreements have been obtained. The non-linear coupled differential equations have been solved by Galerkin finite analysis with three nodded line segments. In the absence of convective heat and mass transfer(N=0,G=0,A1=B1=0) the results are compared Rahman et al[27].

Figs.1a-1d depicts the influence of radiative heat flux (Rd)on the velocity, micro-rotation velocity , temperature and concentration. It is observed that higher the radiative heat flux larger the fluid velocity throughout the boundary layer .This may be attributed to the fact that an increase in thermal radiation parameter increases thickness of the boundary layer. The micro-rotation reduces in the left half and enhances in the right half of the channel with increase in the radiation parameter(Rd). The presence of the thermal radiation is very significant on the variation of temperature. It is seen that the temperature rapidly increases in the presence of thermal radiation parameter throughout the thermal boundary layer. This may be attributed to the fact as the Rosseland radiative absorption parameter R* diminishes the corresponding heat flux diverges and thus rises the rate of radiative heat transfer to the fluid causing a rise in the temperature of the fluid. The thickness of the boundary layer also increases in the presence of Rd. The effect of Rd on concentration is to diminish it in the solutal boundary layer.

Figs.2a-2d shows that an increasing (S_0) enhances the velocity (f′) and temperature (θ) and reduces the concentration. The angular velocity ω reduces with S_0 in the region (0,1) and enhances far away from the plate.

Figs.3a-3d shows the influence of dissipation on the velocity, micropolar velocity, temperature and concentration. It is pointed out that the presence of Eckert number increases the velocity (figs.3a).This is due to the fact that the thermal energy is reserved in the fluid on account of friction heating. Hence the velocity and rises in the entire boundary layer .The variation of micro-rotation with Ec shows that higher the dissipation smaller the micro-rotation in the vicinity of the wall and enhances in the region far away from the wall (fig.3b). However, the temperature and mass concentration increases with increase in Ec (fig.3c,3D).

From figs.4a-4d represent the variation of f’,ω,θ and C with surface boundary parameter(S).It can be seen from the profiles that an increase in S reduces the velocity and concentration while the micropolar velocity and temperature and concentration enhances in the flow region.

Figs.5a-5d represent the impact of space dependent heat source (A1) on f’,ω,θ and C. It can be seen from the profiles that the velocity and the temperature reduces with in crease in the strength of the space dependent heat generating source (A1>0)and in the case of space dependent heat absorbing source (A1<0) they experience an enhancement in the flow region(figs.5a&5c).The micro-rotation and concentration increases with A1>0 and reduces with A1<0 in the entire flow region(figs.5b&5d).

Figs.6a-6d represent f’,ω,θ and C with temperature dependent heat source(B1).It can be seen from the profiles that the velocity reduces with increase in the strength of heat generating source and enhances with of heat absorbing source(6a).The temperature reduces and the concentration enhances with B1>0 and for B1<0,the temperature enhances and the concentration reduces in the flow region(6c&d).The micro-rotation enhances with increase in B1>0 and reduces with B1<0(fig.6b).

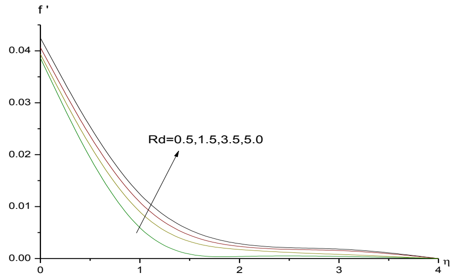
Figs.7a-7d exhibit the variation of f’,ω,θ and C with convective heat transfer constant Bi. We find from the profiles that an increase in Bi enhances the velocity, temperature and concentration while the micro-rotation reduces in the flow region (0,1) and enhances in the region(1.1,4.0).Thus the momentum thermal and solutal boundary layers increase with Bi.

Figs.8a-8d exhibit the variation of f’,ω,θ and C with convective mass transfer constant Bc. We find from the profiles that an increase in Bc enhances the velocity, temperature and concentration while the micro-rotation reduces in the flow region (0,1) and enhances in the region(1.1,4.0).Thus the momentum thermal and solutal boundary layers increase with Bc.

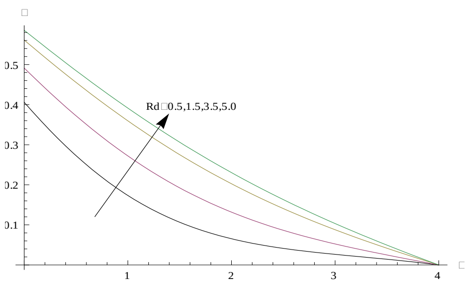
Figs.9a-16d shows the variation of f’,ω,θ and C with electric parameter E1.Higher the values of electric parameter results in a raise in velocity ,temperature and a fall in concentration. The micro-rotation reduces with E1 in the vicinity of the wall and enhances far away from the wall. In fact higher the values of the electric parameter E1 provided less resistance in the direction of flow fluid and as a result large heat is produced. Therefore the temperature distribution increases (9d).

Figs.10a-10d represent the variation of f’,ω,θ and C with slip parameter (A).It can be seen from the profiles that an increase in the slip parameter (A) reduces thee velocity and micropolar velocity (figs.10a&10b) .Also an increase in A enhances the temperature and reduces the concentration in the flow region(figs.10c&10d).

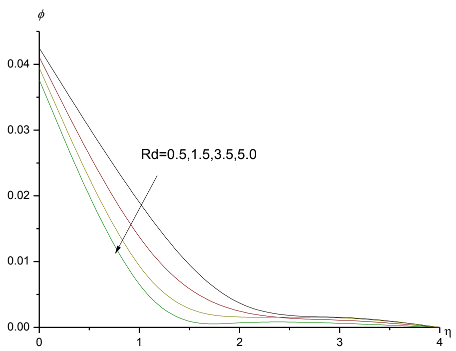
Figs.11a-11d show the variation of f’,ω,θ and C with microrotation parameter(Δ). From the profiles we find that the velocity reduce with increase in Δ.The micro-rotation reduces in the region adjacent to the wall and enhances far away from the wall(fig.11b).The temperature ,concentration increase in the flow region.



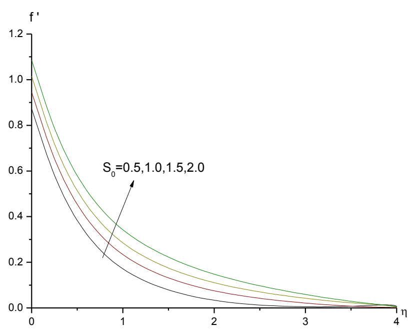
Fig,1a Variation of f′ with Rd
So=0.5,A=0.5,Δ=1, Ec=0.01



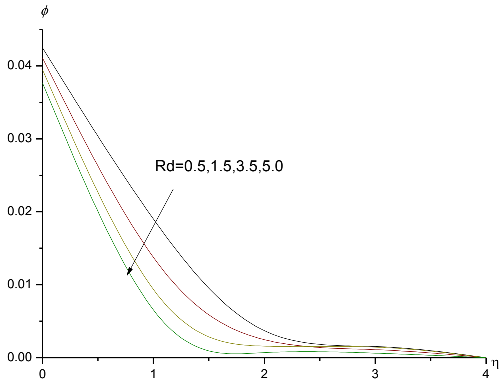
Fig,1c Variation of θ with Rd
So=0.5,A=0.5,Δ=1, Ec=0.01



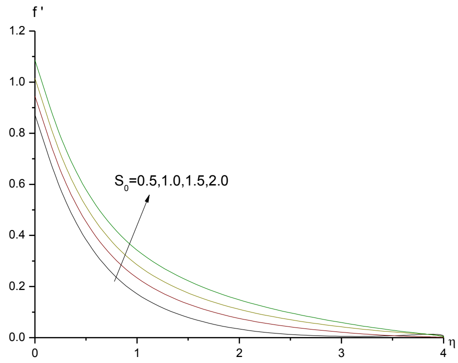
Fig,1d Variation of ϕ with Rd
So=0.5,A=0.5,Δ=1, Ec=0.01



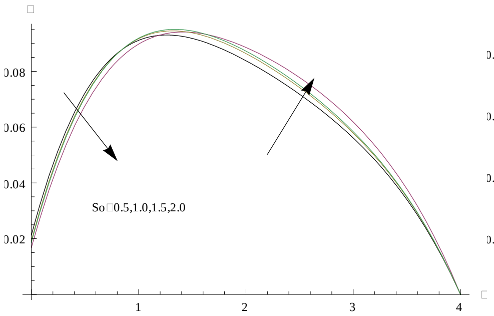
Fig,2a Variation of f′ with So
A=0.5,Δ=1,Rd=0.5,Ec=0.01



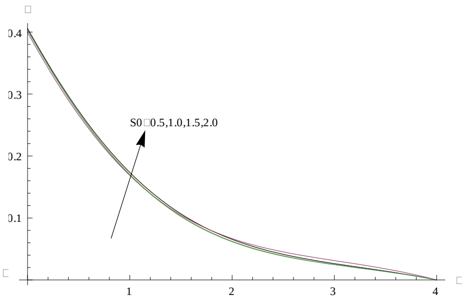
Fig,1d Variation of ϕ with Rd
So=0.5,A=0.5,Δ=1, Ec=0.01



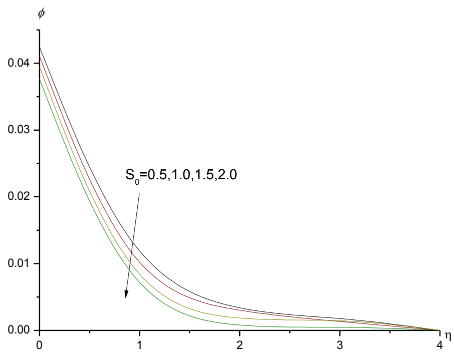
Fig,2a Variation of f′ with So
A=0.5,Δ=1,Rd=0.5,Ec=0.01



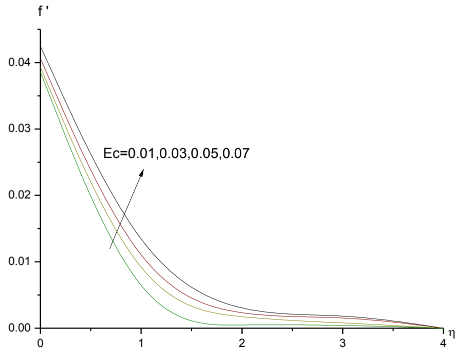
Fig,2b Variation of ω with So
A=0.5,Δ=1,Rd=0.5,Ec=0.01



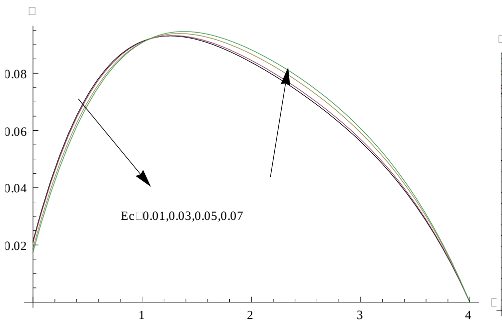
Fig,2c Variation of θ with So
A=0.5,Δ=1,Rd=0.5,Ec=0.01



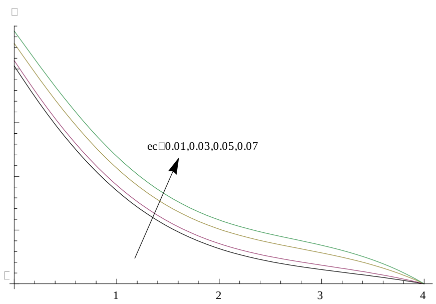
Fig,2d Variation of ϕ with So
A=0.5,Δ=1,Rd=0.5,Ec=0.01



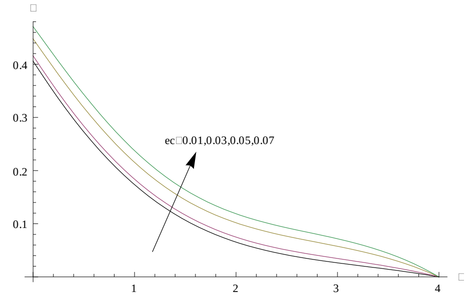
Fig,3a Variation of f′ with Ec
So=0.5,A=0.5,Δ=1,Rd=0.5



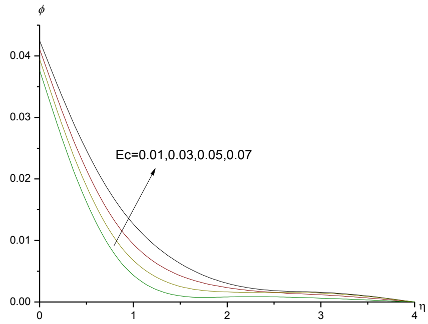
Fig,3b Variation of ω with Ec
So=0.5,A=0.5,Δ=1,Rd=0.5



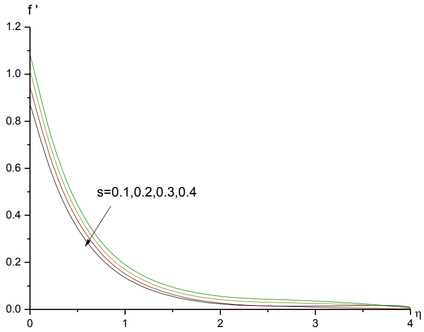
Fig,3c Variation of θ with Ec
So=0.5,A=0.5,Δ=1,Rd=0.5



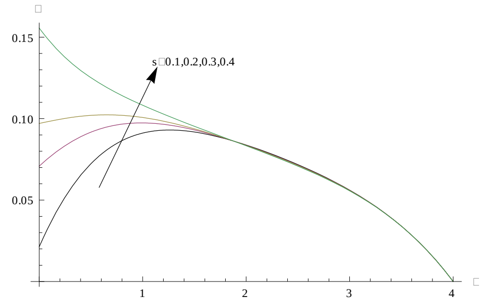
Fig,3c Variation of θ with Ec
So=0.5,A=0.5,Δ=1,Rd=0.5



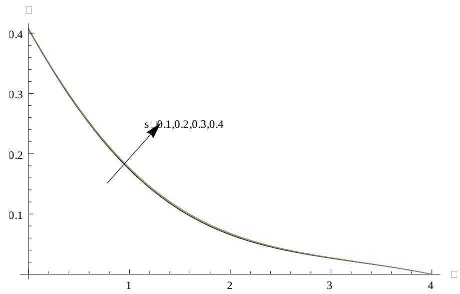
Fig,3d Variation of ϕ with Ec
So=0.5,A=0.5,Δ=1,Rd=0.5



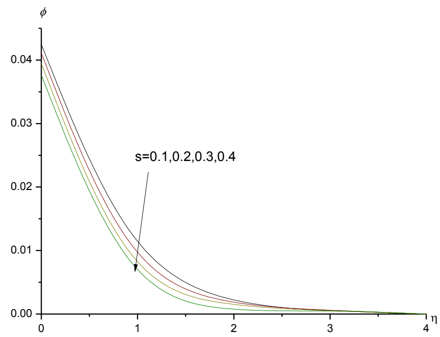
Fig,4a Variation of f′ with s
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01



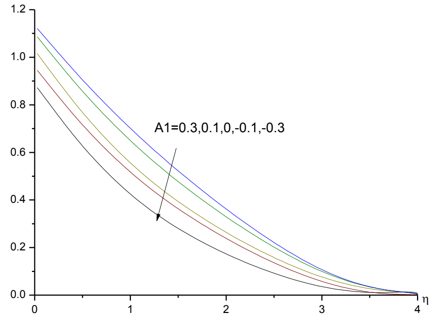
Fig,4b Variation of ω with s
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01



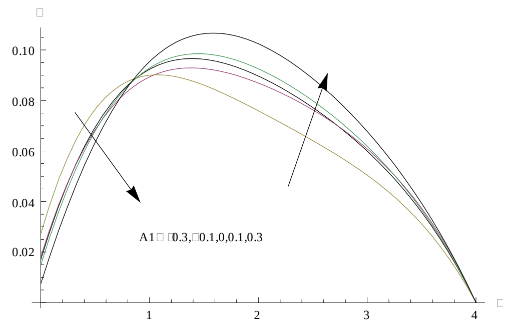
Fig,4c Variation of θ with s
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01



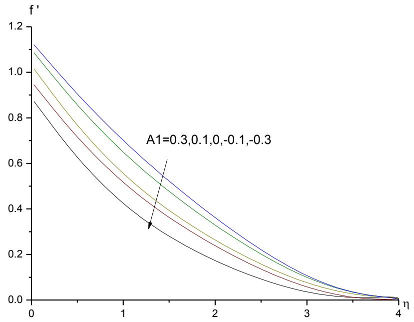
Fig,4d Variation of ϕ with s
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01



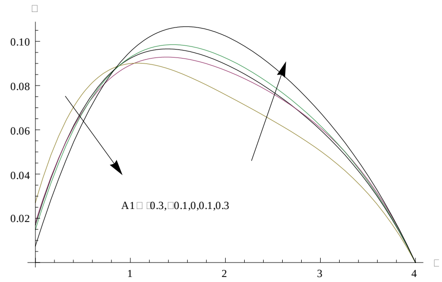
Fig,5a Variation of f′ with A1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, s=0.1



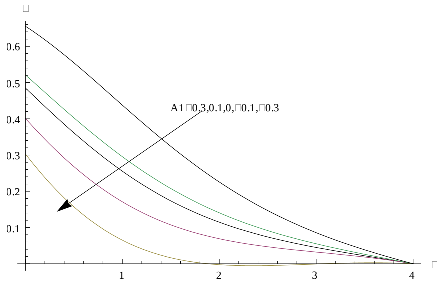
Fig,5b Variation of ω with A1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, s=0.1



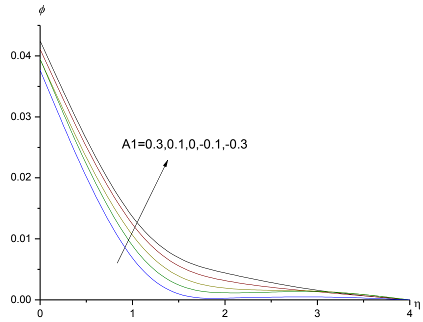
Fig,5a Variation of f′ with A1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, s=0.1



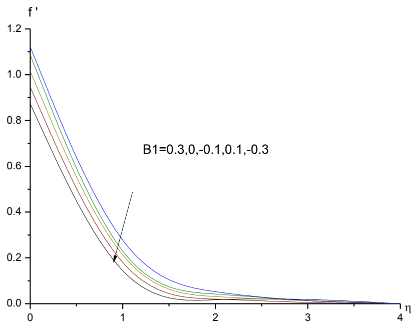
Fig,5b Variation of ω with A1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, s=0.1



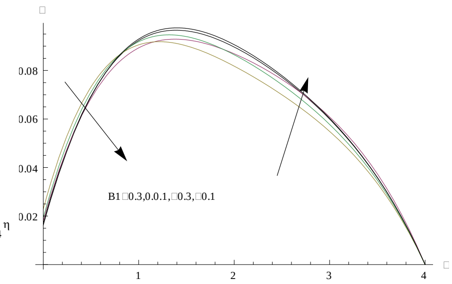
Fig,5c Variation of θ with A1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, s=0.1



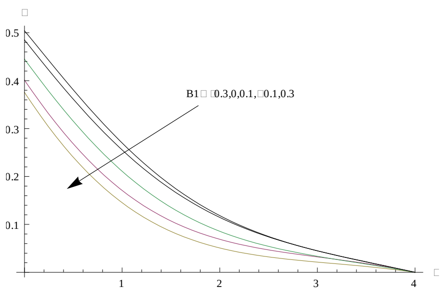
Fig,5d Variation of ϕ with A1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, s=0.1



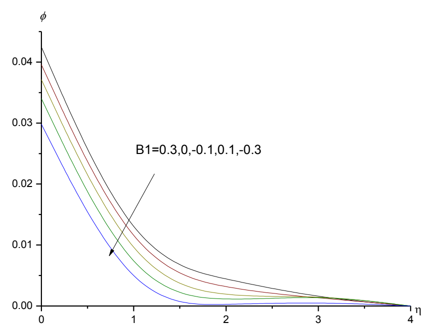
Fig,6a Variation of f′ with B1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,s=0.1



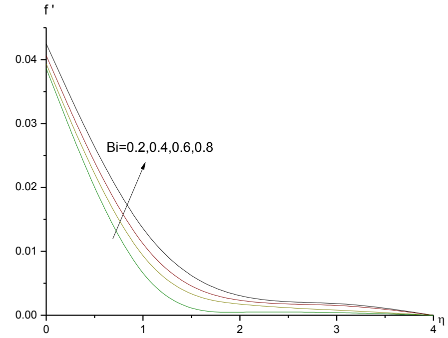
Fig,6b Variation of ω with B1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,s=0.1



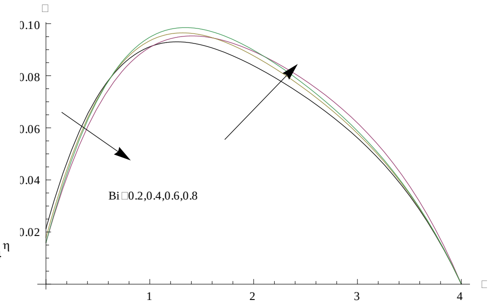
Fig,6c Variation of θ with B1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,s=0.1



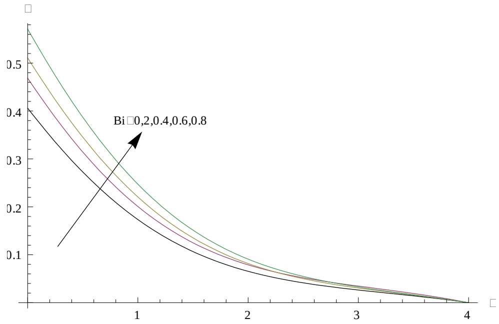
Fig,6d Variation of ϕ with B1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,s=0.1



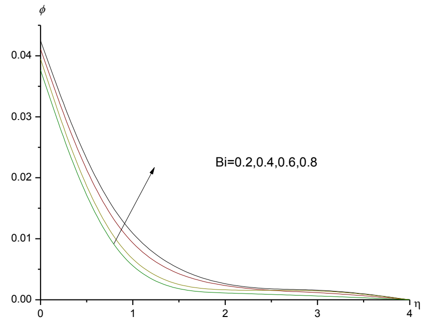
Fig,7a Variation of f′ with Bi
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01,A1=-0.1,B1=-0.1



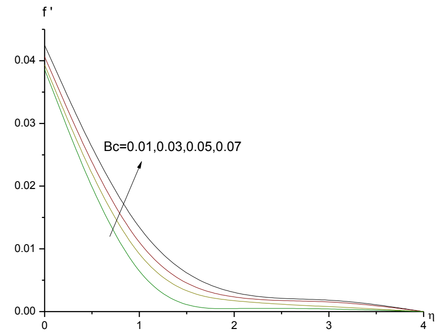
Fig,7b Variation of ω with Bi
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01,A1=-0.1,B1=-0.1



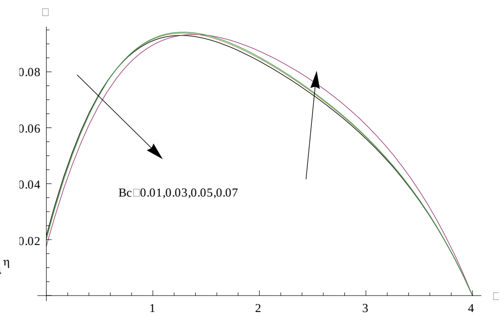
Fig,7c Variation of θ with Bi
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01,A1=-0.1,B1=-0.1



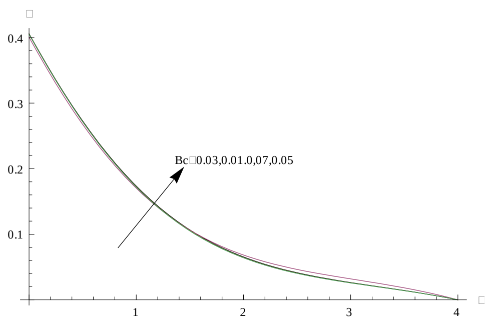
Fig,7d Variation of ϕ with Bi
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01,A1=-0.1,B1=-0.1



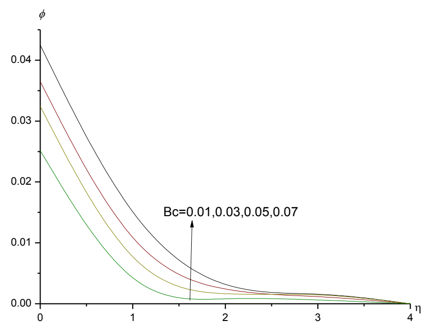
Fig,8a Variation of f′ with Bc
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,B1=-0.1



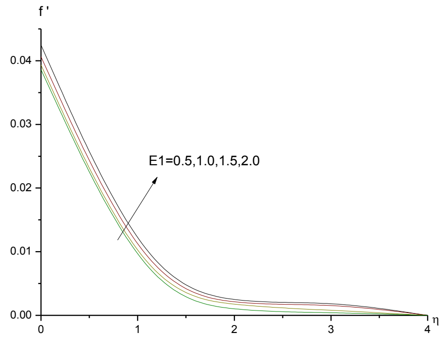
Fig,8b Variation of ω with Bc
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,B1=-0.1



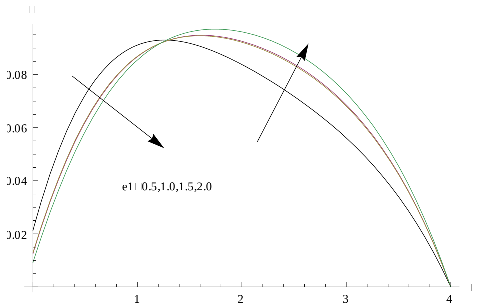
Fig,8c Variation of θ with Bc
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,B1=-0.1



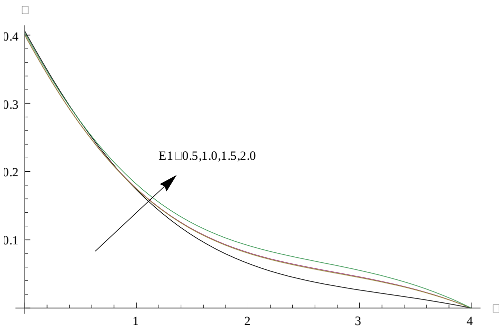
Fig,8d Variation of ϕ with Bc
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,B1=-0.1



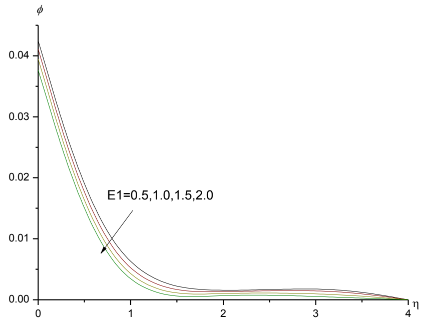
Fig,9a Variation of f′ with E1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, A1=-0.1,B1=-0.1



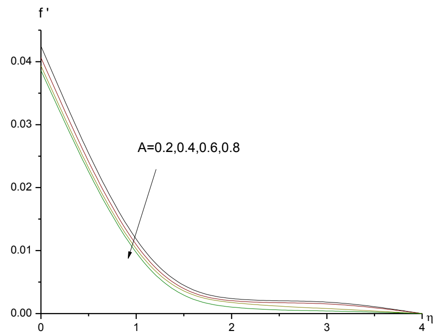
Fig,9b Variation of ω with E1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, A1=-0.1,B1=-0.1



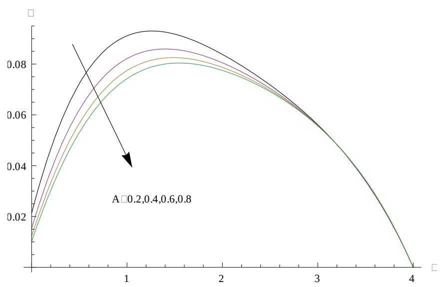
Fig,9c Variation of θ with E1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, A1=-0.1,B1=-0.1



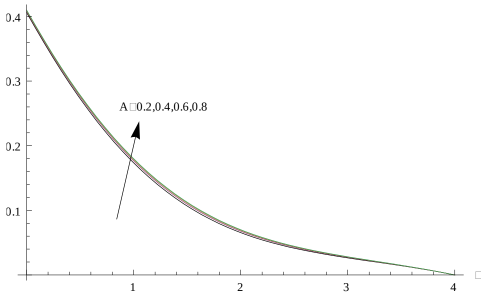
Fig,9d Variation of ϕ with E1
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, A1=-0.1,B1=-0.1



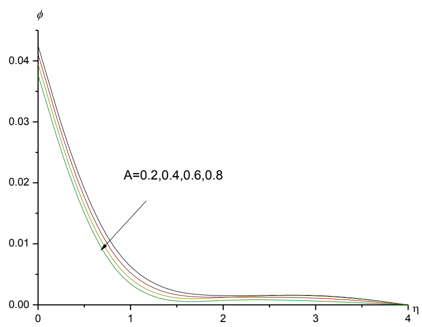
Fig,10a Variation of f′ with A
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, A1=-0.1,B1=-0.1



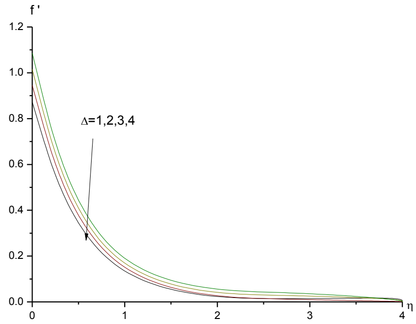
Fig,10b Variation of ω with A
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, A1=-0.1,B1=-0.1



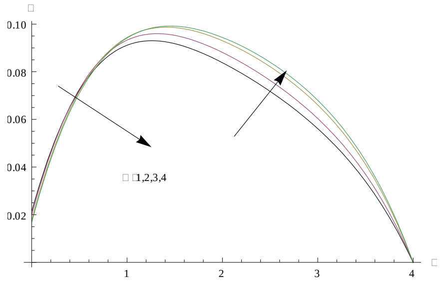
Fig,10c Variation of θ with A
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, A1=-0.1,B1=-0.1



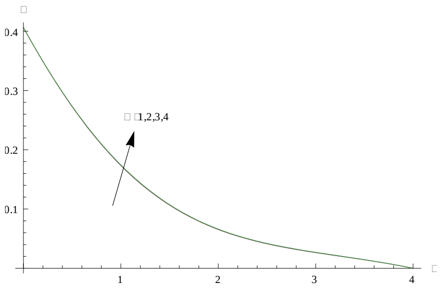
Fig,10d Variation of ϕ with A
So=0.5,A=0.5,Δ=1,Rd=0.5,Ec=0.01, A1=-0.1,B1=-0.1



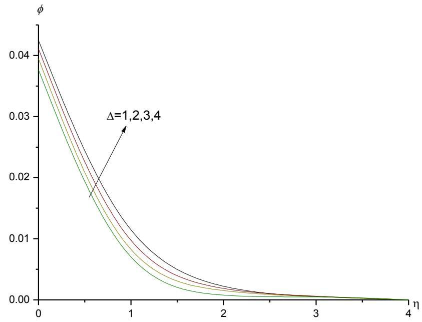
Fig,11a Variation of f′ with Δ
So=0.5,s=0.1,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,B1=-0.1



Fig,11b Variation of ω with Δ
So=0.5,s=0.1,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,B1=-0.1



Fig,11c Variation of θ with Δ
So=0.5,s=0.1,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,B1=-0.1



Fig,11d Variation of ϕ with Δ
So=0.5,s=0.1,Δ=1,Rd=0.5,Ec=0.01A1=-0.1,B1=-0.1

The skin friction (τ) at the wall η=0 is shown in table.1 for different values of the parameters S_0,A1,B1,Rd,Ec,A,Bi,Bc,S,Δ,E1. The variation of skin friction with space dependent heat source(A1) shows that the skin friction enhances with A1>0 and reduces with A1<0.An increasing in the heat generating source increases τ and decreases in the case heat absorbing source. Higher the thermo-diffusion effects smaller the skin friction on the wall. The skin friction decreases with increases in slip parameter A or micropolar parameter Δ or electric parameter E1 .The skin friction enhances in degenerating/generating chemical reaction cases. An increase in S enhances τ and reduces with Bi and Bc on the wall.

The couple stress (Cw) at the wall η=0 is shown in table.2 for different values of the parameters So,A1,B1,Rd,Ec,A,Bi,Bc, S,Δ,E1. With respect to radiation parameter Rd, we find that higher the radiative heat flux smaller the couple stress at the wall. . Higher the space dependent heat source smaller w and larger Cw at the wall with space dependent absorbing source. An increasing in the heat generating source enhances Cw and decreases with heat absorbing source. The couple stress increases with convection heat and mass transfer constants Bi&Bc, micropolar parameters Δ at the wall. An increase in electric parameter E1 or slip parameter A leads to a depreciation in the couple stress at the wall. When the molecular buoyancy force dominates over the thermal force the couple stress reduces when the buoyancy forces are in the same direction and for the forces acting in opposite directions, it enhances on the wall. The couple stress enhances in both degenerating/generating chemical reaction cases.

The rate of heat transfer(Nusselt number) at the wall η=0 is evaluated for different parametric variations. Higher the electric parameter E1 /slip parameter A / radiation parameter Rd/dissipation parameter Ec, smaller the Nusselt number at the wall. Higher the convective heat transfer constant Bi lesser the Nusselt number at the wall while a reversed effect is noticed in the case of mass transfer convective transfer constant Bc. Higher the space dependent heat source smaller the rate of heat transfer at the wall. With reference to the temperature dependent heat source parameter, we find that the rate of heat transfer enhances with increase in heat generating source and smaller Nu in the case of heat absorbing source. The rate of heat transfer experiences a depreciation at =0 in both degenerating/generating chemical reaction cases. Higher the thermo-diffusion effects larger the rate of heat transfer at the wall. The Nusselt number enhances with increase in micropolar parameters $\Delta \leq 2$ and reduces with higher $\Delta \geq 3$. An increase in S decreases the rate of heat transfer at the wall.

The rate of mass transfer(Sherwood number) at the wall $\eta=0$ is evaluated with variations in different parameters. Higher the surface boundary parameter S/electric parameter E1 larger the rate of mass transfer at the wall. Higher values of convective heat and mass transfer constants Bi&Bc/slip parameter(A)smaller Sh at the wall. Lesser the molecular diffusivity smaller the rate of mass transfer at the wall. An increase in the strength of the heat generation source leads to a depreciation in the Sh and enhances in the case of heat absorbing source. An increase in the micropolar parameters $\Delta \leq 2$,larger Sh and for still higher $\Delta \geq 3$,smaller the rate of mass transfer at the wall. When the molecular buoyancy force dominates over the thermal buoyancy force Sh enhances when the buoyancy forces are in the same direction and for the forces acting in opposite directions, it reduces on the wall. Sh increases on the wall in the degenerating chemical reaction case and decreases in the generating case.

Table2.

Parameters		τ(0)	Cw(0)	Nu(0)	Sh(0)
A1	0	-0.169325	0.142242	2.06182	30.309
	-0.1	-0.180702	0.139253	2.49326	23.6502
	-0.3	-0.270725	0.150702	3.30093	18.0235
	0.1	-0.148366	0.140483	1.91883	34.6624
	0.3	-0.0704935	0.135583	1.50914	75.4925
B1	0	-0.169325	0.142242	2.06182	30.309
	-0.1	-0.180702	0.139253	2.49326	23.6502
	-0.3	-0.227903	0.146314	2.66177	21.6127
	0.1	-0.192669	0.144308	2.24755	26.5277
	0.3	-0.16151	0.142425	1.98354	32.6286
Rd	0.5	-0.21272	0.145485	2.46322	23.5441
	1.5	-0.13085	0.135121	2.05739	30.7461
	3.5	-0.114731	0.134923	1.78186	40.3315
	5	-0.0976985	0.132704	1.70763	45.0129
So	0.5	-0.21272	0.145485	2.46322	23.5441
	1	-0.206183	0.145064	2.46741	14.3118
	1.5	-0.19965	0.144648	2.47145	10.2876
	2	-0.186601	0.143836	2.4791	6.59447
Ec	0.1	-0.21272	0.145485	2.46322	23.5441
	0.3	-0.173402	0.137915	2.43346	24.369
	0.5	-0.187402	0.142107	2.23245	26.9545
	0.7	-0.16035	0.144339	2.01428	32.1956
S	0.1	-0.161272	0.147485	2.46322	23.5441
	0.2	-0.187376	0.102029	2.45074	23.6242
	0.3	-0.228386	0.0593336	2.44892	23.7465
	0.4	-0.236678	0.0138062	2.41652	23.94243
Bi	0.5	-0.21272	0.145485	2.46322	23.5441
	1.0	-0.178438	0.146792	1.95764	20.8743
	1.5	-0.155228	0.147808	1.71627	19.3761
	2	-0.124334	0.149321	1.47199	17.6754
Bc	0.05	-0.21272	0.145485	2.46322	23.5441
	0.1	-0.208074	0.145592	2.46545	13.9689
	0.15	-0.20392	0.14569	2.46739	10.2412
	0.2	-0.200006	0.145786	2.46919	8.18192
A	0.2	-0.21272	0.145485	2.46322	23.5441
	0.4	-0.129876	0.123817	2.47936	23.4501
	0.6	-0.122252	0.117652	2.44445	23.1067
	0.8	-0.101694	0.111231	2.43886	23.0203
□	1	-0.21272	0.145485	2.46322	23.5441

Parameters		τ(0)	Cw(0)	Nu(0)	Sh(0)
	2	-0.199717	0.146351	2.49092	23.6252
	3	-0.185805	0.151915	2.46238	23.4771
	4	-0.175839	0.152235	2.46157	23.4411
E1	0.5	-0.21272	0.145485	2.46322	23.5441
	1.0	-0.128856	0.129892	2.44995	24.2058
	1.5	-0.0535674	0.117674	2.44283	25.6399
	2	-0.018414	0.0879603	2.05131	34.7545

5. CONCLUSION

The non linear governing equations have been solved by using finite element technique. The effect radiation dissipation and convective boundary conditions on the flow characteristics have been analysed. It is found that an increasing thermal radiation / Eckert number enhances the velocity, temperature, while reduces the angular velocity. The concentration reduces with Rd and enhances with Ec. An increasing convective heat and mass transfer constants enhances a velocity, temperature and concentration and reduces the angular velocity.

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