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Hyper Wiener Index of Concatenated Pentagons in Two Rows

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Abstract: *The major task of QSAR research is to transfer the chemical formula of a molecular graph into numerical format. Topological index is an effective tool to do this. Pentachains in two rows include chemical graph of Dodecahydropentaleno[1,6-cd]pentalene. In this paper, the explicit formulae for hyper wiener index of pentachains in two rows of different lengths are established.*

Keywords: *Topological index, Hyper wiener.*

1. Introduction

A pentachain is a graph formed of concatenated 5-cycles. Ivan Gutman and others [1] in 2005 presented the study of concatenated 5-cycles in one row and obtained explicit formulas for Schultz and modified Szeged indices of pentachains in one row and two rows.

The Hyper wiener index of a graph G= (V, E) is defined as $WW(G) = \frac{1}{2}\sum_{i<j}\left(d_{ij} + d_{ij}^2\right)$.

2. Notation

5-cycles can be concatenated in two rows as shown in figures 1,2 (we call these cases as Straight chaining cases) or as in figure 3 (we call this case as Alternate chaining case).

In straight chaining (first case) the graph consisting of 5-cycles in two rows with 'b' cycles in row 1 and 'a' cycles in row 2 denoted by G(a,b,S_1) as shown below.

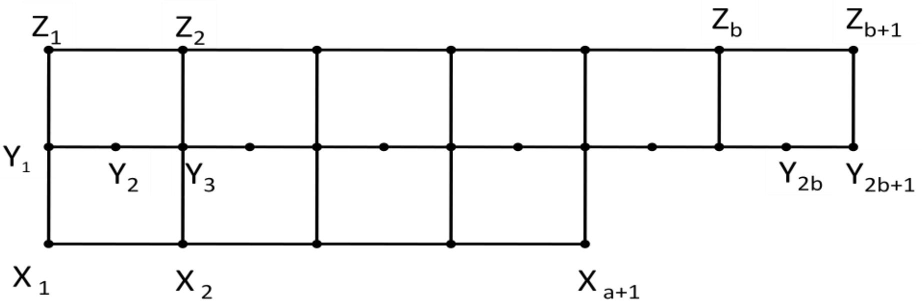

Figure 1. Graph of G (a, b, S_1)

In straight chaining (second case) the graph consisting of 5-cycles in two rows with 'b' cycles in row 1 and 'a' cycles in row 2 denoted by G(a,b,S_2) is as shown below.

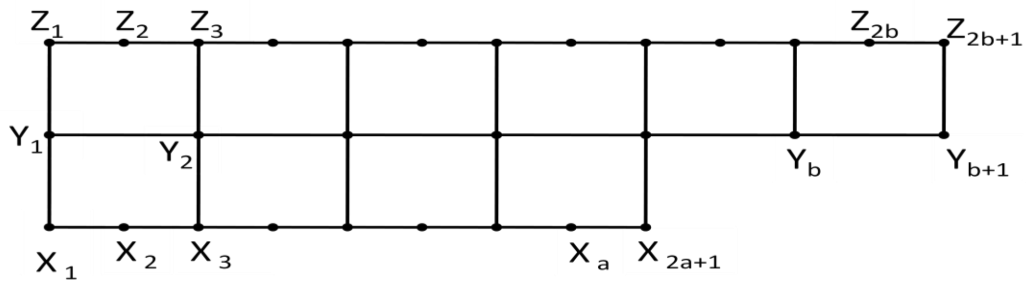

Figure 2. Graph of G (a, b, S_2)

In alternate chaining we denote the graph consisting of 5-cycles in two rows with 'b' cycles in row 1 and 'a' cycles in row 2, as shown below by G(a,b,A).

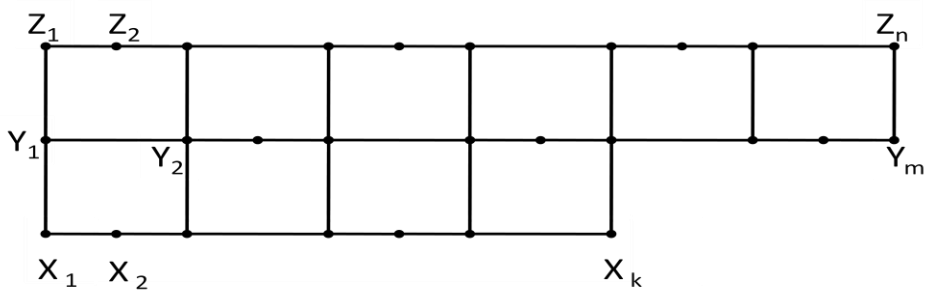

Figure 3. Graph of G (a, b, A)

Here, $k=\dfrac{3a+2}{2}, m=n=\dfrac{3a+2}{2}$, if a is even and b is even;

$k=\dfrac{3a+2}{2}, m=\dfrac{3a+1}{2}, n=\dfrac{3a+3}{2}$, If a is even and b is odd;

$k=\dfrac{3a+3}{2}, m=n=\dfrac{3a+2}{2}$, If a is odd and b is even;

$k=\dfrac{3a+3}{2}, m=\dfrac{3a+1}{2}, n=\dfrac{3a+2}{2}$, if a is odd and b is odd.

3. Algorithm

First we explain the algorithm we adopted for the computation of Hyper wiener index of G(a,b,S_1) with b>a.

1. Form the set of vertices $A=\{x_1,x_2,...,x_{a+1}\}, B=\{y_1,y_2,...,y_{2a+1}\}, C=\{z_1,z_2,...,z_{a+1}\}$ and take $WW=0$.
2. Compute the Hyper wiener index (say n_1) for the sub graph generated by $B\cup C$ using the formula $WW(G(a,s))$ and $WW=WW+n_1$.
3. Compute $\sum_{u,v\in A} d(u,v)$, the sum of the shortest distances between vertices of A (say n_2), $WW=WW+\dfrac{n_2}{2}$.

4. Compute $\sum_{u\in A, v\in B} d(u,v)$, the sum of the shortest distances between vertices of A and B (say n_3), $WW = WW + \frac{n_3}{2}$.

5. Compute $\sum_{u\in A, v\in C} d(u,v)$, the sum of the shortest distances between vertices of A and C (say n_4), $WW = WW + \frac{n_4}{2}$.

6. Compute $\sum_{u,v\in A} d^2(u,v)$, the squares of the sum of the shortest distances between vertices of A (say n_5), $WW = WW + \frac{n_5}{2}$.

7. Compute $\sum_{u\in A, v\in B} d^2(u,v)$, the squares of the sum of the shortest distances between vertices of A and B (say n_6), $WW = WW + \frac{n_6}{2}$.

8. Compute $\sum_{u\in A, v\in C} d^2(u,v)$, the squares of the sum of the shortest distances between vertices of A and C (say n_7), $WW = WW + \frac{n_7}{2}$.

9. WW is the Hyper wiener index of the graph $G(a,b,S_1)$.

By modifying the above algorithms suitably, Hyper wiener index for all other cases can be computed.

4. Main Results

Theorem 4.1.

$$
\begin{aligned}
WW(G(a,b,S_1)) &= 53 \; if \; a=b=1. \\
&= 165 \; if \; a=1, b=2. \\
&= \frac{1}{6}\left(4a^4+44a^3+182a^2-62a+180\right) if \; a=b\geq 2. \\
&= \frac{1}{6}\left(4a^4+56a^3+305a^2+361a+264\right) if \; b=a+1, a\geq 2. \\
&= \frac{1}{8}\left(3b^4+42b^3+221b^2+54b-144\right) if \; a=1, b>2.
\end{aligned}
$$

Proof: For a=b=1 and b=a+1=2 the results are trivial. From the following lemmas (4.2 to 4.4) the proof of the theorem is clear.

Lemma 4.2. $WW(G(a,a,S_1)) = \frac{1}{6}\left(4a^4+44a^3+182a^2-62a+180\right) if \; a\geq 2.$

Proof: Let the vertex set of $G(a,a,S_1)$ be $A\cup B\cup C$, where

$A=\{x_1, x_2, ..., x_{a+1}\}, B=\{y_1, y_2, ..., y_{2a+1}\}, C=\{z_1, z_2, ..., z_{a+1}\}.$

Now

$$WW(G(a,a,S_1)) = WW(G(a,S)) + \frac{1}{2}\{\sum_{u,v\in A} d(u,v) + \sum_{u\in A,v\in B} d(u,v) + \sum_{u\in A,v\in C} d(u,v)$$
$$+ \sum_{u,v\in A} d^2(u,v) + \sum_{u\in A,v\in B} d^2(u,v) + \sum_{u\in A,v\in C} d^2(u,v)\}.$$
$$= \frac{1}{8}\left(3a^4 + 34a^3 + 145a^2 - 190a + 208\right) + \frac{1}{2}(\{\sum_{i=1}^{a}\left(a-(i-1)\right)(i)\} + \{(a+1)(1) +$$
$$\sum_{i=2}^{a}\left(a-(i-2)\right)(4i) + 4(a+1)\} + \{(a+1)(2) + \sum_{i=3}^{a+1}\left(2a-(2i-6)\right)(i) + 2(a+2)\}$$
$$+\{\sum_{i=1}^{a}\left(a-(i-1)\right)(i)^2\} + \{(a+1)(1)^2 + 4\sum_{i=2}^{a}\left(a-(i-2)\right)(i)^2 + 4(a+1)^2\}$$
$$+\{(a+1)(2)^2 + \sum_{i=3}^{a+1}\left(2a-(2i-6)\right)(i)^2 + 2(a+2)^2\}).$$
$$= \frac{1}{8}\left(3a^4 + 34a^3 + 145a^2 - 190a + 208\right) + \frac{1}{2}\{\frac{1}{6}\left(a^3 + 3a^2 + 2a\right) +$$
$$\frac{1}{3}\left(2a^3 + 12a^2 + 13a + 3\right) + \frac{1}{3}\left(a^3 + 9a^2 + 14a + 6\right) + \frac{1}{12}\left(a^4 + 4a^3 + 5a^2 + 2a\right) +$$
$$\frac{1}{3}\left(a^4 + 8a^3 + 23a^2 + 19a + 3\right) + \frac{1}{6}\left(a^4 + 12a^3 + 53a^2 + 66a + 24\right)\}.$$
$$= \frac{1}{6}\left(4a^4 + 44a^3 + 182a^2 - 62a + 180\right).$$

Lemma 4.3. $WW(G(a,a+1,S_1)) = \frac{1}{6}\left(4a^4 + 56a^3 + 305a^2 + 361a + 264\right) \textit{if } b = a+1,\ a \geq 2.$

Proof: Let the vertex set of $G(a,a+1,S_1)$ be $A \cup B \cup C,$ where

$A = \{x_1, x_2, ..., x_{a+1}\}, B = \{y_1, y_2, ..., y_{2a+1}\}, C = \{z_1, z_2, ..., z_{a+1}\}.$

Now

$$WW(G(a,a+1,S_1)) = WW(G(b,S)) + \frac{1}{2}\{\sum_{u,v\in A} d(u,v) + \sum_{u\in A,v\in B} d(u,v) + \sum_{u\in A,v\in C} d(u,v)$$
$$+ \sum_{u,v\in A} d^2(u,v) + \sum_{u\in A,v\in B} d^2(u,v) + \sum_{u\in A,v\in C} d^2(u,v)\}.$$
$$= \frac{1}{8}\left(3a^4 + 34a^3 + 145a^2 - 190a + 208\right) + \frac{1}{2}(\{\sum_{i=1}^{a}\left(a-(i-1)\right)(i)\} + \{(a+1)(1)$$
$$+(4a+1)(2) + \sum_{i=3}^{a+2}\left(4a-(4i-10)\right)(i) + (1)(a+3)\} + \{(a+1)(2) +$$
$$+\sum_{i=3}^{a+1}\left(2a-(2i-7)\right)(i)\} + \{\sum_{i=1}^{a}\left(a-(i-1)\right)(i)^2\} + \{(a+1)(1)^2 + (4a+1)\left(2^2\right)$$
$$+\sum_{i=3}^{a+2}\left(4a-(4i-10)\right)(i)^2 + 1(a+3)^2\} + \{(a+1)(2)^2 + \sum_{i=3}^{a+1}\left(2a-(2i-7)\right)(i)^2\}).$$

$$=\frac{1}{8}\left(3a^4+34a^3+145a^2-190a+208\right)+\frac{1}{2}\{\frac{1}{6}\left(a^3+3a^2+2a\right)$$
$$+\frac{1}{3}\left(2a^3+15a^2+31a+18\right)+\frac{1}{3}\left(8a^3+72a^2+124a+81\right)+\frac{1}{12}\left(a^4+4a^3+5a^2+2a\right)$$
$$+\frac{1}{3}\left(a^4+10a^3+41a^2+74a+42\right)+\frac{1}{6}\left(a^4+14a^3+74a^2+139a+78\right)\}.$$
$$=\frac{1}{6}\left(4a^4+56a^3+305a^2+361a+264\right).$$

Lemma 4.4. $WW(G(1,b,S_1))=\frac{1}{8}\left(3b^4+42b^3+221b^2+54b-144\right) if\ a=1,\ b>2.$

Proof: Let the vertex set of $G(1,b,S_1)$ be $A\bigcup B\bigcup C,$ where

$A=\{x_1,x_2\},B=\{y_1,y_2,...,y_{2b+1}\},C=\{z_1,z_2,...,z_{b+1}\}$.

Now

$$WW(G(1,b+1,S_1))=WW(G(b,S))+\frac{1}{2}\{\sum_{u,v\in A}d(u,v)+\sum_{u\in A,v\in B}d(u,v)+\sum_{u\in A,v\in C}d(u,v)$$
$$+\sum_{u,v\in A}d^2(u,v)+\sum_{u\in A,v\in B}d^2(u,v)+\sum_{u\in A,v\in C}d^2(u,v)\}.$$
$$=\frac{1}{8}\left(3a^4+34a^3+145a^2-190a+208\right)+\frac{1}{2}(1+\{2(1)+5(2)+2\sum_{i=3}^{5}i+3(6)$$
$$+4\sum_{i=7}^{b+2}i+2(b+3)\}+\{2\sum_{i=1}^{b+1}i+(b+3)\}+\{2(1)^2+5(2)^2+2\sum_{i=3}^{5}i^2+3(6)^2$$
$$+4\sum_{i=7}^{b+2}i^2+2(b+3)^2\}+\{2(2)^2+3(3)^2+2\sum_{i=4}^{b+1}i^2+(b+2)^2\}.$$
$$=\frac{1}{8}\left(3a^4+34a^3+145a^2-190a+208\right)+\frac{1}{2}\{(1+\left(2b^2+12b-12\right)+\left(b^2+4b+5\right)$$
$$+1+\frac{1}{3}\left(4b^3+36b^2+110b-288\right)+\frac{1}{3}\left(2b^3+12b^2+25b+39\right).$$
$$=\frac{1}{8}\left(3b^4+42b^3+221b^2+54b-144\right).$$

Theorem 4.5.

$$WW(G(a,b,S_2))=91\ if\ a=b=1.$$
$$=\frac{1}{24}\left(25a^4+310a^3+1427a^2-658a+1320\right)\ if\ a=b\geq 2.$$
$$=\frac{1}{8}\left(3b^4+46b^3+229b^2+10b+480\right)\ if\ a=1,b>1.$$
$$=\frac{1}{8}\left(3b^4+56b^3+257b^2+46b+1264\right)\ if\ a=2,b>1.$$

Proof: For a=b=1 the result is trivial. From the following lemmas (4.6 to 4.8) the proof of the theorem is clear.

Lemma 4.6. $WW(G(a,a,S_2)) = \frac{1}{24}\left(25a^4 + 310a^3 + 1427a^2 - 658a + 1320\right)$ *if* $a \geq 2.$

Proof: Let the vertex set of $G(a,a,S_2)$ be $A \cup B \cup C,$ where

$$A = \{x_1, x_2, ..., x_{2a+1}\}, B = \{y_1, y_2, ..., y_{a+1}\}, C = \{z_1, z_2, ..., z_{2a+1}\}.$$

Now

$$\begin{aligned}
WW(G(a,a,S_2)) &= WW(G(a,S)) + \frac{1}{2}\{\sum_{u,v\in A} d(u,v) + \sum_{u\in A, v\in B} d(u,v) + \sum_{u\in A, v\in C} d(u,v) \\
&\quad + \sum_{u,v\in A} d^2(u,v) + \sum_{u\in A, v\in B} d^2(u,v) + \sum_{u\in A, v\in C} d^2(u,v)\}. \\
&= \frac{1}{8}\left(3a^4 + 34a^3 + 145a^2 - 190a + 208\right) + \frac{1}{2}(\{\sum_{i=1}^{4}\left(2a - (i-1)\right)(i) + (3a-6)(5) \\
&\quad + \sum_{i=6}^{a+1}\left(4a - (4i-12)\right)(i) + 4(a+2)\} + \{(a+1)(1) + 4\sum_{i=2}^{a+1}\left(a - (i-2)\right)(i)\} \\
&\quad + \{(a+1)(2) + 6a(3) + (9a-8)(4) + 8\sum_{i=5}^{a+2}\left(a - (i-3)\right)(i)\} + \{\sum_{i=1}^{4}\left(2a - (i-1)\right)(i)^2 \\
&\quad + (3a-6)(5)^2 + \sum_{i=6}^{a+1}\left(4a - (4i-12)\right)(i)^2 + 4(a+2)^2\} + \{(a+1)(1)^2 + \\
&\quad \sum_{i=2}^{a+2} 4\left(a - (i-2)\right)(i)^2\} + \{(a+1)(2)^2 + 6a(3)^2 + (9a-8)(4)^2 + 8\sum_{i=5}^{a+2}\left(a - (i-3)\right)(i)^2\}). \\
&= \frac{1}{8}\left(3a^4 + 34a^3 + 145a^2 - 190a + 208\right) + \frac{1}{2}\{\frac{1}{6}\left(2a^3 + 18a^2 - 23a + 18\right) \\
&\quad + \frac{1}{3}\left(2a^3 + 12a^2 + 13a + 3\right) + \frac{1}{3}\left(4a^3 + 36a^2 + 32a + 6\right) + \frac{1}{3}\left(a^4 + 12a^3 + 53a^2 - 153a + 132\right) \\
&\quad + \frac{1}{3}\left(a^4 + 8a^3 + 23a^2 + 19a + 3\right) + \frac{1}{3}\left(2a^4 + 24a^3 + 106a^2 + 90a + 12\right)\}. \\
&= \frac{1}{24}\left(25a^4 + 310a^3 + 1427a^2 - 658a + 1320\right).
\end{aligned}$$

Lemma 4.7. $WW(G(1,b,S_2)) = \frac{1}{8}\left(3b^4 + 46b^3 + 229b^2 + 10b + 480\right)$ *if* $b > 1.$

Proof: Let the vertex set of $G(1,b,S_2)$ be $A \cup B \cup C,$ where

$$A = \{x_1, x_2, x_3\}, B = \{y_1, y_2, ..., y_{b+1}\}, C = \{z_1, z_2, ..., z_{2b+1}\}.$$

Now

$$\begin{aligned}
WW(G(1,b,S_2)) &= WW(G(b,S)) + \frac{1}{2}\{\sum_{u,v\in A} d(u,v) + \sum_{u\in A, v\in B} d(u,v) + \sum_{u\in A, v\in C} d(u,v) \\
&\quad + \sum_{u,v\in A} d^2(u,v) + \sum_{u\in A, v\in B} d^2(u,v) + \sum_{u\in A, v\in C} d^2(u,v)\}. \\
&= \frac{1}{8}\left(3a^4 + 34a^3 + 145a^2 - 190a + 208\right) + \frac{1}{2}(4 + \{2(1) + 5(2) + 3\sum_{i=3}^{b}(i) + 2(b+1)\}
\end{aligned}$$

$$+\{2(2)+8(3)+7(4)+6\sum_{i=5}^{b+1}(i)+4(b+2)\}+\{6+2(1)^2+5(2)^2+\sum_{i=3}^{b}i^2+2(b+1)^2\}$$
$$+\{2(2)^2+8(3)^2+7(4)^2+6\sum_{i=5}^{b+1}i^2+4(b+2)^2\}).$$
$$=\frac{1}{8}(3a^4+34a^3+145a^2-190a+208)+\frac{1}{2}\{4+\frac{1}{2}(3b^2+7b+10)+(3b^2+13b+10)$$
$$+(6)+\frac{1}{2}(2b^3+7b^2+9b+18)+(2b^3+13b^2+29b+34)\}.$$
$$=\frac{1}{8}(3b^4+46b^3+229b^3+10b+480).$$

Lemma 4.8. $WW(G(2,b,S_2))=\frac{1}{8}(3b^4+56b^3+257b^2+46b+1264)$ *if* $b>1.$

Proof: Let the vertex set of ` be $A\bigcup B\bigcup C,$ where

$A=\{x_1,x_2,...,x_5\},B=\{y_1,y_2,...,y_{b+1}\},C=\{z_1,z_2,...,z_{2b+1}\}.$

Now

$$WW(G(2,b,S_2))=WW(G(b,S))+\frac{1}{2}\{\sum_{u,v\in A}d(u,v)+\sum_{u\in A,v\in B}d(u,v)+\sum_{u\in A,v\in C}d(u,v)$$
$$+\sum_{u,v\in A}d^2(u,v)+\sum_{u\in A,v\in B}d^2(u,v)+\sum_{u\in A,v\in C}d^2(u,v)\}.$$
$$=\frac{1}{8}(3a^4+34a^3+145a^2-190a+208)+\frac{1}{2}(\{20\}+\{(a+1)(1)+(4a+1)(2)$$
$$+(4a-1)(3)+5\sum_{i=4}^{b-1}(i)+4b+2(b+1)\}+\{(a+1)(2)+7a(3)+8a(4)$$
$$+10\sum_{i=5}^{b}(i)+8(b+1)+4(b+2)\}+\{\sum_{i=1}^{4}(2b-(i-1))(i)^2+(3b-6)(5)^2$$
$$+\sum_{i=6}^{b+1}(4b-(4i-12))(i)^2+4(b+2)^2\}+\{(a+1)(1)^2+(4a+1)(2)^2$$
$$+(4a-1)(3)^2+5\sum_{i=4}^{b-1}(i)^2+4b^2+2(b+1)^{2`}\}+\{(a+1)(2)^2+7a(3)^2$$
$$+8a(4)^2+10\sum_{i=5}^{b}(i)^2+8(b+1)^2+4(b+2)^2\}).$$
$$=\frac{1}{8}(3a^4+34a^3+145a^2-190a+208)+\frac{1}{2}\{20+\frac{1}{2}(5b^2+7b+28)$$
$$+(5b^2+17b+28)+(50)+\frac{1}{6}(10b^3+21b^2+29b+204)$$
$$+\frac{1}{3}(10b^3+51b^2+101b+354)\}.$$
$$=\frac{1}{8}(3b^4+54b^3+257b^2+46b+1264).$$

Theorem 4.9.

$$WW(G(a,b,A)) = \frac{1}{128}\left(243a^4 + 1368a^3 + 2826a^2 + 7784a - 573\right), \textit{where } a = b \textit{ is odd}.$$
$$= \frac{1}{128}\left(243a^4 + 1260a^3 + 2340a^2 + 7088a - 2432\right), \textit{where } a = b \textit{ is even.}$$

Proof: The proof is clear from the following lemmas (4.10 to 4.11).

Lemma 4.10. $WW(G(a,a,A)) = \frac{1}{128}\left(243a^4 + 1368a^3 + 2826a^2 + 7784a - 573\right)$, where a is odd.

Proof: Let the vertex set of $G(a,a,A)$ be The $A \cup B \cup C$, where

$$A = \left\{x_1, x_2, \ldots, x_{\frac{3a+3}{2}}\right\}, B = \left\{y_1, y_2, \ldots, y_{\frac{3a+1}{2}}\right\}, C = \left\{z_1, z_2, \ldots, z_{\frac{3a+3}{2}}\right\}.$$

Now

$$WW(G(a,a,A)) = WW(G(a,A)) + \frac{1}{2}\{\sum_{u,v\in A} d(u,v) + \sum_{u\in A, v\in B} d(u,v) + \sum_{u\in A, v\in C} d(u,v)$$
$$+ \sum_{u,v\in A} d^2(u,v) + \sum_{u\in A, v\in B} d^2(u,v) + \sum_{u\in A, v\in C} d^2(u,v)\}.$$
$$= \frac{1}{32}\left(27a^4 + 126a^3 + 204a^2 + 354a - 71\right) + \frac{1}{2}(\{\sum_{i=1}^{\frac{3a+1}{2}}\left(\frac{3a+1}{2} - (i-1)\right)(i)\} + \{(a+1)(1)$$
$$+ 4a(2) + (4a-4)(3) + \sum_{i=4}^{\frac{3a+1}{2}}\left((3a-5) - (2i-8)\right)(i)\} + \{(a+1)(2) + (4a+2)(3)$$
$$+ \frac{1}{2}(9a-7)(4) + (4a-6)(5) + \sum_{i=6}^{\frac{3a+3}{2}}\left((3a-7) - (2i-12)\right)(i)\} + \{\sum_{i=1}^{\frac{3a+1}{2}}\left(\frac{3a+1}{2} - (i-1)(i)^2\right)\}$$
$$+ \{(a+1)(1)^2 + 4a(2)^2 + (4a-4)(3)^2 + \sum_{i=4}^{\frac{3a+1}{2}}\left((3a-5) - (2i-8)(i)^2\right)\} + \{(a+1)(2)^2$$
$$+ (4a+2)(3)^2 + \frac{1}{2}(9a-7)(4)^2 + (4a-6)(5)^2 + \sum_{i=6}^{\frac{3a+3}{2}}\left((3a-7) - (2i-12)\right)(i)^2\}).$$
$$= \frac{1}{32}\left(27a^4 + 126a^3 + 204a^2 + 354a - 71\right) + \frac{1}{2}\{\frac{1}{16}\left(9a^3 + 27a^2 + 23a + 5\right)$$
$$+ \frac{1}{8}\left(9a^3 + 27a^2 + 47a - 3\right) + \frac{1}{8}\left(9a^3 + 45a^2 + 127a + 27\right)$$
$$+ \frac{1}{64}\left(27a^4 + 108a^3 + 150a^2 + 84a + 15\right) + \frac{1}{32}\left(27a^4 + 108a^3 + 438a^2 + 1964a - 113\right)\}.$$
$$= \frac{1}{128}\left(243a^4 + 1368a^3 + 2826a^2 + 7784a - 573\right).$$

Lemma 4.11. $WW(G(a,a,A)) = \frac{1}{128}\left(243a^4 + 1260a^3 + 2340a^2 + 7088a - 2432\right)$, where a is even.

Proof: Let the vertex set of $G(a,a,A)$ be The $A \cup B \cup C$, where

$$A = \left\{x_1, x_2, \ldots, x_{\frac{3a+2}{2}}\right\}, B = \left\{y_1, y_2, \ldots, y_{\frac{3a+2}{2}}\right\}, C = \left\{z_1, z_2, \ldots, z_{\frac{3a+2}{2}}\right\}.$$

Now

$$WW(G(a,a,A)) = WW(G(a,A)) + \frac{1}{2}\{\sum_{u,v\in A} d(u,v) + \sum_{u\in A,v\in B} d(u,v) + \sum_{u\in A,v\in C} d(u,v)$$
$$+ \sum_{u,v\in A} d^2(u,v) + \sum_{u\in A,v\in B} d^2(u,v) + \sum_{u\in A,v\in C} d^2(u,v)\}.$$
$$= \frac{1}{32}\left(27a^4+126a^3+204a^2+360a-64\right) + \frac{1}{2}(\{\sum_{i=1}^{\frac{3a}{2}}\left(\frac{3a}{2}-(i-1)\right)(i)\} + \{(a+1)(1)$$
$$+4a(2)+(4a-4)(3)+\sum_{i=4}^{\frac{3a-4}{2}}\left((3a)-(2i-3)\right)(i)+5\left(\frac{3a-2}{2}\right)+3\left(\frac{3a}{2}\right)+\left(\frac{3a+2}{2}\right)\}$$
$$+\{(a+1)(2)+(4a)(3)+\left(\frac{9a-8}{2}\right)(4)+(4a-8)(5)+\sum_{i=6}^{\frac{3a+2}{2}}\left((3a-8)-((2i-12))(i)\right)\}$$
$$+\{\sum_{i=1}^{\frac{3a}{2}}\left(\frac{3a}{2}-(i-1)\right)(i)^2\}+\{(a+1)(1)^2+4a(2)^2+(4a-4)(3)^2+$$
$$+\sum_{i=4}^{\frac{3a-4}{2}}\left(3a-(2i-3)(i)^2\right)+5\left(\frac{3a-2}{2}\right)^2+\left(\frac{3a}{2}\right)^2+\left(\frac{3a+2}{2}\right)^2\}+\{(a+1)(1)^2+4a(2)^2$$
$$+(4a-4)(3)^2+\sum_{i=4}^{\frac{3a-4}{2}}\left(3a-(2i-3)\right)(i)^2\}+5\left(\frac{3a-2}{2}\right)^2+3\left(\frac{3a}{2}\right)^2+\left(\frac{3a+2}{2}\right)^2\}$$
$$+\{(a+1)(2)^2+4a(3)^2+\left(\frac{9a-8}{2}\right)(4)^2+(4a-8)(5)^2+\sum_{i=6}^{\frac{3a+2}{2}}\left((3a-8)-(2i-12)\right)(i)^2\}).$$
$$= \frac{1}{32}\left(27a^4+126a^3+204a^2+360a-64\right)+\frac{1}{2}\{\frac{1}{16}\left(9a^3+18a^2+8a\right)+\frac{1}{8}\left(9a^3+27a^2+50a\right)$$
$$+\frac{1}{8}\left(9a^3+36a^2+100a-16\right)+\frac{1}{64}\left(27a^4+72a^3+60a^2+16a\right)+$$
$$+\frac{1}{32}\left(27a^4+108a^3+168a^2+472a-128\right)+\frac{1}{32}\left(27a^4+144a^3+276a^2+1728a-896\right)\}.$$
$$= \frac{1}{128}\left(243a^4+1260a^3+2340a^2+7088a-2432\right).$$

5. Conclusion

The effective explicit formulas for Hyper Wiener Index of pentachains are determined successfully in two rows of different lengths. It is also established efficiently the transformation of chemical formula into numerical formula for the purpose of an analytical study. Moreover the pentachains in two rows has been established with the chemical graph of Dodecahydropentaleno[1,6-cd]pentalene.

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