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# A Production Inventory Model for Weibull Deteriorating Items with Two Components of Demand Rate

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**Abstract:** *The present paper deals a production inventory model for weibull deteriorating items with constant and exponential demand rate. The products of this model have been considered to have finite life and small amount of decay. The rate of production is constant. The production starts with zero inventory level and stop at a desired maximum inventory level. The purpose of our study is to optimizing the total variable inventory cost. A numerical example is also given to demonstrate the developed model.*

**Keywords:** *Production Inventory, Weibull Deterioration, Two Component Demand Rate and Constant Production Rate.*

## 1. Introduction

Academicians as well as industrialists have great interest in the development of inventory control and their uses. There are many goods that are either deteriorates or become obsolete with passage of time, such perishable products have different modeling. Perishable inventory forms a small portion of the total inventory. The perishable inventory items can be classified into three categories: (1) deterioration (2) obsolescence (3) no deterioration or no obsolescence. Obsolescence occurs due to the arrival of new and better products in the market.

In the existing literature several inventory models were constructed by several researchers based on the market demand. They have assumed that the demand rate is either constant or an increasing or decreasing function of time or stock dependent. The demand of newly launched products, such as fashionable garments, electronic items, mobiles, motor vehicles etc. increases with time and later it becomes constant.

Rosenberg [1979] presented a lot-size inventory model for deteriorating items with partial backlogging. Billington [1987] developed an economic production quantity model by considering ordering cost as a function of capital expenditure. Urban [1995] presented an inventory model of deteriorating items with stock dependent demand rate by allowing shortages. Abad [1996] proposed an inventory model and studied the optimal pricing and lot sizing conditions of deterioration and partial backlogging. Kishan and Mishra [1996] presented an inventory model with exponential demand rate. They also considered constant deterioration rate and shortages in their inventory model. Teng et al. [1999] proposed a lot-size inventory model for deteriorating items with fluctuating demand and allowing shortages. Skouri and Papachristos [2002] developed a continuous review inventory model for deteriorating items with time dependent demand rate and partial backlogging.

Ouyang and Cheng [2005] presented an inventory model of deteriorating items with exponential declining demand and partial backlogging. Ouyang et al. [2005] proposed an inventory model for deteriorating items with exponential declining demand by allowing shortages. Chund and Wee [2008] discussed the scheduling and replenishment plan for an integrated inventory model of deteriorating items with stock dependent selling rate. Jain et al. [2008] developed an inventory model of deteriorating items with inventory level declining demand rate and allowing shortages. Mishra et al. [2013] proposed an inventory model for deteriorating items with time dependent demand rate and holding cost by allowing shortages. Dash et al. [2014] presented an inventory model of deteriorating items with exponential declining demand rate and time varying holding cost. Raj et al. [2015] discussed an inventory model for deteriorating items with exponential demand rate and partial backlogging. Islam [2015, 2016] developed two inventory models. One of them is a production inventory model of deteriorating items with constant demand rate and three types of production rate and other is an inventory model with exponential demand rate and constant production rate for the products having finite shelf-life. Islam et al. [2015] presented a production inventory model with constant production rate and different components of demand. They also considered the product's shelf -life is finite in their inventory model. Ukil et al. [2015] discussed a production inventory model with constant production rate and time dependent power demand by considering the finite shelf -life of products. Lakshmidevi and Maragatham [2015] developed an inventory model with three rates of production and time dependent demand rate and deterioration rate. Ukil and Uddin [2016] proposed a production inventory model with constant production rate and linear trend in demand.

In the present paper, we have developed a production inventory model for weibull deteriorating items with constant and exponential demand rate.

## 2. Assumptions and Notations

We consider the following assumptions and notations corresponding to the developed model.

1. The demand rate is $R(t)=\begin{cases} a, & 0\le t\le T_1 \\ a e^{bt}, & T_1\le t\le T \end{cases}$

   where $a$ and $b$ are constants.
2. The deterioration rate is $\theta(t)=\alpha\beta t^{\beta-1}$, where $\alpha$ and $\beta$ are parameters.
3. The production rate is $P(t)=\lambda$, where $\lambda$ is a constant.
4. $o_C$ is the ordering cost per order.
5. $h_C$ is the holding cost per unit per cycle.
6. $d_C$ is the deterioration cost per unit per cycle.
7. T is the cycle length.
8. $T_1$ is the time at which production level reaches maximum.
9. The replenishment rate is finite.
10. The lead time is zero.
11. $TC(T_1,T)$ is the total cost per cycle.
12. $I(t)$ is the inventory level at any time t.

## 3. Mathematical Formulation

Suppose the production starts at time $T=0$ with zero inventory level and becomes maximum inventory level Q at time $T=T_1$. The instantaneous inventory level at any time t in $[0,T]$ is given by the following differential equations

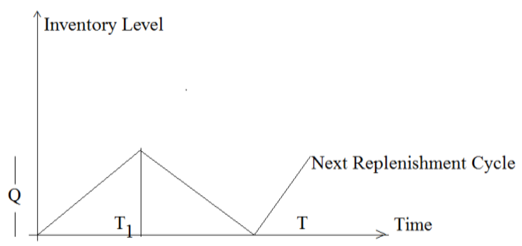



Figure 1 Inventory Model

$\frac{dI}{dt}+\theta(t)\,I=P(t)-R(t)$, putting the values of *P(t)* and *R(t)* in this equation, we obtain

$$\frac{dI}{dt}+\alpha\,\beta\,t^{\beta-1}\,I=(\lambda-a),\qquad 0\le t\le T_1 \tag{1}$$

With initial boundary condition

$I(0)=0$

$$\frac{dI}{dt}+\alpha\,\beta\,t^{\beta-1}\,I=a\,e^{bt},\qquad T_1\le t\le T \tag{2}$$

With initial boundary condition

$I(T_1)=Q$

The solutions of equations (1) and (2) are given by the equations (3) and (4) respectively.

$$I(t)=(\lambda-a)t-\alpha(\lambda-a)t^{\beta+1},\qquad 0\le t\le T_1 \tag{3}$$

$$I(t)=a[T_1+\left(\frac{b}{2}\right)T_1^2+\left(\frac{\alpha}{\beta+1}\right)T_1^{\beta+1}+\left(\frac{\alpha b}{\beta+2}\right)T_1^{\beta+2}-\alpha T_1 t^{\beta}-\left(\frac{\alpha b}{2}\right)T_1^2 t^{\beta}]+Q(1+\alpha T_1^{\beta}-\alpha t^{\beta})$$

$$-a[t+\left(\frac{b}{2}\right)t^2+\left(\frac{\alpha}{\beta+1}\right)t^{\beta+1}+\left(\frac{\alpha b}{\beta+2}\right)t^{\beta+2}-\alpha t^{\beta+1}-\left(\frac{\alpha b}{2}\right)t^{\beta+2}] \tag{4}$$

The maximum inventory level Q is obtained by putting $t=T_1$ in equation (3). We have

$$Q=(\lambda-a)T_1-\alpha(\lambda-a)T_1^{\beta+1}, \tag{5}$$

Putting the value of Q in equation (4),the equation (4) becomes

$$I=-a\left[t+\left(\frac{b}{2}\right)t^2-\left(\frac{\alpha\beta}{\beta+1}\right)t^{\beta+1}-\left(\frac{b\alpha\beta}{2(\beta+2)}\right)t^{\beta+2}\right]+\lambda T_1+\left(\frac{ab}{2}\right)T_1^2+\left(\frac{a\alpha\,(2\beta+3)}{\beta+1}\right)T_1^{\beta+1}$$

$$-2\alpha\lambda T_1^{\beta+1}+\left(\frac{ab\alpha}{\beta+2}\right)T_1^{\beta+2}-\left(\frac{ab\alpha}{2}\right)T_1^2 t^{\beta}-\alpha\lambda T_1 t^{\beta},\qquad T_1\le t\le T \tag{6}$$

The ordering cost per cycle is

$$O_C=o_C \tag{7}$$

The holding cost per cycle is

$$H_C=h_C\left[\int_0^{T_1}I(t)\,dt+\int_{T_1}^{T}I(t)\,dt\right]$$

Or

$$H_C=h_C\Bigg[-\left(\frac{\lambda}{2}\right)T_1^2-\left(\frac{a}{2}\right)T^2+\lambda T\,T_1-\left(\frac{ab}{3}\right)T_1^3-\left(\frac{ab}{6}\right)T^3+\left(\frac{ab}{2}\right)T\,T_1^2-\left(\frac{\alpha\lambda}{\beta+1}\right)T_1T^{\beta+1}$$

$$+\left\{\frac{a\alpha(2\beta+3)-2\alpha\lambda(\beta+1)}{\beta+1}\right\}T\,T_1^{\beta+1}-\left\{\frac{\alpha(a-\lambda)(16\beta+5)}{7\beta+2}\right\}T_1^{\beta+2}+\left(\frac{a\alpha\beta}{3\beta+2}\right)T^{\beta+2}$$

$$-\left(\frac{ab\alpha\beta}{7\beta+3}\right)T_1^{\beta+3}+\left(\frac{ab\alpha\beta}{2(5\beta+6)}\right)T^{\beta+3}+\left(\frac{ab\alpha}{\beta+2}\right)T\,T_1^{\beta+2}-\left(\frac{a\,b\alpha}{2(\beta+1)}\right)T_1^2\,T^{\beta+1}\Bigg] \tag{8}$$

The deterioration cost per cycle is

$$D_C=d_C\left[\int_0^{T_1}\theta(t)I(t)\,dt+\int_{T_1}^{T}\theta(t)I(t)\,dt\right]$$

Or

$$D_C = d_C\left[-\left(\frac{\alpha\lambda}{\beta+1}\right)T_1^{\beta+1} - \left(\frac{a\alpha\beta}{\beta+1}\right)T^{\beta+1} - \left(\frac{ab\alpha}{\beta+2}\right)T_1^{\beta+2} - \left(\frac{ab\alpha\beta}{2(\beta+2)}\right)T^{\beta+2} + \alpha\lambda T_1 T^{\beta} + \left(\frac{ab\alpha}{2}\right)T_1^2 T^{\beta}\right] \tag{8}$$

The total cost per cycle is

$$TC(T_1,T) = \left(\frac{1}{T}\right)\left[O_C + H_C + D_C\right] \tag{9}$$

Putting the values of $O_C$, $H_C$ and $D_C$ in equation (9), we have

$$\begin{aligned} TC(T_1,T) = \left(\frac{1}{T}\right)\Bigg[ & O_C - \left(\frac{\lambda h_C}{2}\right)T_1^2 - \left(\frac{a h_C}{2}\right)T^2 + \lambda h_C T T_1 - \left(\frac{ab h_C}{3}\right)T_1^3 - \left(\frac{ab h_C}{6}\right)T^3 + \left(\frac{ab h_C}{2}\right)T T_1^2 \\ & - \left(\frac{\alpha\lambda d_C}{\beta+1}\right)T_1^{\beta+1} - \left(\frac{a\alpha\beta d_C}{\beta+1}\right)T^{\beta+1} - \left\{\frac{\alpha(a-\lambda)h_C(37\beta+10) + ab\alpha d_C(7\beta+2)}{4(\beta+1)}\right\}T_1^{\beta+2} \\ & + \left\{\frac{a\alpha\beta(4h_C + 2bd_C)}{8(2\beta+1)}\right\}T^{\beta+2} - \alpha\lambda d_C T_1 T^{\beta} - \left(\frac{ab\alpha d_C}{2}\right)T_1^2 T^{\beta} - \left(\frac{\alpha\lambda h_C}{\beta+1}\right)T_1 T^{\beta+1} \\ & + \left\{\frac{a\alpha(2\beta+3) - 2\alpha\lambda(\beta+1)}{\beta+1}\right\}h_C T T_1^{\beta+1} - \left(\frac{ab\alpha\beta h_C}{7\beta+3}\right)T_1^{\beta+3} + \left(\frac{ab\alpha\beta h_C}{2(5\beta+6)}\right)T^{\beta+3} \\ & + \left(\frac{ab\alpha h_C}{\beta+2}\right)T T_1^{\beta+2} - \left(\frac{ab\alpha h_C}{2(\beta+1)}\right)T_1^2 T^{\beta+1}\Bigg] \end{aligned} \tag{10}$$

The necessary conditions for $TC(T_1,T)$ to be minimum are

$$\frac{\partial TC(T_1,T)}{\partial T_1} = 0, \qquad \frac{\partial TC(T_1,T)}{\partial T} = 0 \tag{11}$$

Solving these equations, we find the optimum values of $T_1$ and $T$ for which the total cost is minimum.

The sufficient conditions for $TC(T_1,T)$ are

$$\left(\frac{\partial^2 TC(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 TC(T_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC(T_1,T)}{\partial T_1^2}\right)^2 > 0, \text{ and } \left(\frac{\partial^2 TC(T_1,T)}{\partial T_1^2}\right) > 0 \tag{12}$$

Differentiating equation (10), we obtain

$$\begin{aligned} \frac{\partial TC(T_1,T)}{\partial T_1} = \left(\frac{1}{T}\right)\Bigg[ & -\lambda h_C T_1 + \lambda h_C T - abh_C T_1^2 + abh_C T T_1 - \alpha\lambda d_C T_1^{\beta} - \alpha\lambda d_C T^{\beta} - ab\alpha\lambda d_C T_1 T^{\beta} \\ & - \left\{\frac{\alpha(a-\lambda)h_C(21\beta+5) + ab\alpha d_C(4\beta+1)}{4\beta+1}\right\}T_1^{\beta+1} + \left\{a\alpha(2\beta+3) - 2\alpha\lambda(\beta+1)\right\}h_C T T_1^{\beta} \\ & - \left(\frac{\alpha\lambda h_C}{\beta+1}\right)T^{\beta+1} - \left(\frac{3ab\alpha\beta h_C}{7\beta+3}\right)T_1^{\beta+2} + ab\alpha h_C T T_1^{\beta+1} - \left(\frac{ab\alpha h_C}{\beta+1}\right)T_1 T^{\beta+1}\Bigg] \end{aligned} \tag{13}$$

$$\begin{aligned} \frac{\partial TC(T_1,T)}{\partial T} = \left(\frac{1}{T}\right)\Bigg[ & -ah_C T + \lambda h_C T_1 - \left(\frac{ab h_C}{2}\right)T^2 + \left(\frac{ab h_C}{2}\right)T_1^2 - a\alpha\beta d_C T^{\beta} - \alpha\beta\lambda d_C T_1 T^{\beta-1} \\ & + \left\{\frac{a\alpha\beta(2h_C + bd_C)}{2(2\beta+1)}\right\}T^{\beta+1} - \left(\frac{ab\alpha\beta d_C}{2}\right)T_1^2 T^{\beta-1} + \left\{\frac{a\alpha(2\beta+3) - 2\alpha\lambda(\beta+1)}{(\beta+1)}\right\}h_C T_1^{\beta+1} \end{aligned}$$

$$-\alpha\lambda h_C T_1 T^{\beta}+\left(\frac{3ab\alpha\beta h_C}{2(5\beta+6)}\right)T^{\beta+2}+\left(\frac{ab\alpha h_C}{\beta+2}\right)T_1^{\beta+2}-\left(\frac{ab\alpha h_C}{2}\right)T_1^2T^{\beta}\Bigg]-\left(\frac{1}{T^2}\right)\Big[O_C$$
$$-\left(\frac{\lambda h_C}{2}\right)T_1^2-\left(\frac{a h_C}{2}\right)T^2+\lambda h_C TT_1-\left(\frac{ab h_C}{3}\right)T_1^3-\left(\frac{ab h_C}{6}\right)T^3+\left(\frac{ab h_C}{2}\right)TT_1^2-\left(\frac{\alpha\lambda d_C}{\beta+1}\right)T_1^{\beta+1}$$
$$-\left(\frac{a\alpha\beta d_C}{\beta+1}\right)T^{\beta+1}-\left\{\frac{\alpha(a-\lambda)h_C(37\beta+10)+ab\alpha d_C(7\beta+2)}{4(4\beta+1)}\right\}T_1^{\beta+2}-\alpha\lambda d_C T_1 T^{\beta}$$
$$+\left\{\frac{a\alpha\beta(2h_C+bd_C)}{4(2\beta+1)}\right\}T^{\beta+2}-\left(\frac{ab\alpha d_C}{2}\right)T_1^2T^{\beta}+\left\{\frac{a\alpha(2\beta+3)-2\alpha\lambda(\beta+1)}{\beta+1}\right\}h_C TT_1^{\beta+1}$$
$$-\left(\frac{\alpha\lambda h_C}{\beta+1}\right)T_1T^{\beta+1}-\left(\frac{ab\alpha\beta h_C}{7\beta+3}\right)T_1^{\beta+3}+\left(\frac{ab\alpha\beta h_C}{2(5\beta+6)}\right)T^{\beta+3}+\left(\frac{ab\alpha h_C}{\beta+2}\right)TT_1^{\beta+2}$$
$$-\left(\frac{ab\alpha h_C}{2(\beta+1)}\right)T_1^2T^{\beta+1}\Bigg] \tag{14}$$

$$\frac{\partial^2 TC(T_1,T)}{\partial T_1^2}=\left(\frac{1}{T}\right)\Big[-\lambda h_C-2abh_C T_1+abh_C T-\alpha\beta\lambda d_C T_1^{\beta-1}-ab\alpha d_C T^{\beta}$$
$$-(\beta+1)\left\{\frac{\alpha(a-\lambda)h_C(21\beta+5)+ab\alpha d_C(4\beta+1)}{4\beta+1}\right\}T_1^{\beta}+\alpha\beta\{a\,(2\beta+3)-2\lambda\,(\beta+1)\}h_C TT_1^{\beta-1}$$
$$-\left(\frac{3ab\alpha\beta(\beta+2)h_C}{7\beta+3}\right)T_1^{\beta+1}-\left(\frac{ab\alpha h_C}{\beta+1}\right)T^{\beta+1}+ab\alpha(\beta+1)h_C TT_1^{\beta}\Big] \tag{15}$$

$$\frac{\partial^2 TC(T_1,T)}{\partial T_1\partial T}=\left(\frac{1}{T}\right)\Big[\lambda h_C+abh_C T_1-\alpha\beta\lambda d_C T^{\beta-1}-ab\alpha\beta d_C T_1T^{\beta-1}+\{a\alpha(2\beta+3)-2\alpha\lambda(\beta+1)\}h_C T_1^{\beta}$$
$$-\alpha\lambda h_C T^{\beta}+ab\alpha h_C T_1^{\beta+1}-ab\alpha h_C T_1T^{\beta}\Big]+\left(\frac{2}{T^3}\right)\Big[-\lambda h_C T_1+\lambda h_C T-abh_C T_1^2$$
$$+abh_C TT_1-\alpha\lambda d_C T_1^{\beta}-(\beta+2)\left\{\frac{\alpha(a-\lambda)(37\beta+10)h_C+ab\alpha d_C(7\beta+2)}{4(4\beta+1)}\right\}T_1^{\beta+1}$$
$$-\alpha\lambda d_C T^{\beta}-ab\alpha d_C T_1T^{\beta}+\{a\alpha(2\beta+3)-2\alpha\lambda(\beta+1)\}h_C TT_1^{\beta}-\left(\frac{\alpha\lambda h_C}{\beta+1}\right)T^{\beta+1}$$
$$-\left(\frac{ab\alpha\beta(\beta+3)h_C}{7\beta+3}\right)T_1^{\beta+2}+ab\alpha h_C TT_1^{\beta+1}-\left(\frac{ab\alpha h_C}{\beta+1}\right)T_1T^{\beta+1}\Big] \tag{16}$$

$$\frac{\partial^2 TC(T_1,T)}{\partial T^2}=\left(\frac{1}{T}\right)\Big[-ah_C-abh_C T-a\alpha\beta^2 d_C T^{\beta-1}+\left(\frac{a\alpha\beta(\beta+1)(2h_C+bd_C)}{2(2\beta+1)}\right)T^{\beta}$$
$$-\alpha\beta\lambda(\beta-1)d_C T_1T^{\beta-2}-\left(\frac{ab\alpha\beta(\beta-1)d_C}{2}\right)T_1^2T^{\beta-2}-\alpha\beta\lambda h_C T_1T^{\beta-1}+\left(\frac{3ab\alpha\beta(\beta+2)h_C}{2(5\beta+6)}\right)T^{\beta+1}$$
$$-\left(\frac{ab\alpha\beta h_C}{2}\right)T_1^2T^{\beta-1}\Big]-\left(\frac{1}{T^2}\right)\Big[-2ah_C T+2\lambda h_C T_1-abh_C T^2+abh_C T_1^2-2a\alpha\beta d_C T^{\beta}$$
$$+\left(\frac{a\alpha\beta(\beta+3)(2h_C+bd_C)}{4(2\beta+1)}\right)T^{\beta+1}-2\alpha\beta\lambda d_C T_1T^{\beta-1}-\left(\frac{ab\alpha\beta d_C}{2}\right)T_1^2T^{\beta-1}$$

$$+2\left\{\frac{a\alpha(2\beta+3)-2\alpha\lambda(\beta+1)}{\beta+1}\right\}h_C T_1^{\beta+1}-2\alpha\lambda h_C T_1 T^{\beta}+\left(\frac{ab\alpha\beta(\beta+6)h_C}{2(5\beta+6)}\right)T^{\beta+2}+\left(\frac{2ab\alpha h_C}{\beta+2}\right)T_1^{\beta+2}$$

$$-ab\alpha h_C T_1^2 T^{\beta}\Big]+\left(\frac{2}{T^3}\right)\Big[o_C-\left(\frac{\lambda h_C}{2}\right)T_1^2-\left(\frac{a h_C}{2}\right)T^2+\lambda h_C T T_1-\left(\frac{ab h_C}{3}\right)T_1^3-\left(\frac{ab h_C}{6}\right)T^3$$

$$+\left(\frac{ab h_C}{2}\right)TT_1^2-\left(\frac{\alpha\lambda d_C}{\beta+1}\right)T_1^{\beta+1}-\left(\frac{a\alpha\beta\lambda d_C}{\beta+1}\right)T^{\beta+1}-\left\{\frac{\alpha(a-\lambda)h_C(37\beta+10)+ab\alpha d_C(7\beta+2)}{4(4\beta+1)}\right\}T_1^{\beta+2}$$

$$+\left(\frac{a\alpha\beta(2h_C+b d_C)}{4(2\beta+1)}\right)T^{\beta+2}-\alpha\lambda d_C T_1 T^{\beta}-\left(\frac{ab\alpha d_C}{2}\right)T_1^2 T^{\beta}-\left(\frac{\alpha\lambda d_C}{\beta+1}\right)T_1 T^{\beta+1}$$

$$+\left\{\frac{a\alpha(2\beta+3)-2\alpha\lambda(\beta+1)}{\beta+1}\right\}h_C T T_1^{\beta+1}-\left(\frac{ab\alpha\beta\lambda h_C}{7\beta+3}\right)T_1^{\beta+3}+\left(\frac{ab\alpha\beta h_C}{2(5\beta+6)}\right)T^{\beta+3}$$

$$+\left(\frac{ab\alpha h_C}{\beta+2}\right)TT_1^{\beta+2}-\left(\frac{ab\alpha\lambda h_C}{2(\beta+1)}\right)T_1^2 T^{\beta+1}\Big] \tag{17}$$

## 4. Numerical Example

Let we consider the following data for parameters of the model in appropriate units

$a=10,\quad b=0.3,\quad \alpha=3,\quad \beta=1,\quad o_C=15,\quad h_C=0.4,\quad d_C=0.4,\quad \lambda=500$

## 5. Tables & Figures

**Table 1.** *variation of total inventory cost with respect to* $\alpha$

| $\alpha$ | $T_1$ | $T$ | $TC(T_1,T)$ |
|---|---|---|---|
| 3 | 13144.30 | 20814.60 | $4.0986\times10^{11}$ |
| 6 | 13169.10 | 20854.80 | $7.7719\times10^{11}$ |
| 9 | 13177.30 | 20868.30 | $1.1941\times10^{12}$ |
| 12 | 13169.90 | 20854.60 | $1.5875\times10^{12}$ |
| 15 | 13171.50 | 20858.70 | $1.9861\times10^{12}$ |

From this table, we see that as we increase the parameter $\alpha$ , then the values of $T_1$, $T$ and $TC(T_1,T)$ are increased.

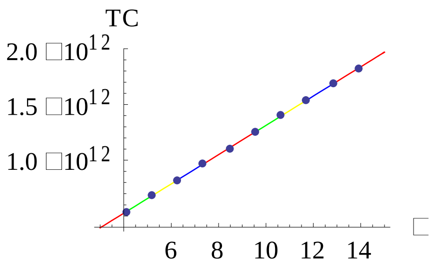



**Figure 2.** *variation in* $TC(T_1,T)$ *with* $\alpha$ ,

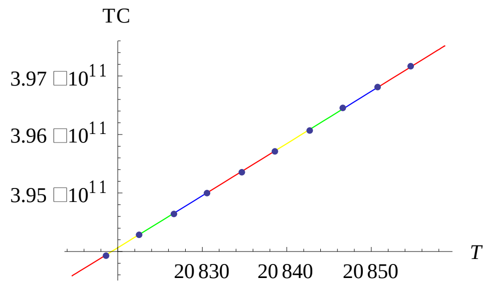



**Figure 3.** *variation in* $TC(T_1,T)$ *with* $T$

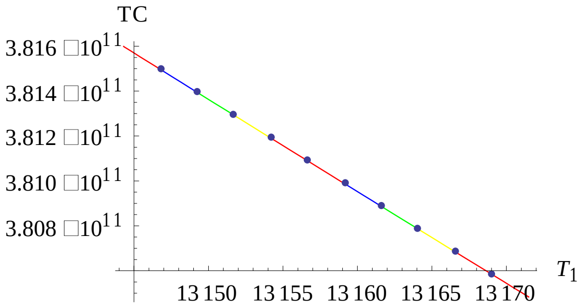
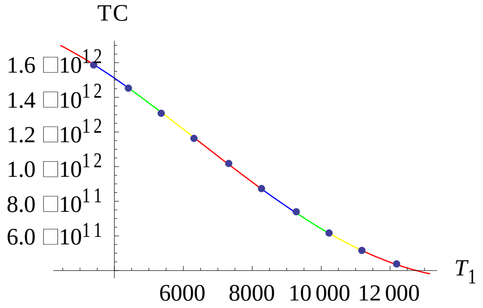



**Figure 4.** variation in $TC(T_1,T)$ with $T_1$

**Table 2.** *variation in total inventory cost with respect to* $a$

| $a$ | $T_1$ | $T$ | $TC(T_1,T)$ |
|---|---|---|---|
| 10 | 13144.30 | 20814.60 | $4.0986\times10^{11}$ |
| 20 | 6466.42 | 10240.60 | $9.4303\times10^{10}$ |
| 30 | 4232.15 | 6702.34 | $3.9498\times10^{10}$ |
| 40 | 3115.02 | 4933.23 | $2.1079\times10^{10}$ |
| 50 | 2444.75 | 3871.77 | $1.2718\times10^{10}$ |

From this table, we see that as we increase the parameter $a$, the values of $T_1$, $T$ and $TC(T_1,T)$ are decreased.



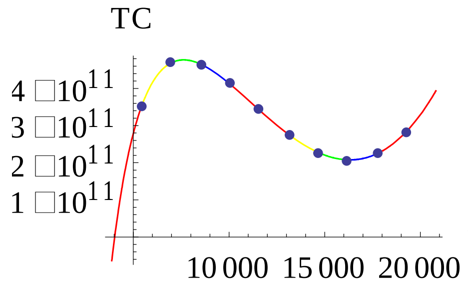
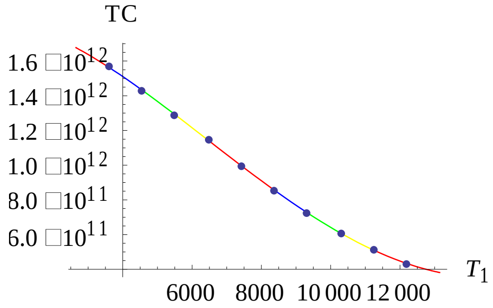




**Figure 5.** *variation in total cost with* $T_1$, **Figure 6.** *variation in total cost with* $T$

**Table 3.** *variation in total inventory cost with respect to b*

| $b$ | $T_1$ | $T$ | $TC(T_1,T)$ |
|---|---|---|---|
| 0.3 | 13144.30 | 20814.60 | $4.0986\times10^{11}$ |
| 0.6 | 6594.29 | 10442.00 | $9.9483\times10^{11}$ |
| 0.9 | 4402.65 | 6970.84 | $4.3301\times10^{10}$ |
| 1.2 | 3306.82 | 5235.29 | $2.4915\times10^{10}$ |
| 1.5 | 2649.33 | 4193.95 | $1.6192\times10^{10}$ |

From this table, we see that as we increase the parameter $b$, then the values of $T_1$, $T$ and $TC(T_1,T)$ are decreased.



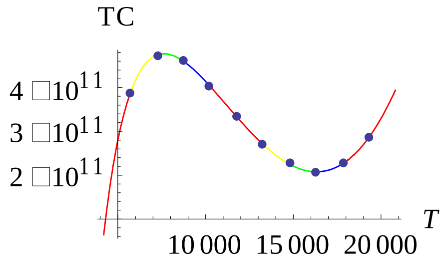




**Figure 7.** *variation in total cost with* $T_1$, **Figure 8.** *variation in total cost with* $T$

These are the numerical values of second order derivatives

$$\frac{\partial^2 TC(T_1,T)}{\partial T_1^2}=46125.30,\qquad \frac{\partial^2 TC(T_1,T)}{\partial T_1 \partial T}=-24397.60,\qquad \frac{\partial^2 TC(T_1,T)}{\partial T^2}=9.3129\times10^6$$

## 6. Conclusion

In this paper, we have developed a production inventory model for weibull deteriorating items with two components of demand rate. We see that the parameters $\alpha$ is more sensitive than the parameters $a$

and $b$. This is the reason that the products having finite life to be sold as soon as possible in the market. Therefore, it is very practical and suitable situation for the products of our daily lives and the firm/retailer wants to increase their demand.

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