Numerical Studies on the Effects of Variable Viscosity and Thermal Conductivity with Joules Dissipation on the MHD Micropolar Fluid Flow, Heat and Mass Transfer Past a Stretching Porous Surface Embedded in a Porous Medium

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Abstract: *The effects of variable viscosity and thermal conductivity on the MHD (Magnetohydrodynamics) fluid flow, heat and mass transfer past a stretching porous surface embedded in a porous medium are investigated numerically taking both viscous and Joules dissipation in to account. The micro-inertia density is considered as variable. Viscosity and thermal conductivity are assumed to be inverse linear function of temperature. Using similarity transformations partial differential equations are converted to ordinary one and the system is then solved using Runge-Kutta method. The effects of various parameters in the problem are shown graphically and skin-friction coefficient and rate of heat transfer are shown in tabular form.*

Keywords: *Micropolar fluid, variable viscosity and thermal conductivity, MHD Flow, porous medium, Joules dissipation, viscous dissipation.*

1. INTRODUCTION

In the mid of 1960, A.C. Eringen [1-3] introduce micropolar fluid theory. As the Navier-Stokes equations fails to describe the behavior of Newtonian fluids with suspended particles, Eringen introduce the concept of micropolar fluid. It is assumed to be illustrated by two independent vectors, the classical velocity and the micro-rotation vector. As well as its usual rigid motion, fluid points contained in a small volume element, can rotate about the centroid of the volume element. Liquid crystals, animal blood, other polymeric fluids, fluids containing minute amount additives, clouds with dust etc are some examples of micropolar fluids.

The problem of MHD flow and heat transfer over a stretching surface is important due to its applications in industry. For example, in the extrusion of a polymer sheet, the properties of the final product considerably depend on the rate of cooling. During the last few decades so many investigations have made. Stagnation flow of micropolar fluids with strong and weak interaction was studied by Guram G.S. *et al.* [4]. Ishak A. *et al.* [5] investigated the heat transfer over a stretching surface with variable heat flux in micropolar fluids. Effects of variable viscosity and thermal conductivity on the MHD flow of micropolar fluid past an accelerated infinite vertical insulated plate was studied by Thakur P.M. *et al.* [6].

Due to several applications like chemical industry, cooling of nuclear reactors etc. the study of heat transfer and the flow of a viscous fluid over a stretching surface have motivated considerable interest. In the presence of non-uniform heat source effects of thermal buoyancy and variable thermal conductivity in a power law fluid past a vertical stretching sheet was studied by Abel M.S. *et al.* [7].Effects of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet was studied by Chen C.H. [8]. Hazarika G.C. *et al.* [9] investigated the effects of variable viscosity and thermal conductivity on MHD mixed convective flow over a stretching surface with radiation. Kelson N.A. *et al.* [10] studied the effects of surface conditions on flow of micropolar fluid driven by a porous stretching sheet. The effects of variable viscosity and thermal conductivity on liquid film on an unsteady stretching surface was studied by Khound P.K. *et al.* [11]. Convective heat

transfer in an electrically conducting fluid at a stretching surface in uniform free stream was studied by Vajravelu K. *et al*. [12]. Ahmed N. *et al*. [13] investigated the MHD free and forced convection and mass transfer flow past a porous vertical plate. In the presence of magnetic field Sarma U. *et al*. [14] investigated the effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate. Effects of variable viscosity and thermal conductivity on magnetohydrodynamic free convection flow was investigated by Hazarika G.C. *et al*. [15] when the fluid past a stretching plate through porous medium with radiation.

The main objective of our present work is to extend the work of Anjali Devi S.P. *et al.* [16] to study the effects of viscous and Joules dissipation on MHD micropolar fluid flow, heat and mass transfer past a stretching porous surface embedded in a porous medium, considering viscosity and thermal conductivity as inverse linear functions of temperature. The problem is solved numerically using fourth order Runge-Kutta shooting algorithm with iteration process after reducing the partial differential equations to ordinary one using similarity transformations.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Two-dimensional, nonlinear, steady, MHD laminar boundary layer flow with heat and mass transfer of a viscous, incompressible and electrically conducting micropolar fluid over a porous surface embedded in a porous medium is considered for investigation. A uniform transverse magnetic field of strength B_0 is applied parallel to y-axis. Both viscous and Joules dissipation are taken into account. Consider a polymer sheet emerging out of a slit at x = 0, y = 0 and subsequently being stretched, as in a polymer extrusion process. Let us assume that the speed at a point in the plate is proportional to the power of its distance from the slit and the boundary layer approximations are applicable. Induced magnetic field, the external electric field and the electric field due to the polarization of charges are assumed to be negligible in writing the following equations.

2.1 Basic Equations

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \qquad \text{... (1)}$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(v \frac{\partial u}{\partial y}\right) + \frac{\kappa}{\rho}\left(\frac{\partial N}{\partial y} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B_0^2}{\rho} u - \frac{v}{K_p} u \qquad \qquad \text{... (2)}$$

Angular momentum equation

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\gamma}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \qquad \qquad \text{... (3)}$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho c_p} u^2 \qquad \qquad \text{... (4)}$$

Species continuity equation

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{1}{S_c} \frac{\partial}{\partial y}\left(v \frac{\partial c}{\partial y}\right) \qquad \qquad \text{... (5)}$$

The boundary conditions are

$$\left.\begin{array}{ll}
u = ax^m, v = v_w(x), \quad , \quad T = T_w(x) = T_\infty + T_0 x^n & \\
N = -n\frac{\partial u}{\partial y}, \quad C = C_w(x) = C_\infty + C_0 x^n & \text{at y = 0} \\
u \to 0, T \to T_\infty, \quad N \to 0, \quad C \to C_\infty & \text{as y} \to \infty
\end{array}\right\} \qquad \text{...(6)}$$

Assume the viscosity as inverse linear functions of temperature, as done by Lai and Kulacki [17], i.e.

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \delta(T - T_\infty)] \text{, or } \frac{1}{\mu} = a(T - T_r) \qquad \text{... (7)}$$

$$\text{where } a = \frac{\delta}{\mu_\infty} \text{ and } T_r = T_\infty - \frac{1}{\delta}$$

where T_r is transformed reference temperature corresponding to viscosity parameter, μ_∞ is the viscosity of the ambient fluid, a and T_∞ are constants whose values are depend upon the reference state and thermal property of the fluid.

Similarly

$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}[1 + \xi(T - T_\infty)] \text{, or } \frac{1}{\lambda} = b(T - T_c) \qquad \text{... (8)}$$

$$\text{where } b = \frac{\xi}{\lambda_\infty} \text{ and } T_c = T_\infty - \frac{1}{\xi}$$

where b and T_c are constants and their values depend on the reference state and thermal properties of the fluid.

Let us introduce the following non-dimensional variables

$$\left. \begin{array}{l} \psi(x,y) = \left[\frac{2v_\infty x U_w}{1+m}\right]^{1/2} f(\eta), \ \ \eta = \left[\frac{(1+m)U_w}{2v_\infty x}\right]^{1/2} y, \ \ N = \left(\frac{(m+1)U_w^3}{2v_\infty x}\right)^{\frac{1}{2}} h(\eta), \\[2mm] j = \frac{v_\infty x}{(m+1)U_w} i(\eta), \ \ v_w(x) = -\lambda_0 \left\{\frac{v_\infty a(1+m)}{2}\right\}^{1/2} x^{\frac{m-1}{2}} \end{array} \right\} \qquad \text{... (9)}$$

and non-dimensional form of temperature and the concentration as

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \ , \ \ g = \frac{C - C_\infty}{C_w - C_\infty} \qquad \text{... (10)}$$

where $\lambda_0 > 0$ for suction at the stretching plate and ψ is the stream function.

The velocity components are given by $u = \frac{\partial \psi}{\partial y}, \ \ v = -\frac{\partial \psi}{\partial x}$.

The equation of continuity (1) is identically satisfied. Substituting above transformations in equations (2) — (5), we get

$$\left(\frac{\theta_r}{\theta - \theta_r} - K\right) f''' = ff'' - \frac{2m}{m+1}f'^2 + \frac{\theta_r}{(\theta - \theta_r)^2}f''\theta' + \frac{1}{R_1}\frac{\theta_r}{\theta - \theta_r}f' + Kh' - Mf' \qquad \text{... (11)}$$

$$\left(1 + \frac{1}{2}K\right)h'' = 2(2h + f'')K_1 + \frac{3m-1}{m+1}hf' - h'f \qquad \text{... (12)}$$

$$\frac{\theta_c}{\theta - \theta_c}\theta'' = \frac{\theta_c}{(\theta - \theta_c)^2}\theta'^2 + P_r f\theta' - \frac{\theta_r}{\theta - \theta_r}E_c P_r f''^2 + E_c P_r Mf'^2 \qquad \text{... (13)}$$

$$g'' = \frac{1}{\theta - \theta_r}\theta'g' + S_c \frac{\theta - \theta_r}{\theta_r}g'f \qquad \text{... (14)}$$

and the boundary conditions (6) become

$$\left. \begin{array}{l} \eta = 0: f = \lambda_0, \ f' = 1, \ g = 1, \ \theta = 1, \ h = -nf'' \\ \eta \to \infty: f' = 0, \ \theta = 0, \ g = 0, h = 0 \end{array} \right\} \qquad \text{... (15)}$$

The important physical quantities of this problem are the skin friction coefficient c_f and local Nusselt number Nu_x which indicates physically the rate of surface shear stress and rate of heat transfer respectively.

The shear stress is given by

$$\tau_w = \left[(\mu + \kappa)\frac{\partial u}{\partial y} + \kappa N\right]_{y=0} \qquad \qquad \dots (16)$$

The skin friction coefficient c_f is defined as

$$c_f = \frac{2\,\tau_w}{\rho U_w^2} \qquad \qquad \dots (17)$$

The rate of heat transfer from the surface is given by

$$q_w = -\lambda\left[\frac{\partial T}{\partial y}\right]_{y=0} \qquad \qquad \dots (18)$$

$$Nu_x = \left(\frac{m+1}{2}\right)^{1/2} \frac{\theta_c}{1-\theta_c}(Re_x)^{1/2}\theta'(0) \qquad \qquad \dots (19)$$

where $Re_x = \dfrac{U_w x}{\nu_\infty}$ is the local Reynolds number.

3. RESULTS AND DISCUSSION

The system of ordinary differential equations (11) - (14) together with the boundary conditions (15) are solved numerically by using the Runge-Kutta fourth order shooting method. The numerical values of different parameters are taken as $M = 1, Pr = 0.7, \theta_c = 4, \theta_r = 3, K = 1,\ m = 1, R_1 = 10, K_1 = 0.5, E_c = 0.05, S_c = 0.22$ unless otherwise stated.

The variation in velocity distribution, angular velocity distribution, temperature distribution and species concentration distribution are illustrated in Figs. (1) – (13) with the variation of different parameters.

Figs. (1) – (3) represents the variations in dimensionless velocity distribution with viscosity parameter θ_r, magnetic parameter M and coupling constant parameter K. From Fig. (1) it is clear that the velocity decreases with the increase of viscosity parameter θ_r. From Fig. (2) it is observed that the velocity decreases due to increase of magnetic parameter M. Application of magnetic field forms a resistive force (Lorentz force) like drag force, retards the velocity. From Fig. (3) we observe that the velocity increase for the increase of coupling constant parameter K. It is due to the fact that viscosity decreases with the increase of coupling constant parameter K and as a result velocity increase.

Figs. (4) – (7) represents the variations in dimensionless angular velocity distribution for various values of coupling constant parameter K_1 and K, magnetic parameter M and viscosity parameter θ_r. We observe that angular velocity increase for the increase of all these parameters K_1, K, M and θ_r.

Figs. (8) – (10) represents the dimensionless temperature profile $\theta(\eta)$ with the variation of viscosity parameter θ_r, thermal conductivity parameter θ_c and magnetic parameter M. Figs. (8) and (10) shows that temperature increase for the increase of viscosity parameter θ_r and magnetic parameter M. As in both cases a resistive force will act against the fluid flow and as a result temperature will increase. From Fig. (9) we observe that the temperature decrease for the increase of the thermal conductivity parameter θ_c.

Figs. (11) – (13) shows the distribution representing species concentration profile within the boundary layer with the variation of θ_r, S_c and M. Figs. (11) and (12) shows that concentration decreases with the increase of θ_r and S_c. S_c connects velocity and concentration profiles. So when S_c increases, molecular mass diffusivity decreases resulting the decreasing of concentration. Fig. 13 shows that the concentration increases with the increasing values of M.
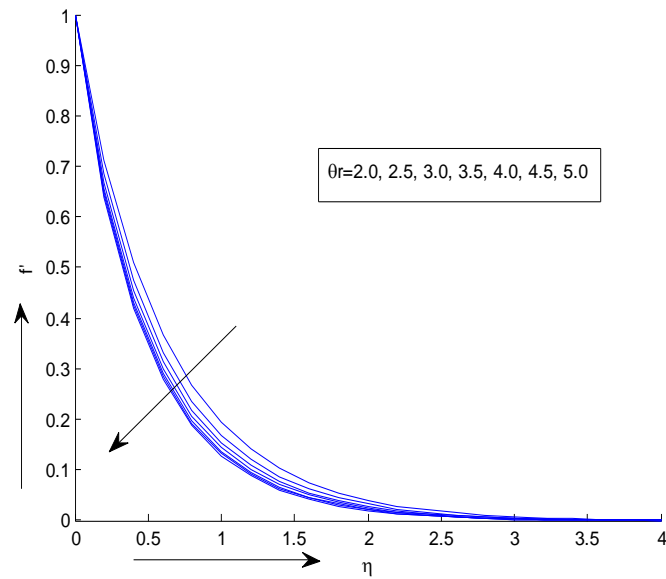
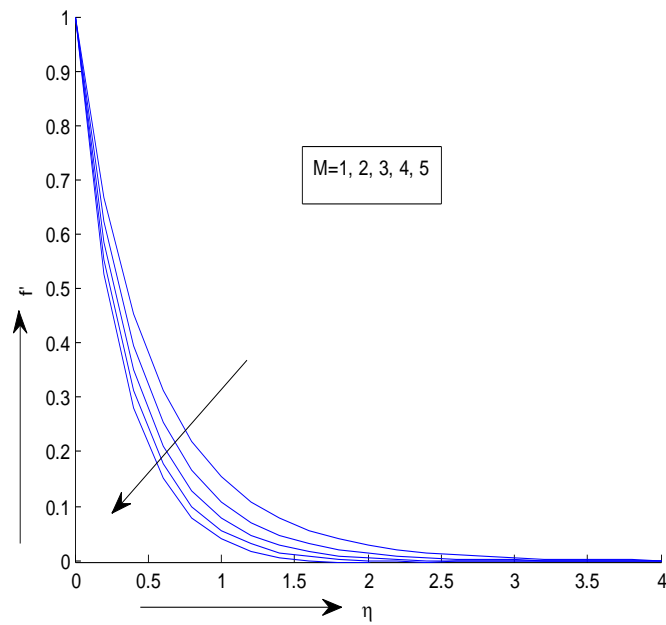## 3.1 Figures

Figure 1. *Velocity profile for various θ_r.*

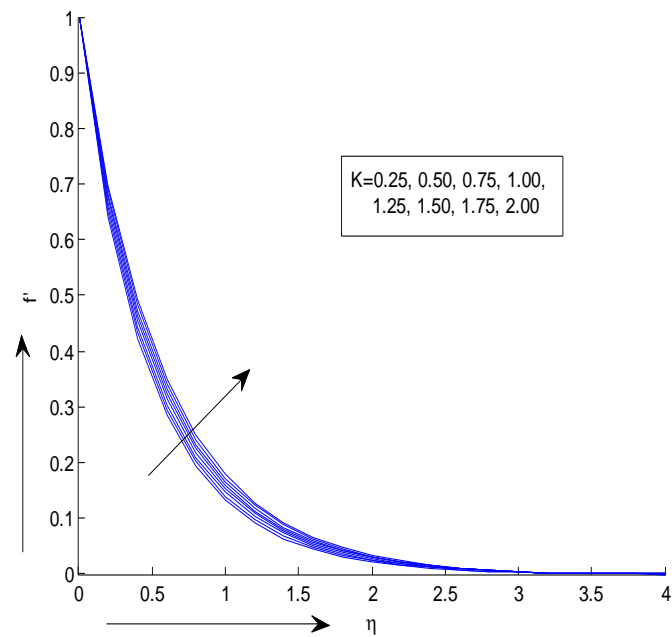**Figure 2.** *Velocity profile for various M*

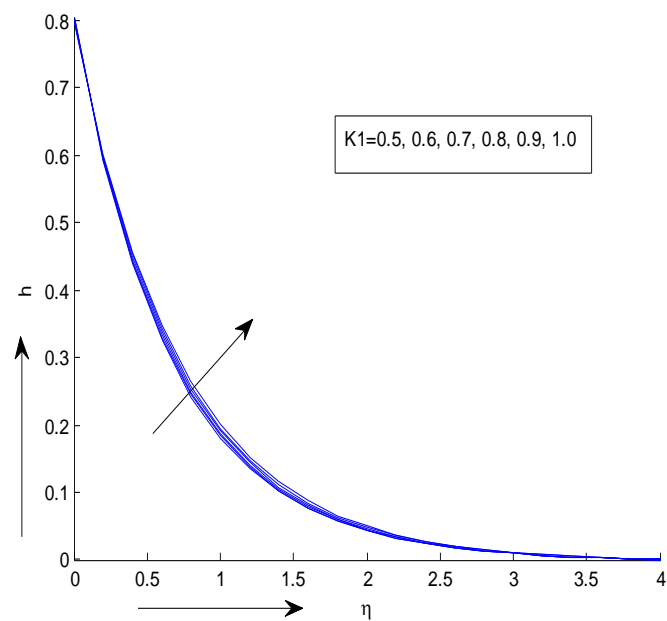**Figure 3.** *Velocity profile for various K.*

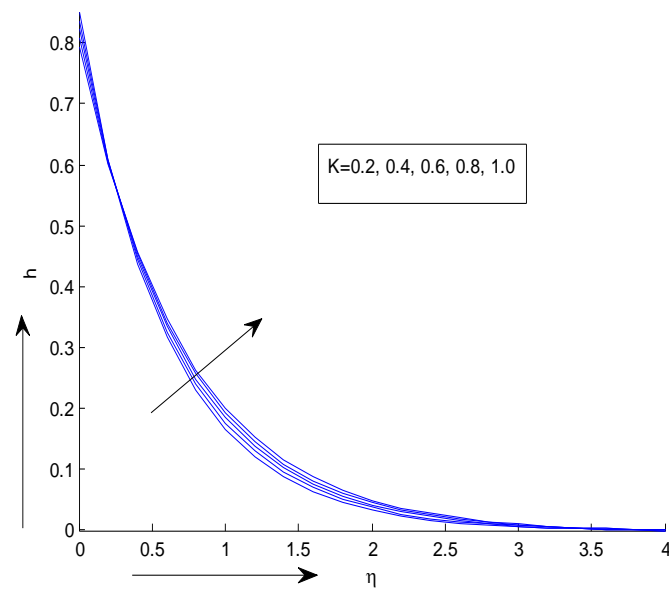**Figure 4.** *Angular velocity profile for various K_1.*
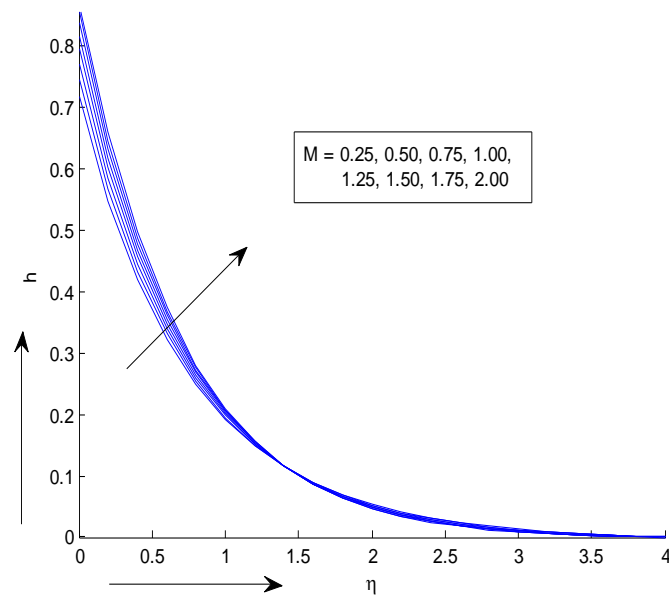
Figure 5. *Angular velocity profile for various K.*

Figure 6. *Angular velocity profile for various M.*

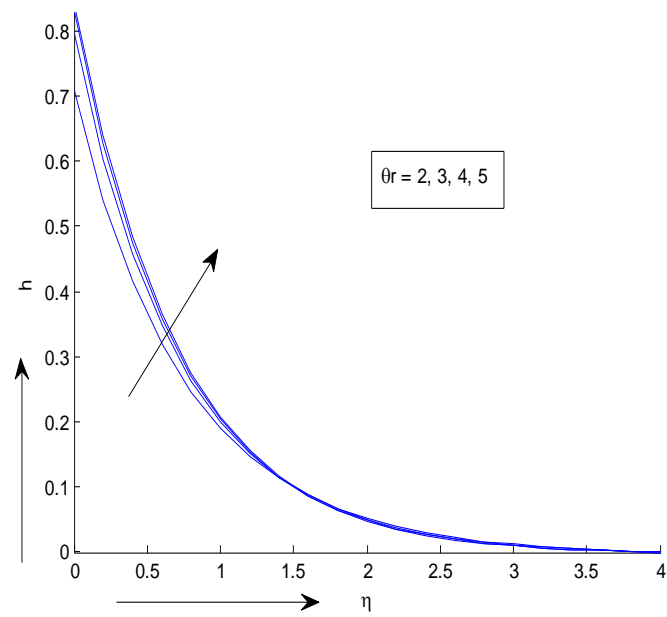**Figure 7.** *Angular velocity profile for various θ_r.*
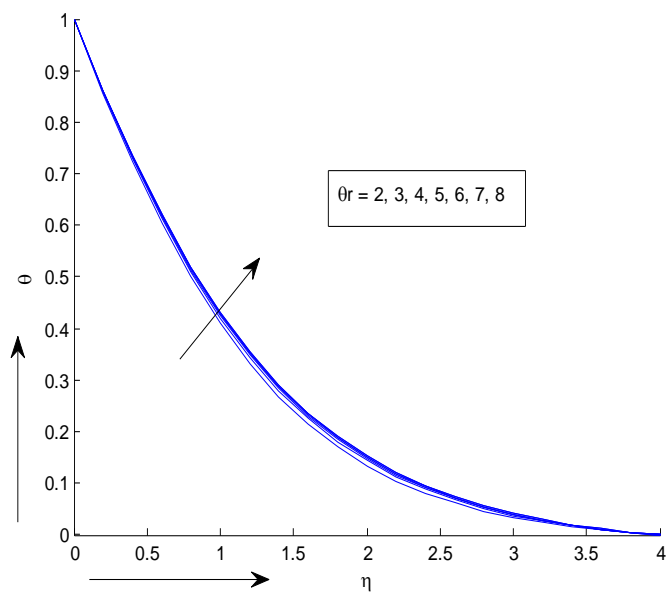
Figure 8. *Temperature profile for various θ_r.*
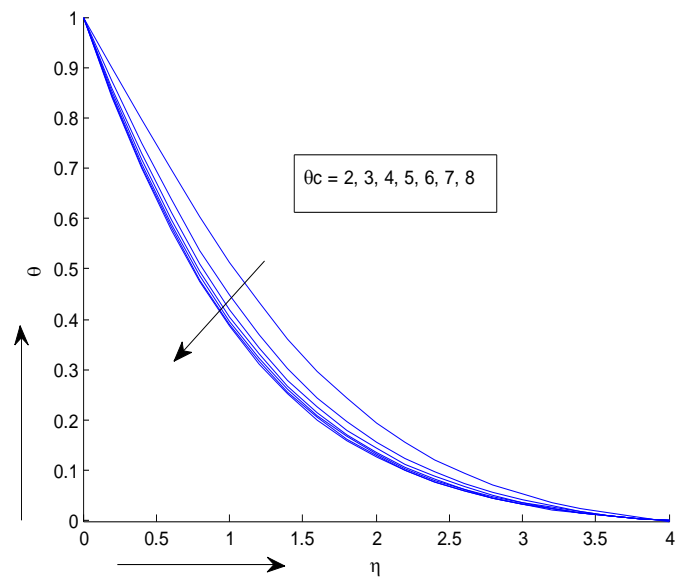
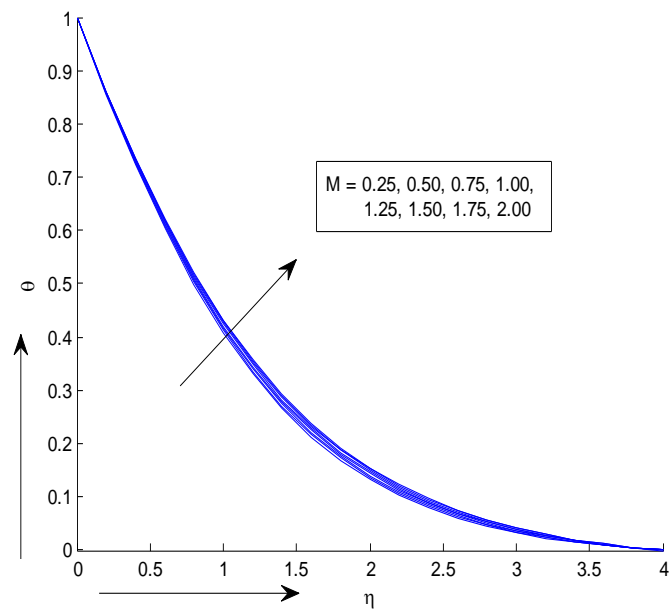**Figure 9.** *Temperature profile for various* θ_c.

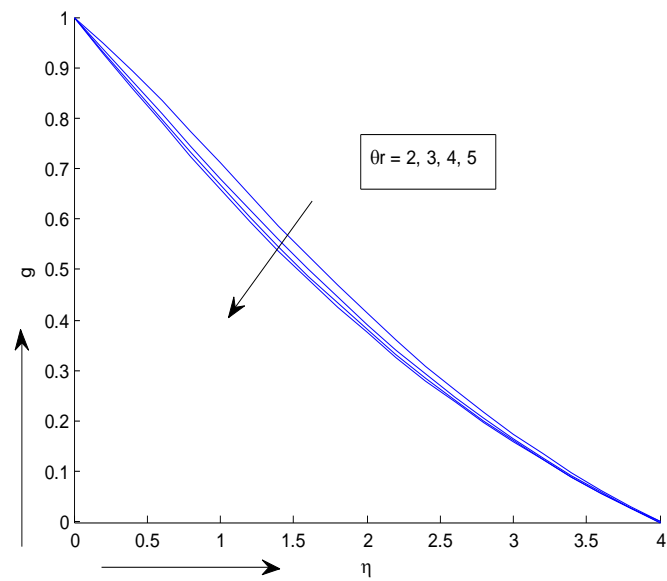**Figure 10.** *Temperature profile for various* M.

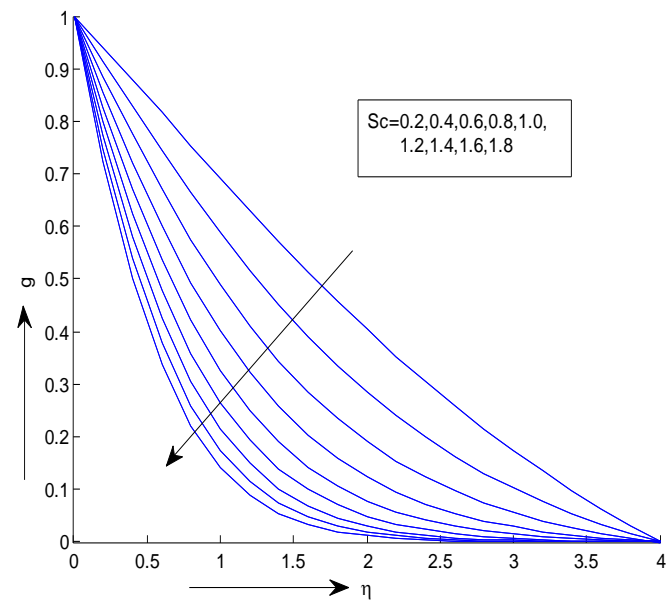**Figure 11.** *Concentration profile for various θ_r .*

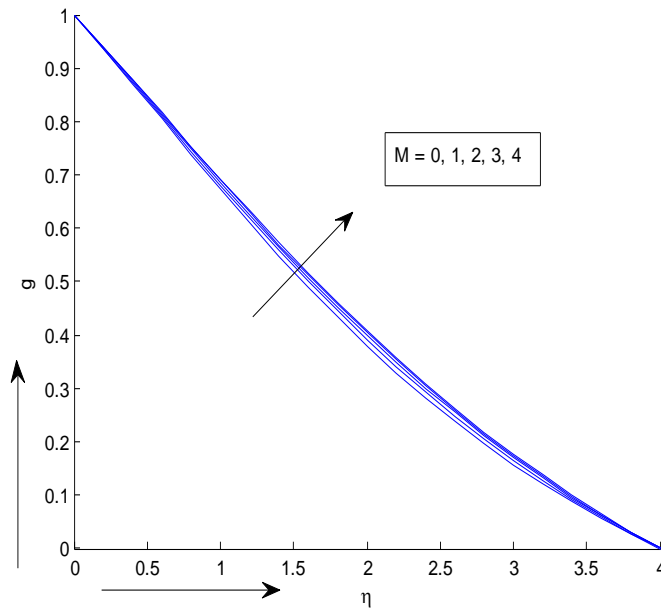**Figure 12.** *Concentration profile for various S_c .*

Figure 13. *Concentration profile for various M.*

3.2 Tables

Table 1. *Missing values and physical quantities with the variation of M and θ_c.*

θ_c	M	c_f	Nu	$f''(0)$	$\theta'(0)$	$h'(0)$	$g'(0)$
2	0.50	1.498964	1.0111	-0.99931	-0.50555	-1.03763	-0.31086
	1.00	1.596479	0.997356	-1.06432	-0.49868	-1.11714	-0.30784
	1.50	1.681777	0.986128	-1.12118	-0.49306	-1.18695	-0.30541
	2.00	1.7581	0.976819	-1.17207	-0.48841	-1.24962	-0.3034
3	0.50	1.493648	0.987639	-0.99577	-0.65843	-1.03209	-0.30871
	1.00	1.590964	0.975415	-1.06064	-0.65028	-1.11151	-0.30566
	1.50	1.676127	0.965368	-1.11742	-0.64358	-1.18129	-0.30322
	2.00	1.752358	0.956993	-1.16824	-0.638	-1.24394	-0.3012
4	0.50	1.491313	0.978146	-0.99421	-0.73361	-1.02965	-0.30786
	1.00	1.588522	0.966567	-1.05902	-0.72493	-1.10901	-0.30481
	1.50	1.673607	0.957028	-1.11574	-0.71777	-1.17876	-0.30235
	2.00	1.74978	0.949061	-1.16652	-0.7118	-1.24139	-0.30032
5	0.50	1.489996	0.972971	-0.99333	-0.77838	-1.02827	-0.3074
	1.00	1.587139	0.961747	-1.05809	-0.7694	-1.10759	-0.30434
	1.50	1.672175	0.952491	-1.11478	-0.76199	-1.17732	-0.30188
	2.00	1.748311	0.944752	-1.16554	-0.7558	-1.23993	-0.29985

Table 2. *Missing values and physical quantities with the variation of M and θ_r*

θ_r	M	c_f	Nu	$f''(0)$	$\theta'(0)$	$h'(0)$	$g'(0)$
2	0.50	0.879632	0.996193	-0.87963	-0.74715	-0.88278	-0.24116
	1.00	0.941322	0.985133	-0.94132	-0.73885	-0.95846	-0.2388
	1.50	0.995484	0.975798	-0.99548	-0.73185	-1.02523	-0.23688
	2.00	1.044001	0.967846	-1.044	-0.72588	-1.08526	-0.23528
3	0.50	1.491313	0.978146	-0.99421	-0.73361	-1.02965	-0.30786
	1.00	1.588522	0.966567	-1.05902	-0.72493	-1.10901	-0.30481
	1.50	1.673607	0.957028	-1.11574	-0.71777	-1.17876	-0.30235
	2.00	1.74978	0.949061	-1.16652	-0.7118	-1.24139	-0.30032
4	0.50	1.733045	0.970531	-1.03983	-0.7279	-1.08732	-0.33943
	1.00	1.842703	0.958862	-1.10562	-0.71915	-1.1678	-0.33606
	1.50	1.938648	0.949348	-1.16319	-0.71201	-1.2385	-0.33336
	2.00	2.024592	0.941465	-1.21476	-0.7061	-1.302	-0.33114

	0.50	1.862685	0.966326	-1.06439	-0.72474	-1.11818	-0.35788
5	1.00	1.97866	0.954641	-1.13066	-0.71598	-1.19919	-0.35432
	1.50	2.080135	0.945167	-1.18865	-0.70888	-1.27035	-0.35147
	2.00	2.171076	0.937351	-1.24062	-0.70301	-1.33429	-0.34914

Table 3. *Missing values and physical quantities with the variation of K and θ_c.*

θ_c	K	c_f	Nu	$f''(0)$	$\theta'(0)$	$h'(0)$	$g'(0)$
2	0.25	-0.85325	0.983003	-1.13767	-0.4915	-1.42877	-0.30577
	0.50	0	0.988099	-1.11193	-0.49405	-1.30757	-0.3065
	0.75	0.81559	0.992844	-1.08745	-0.49642	-1.20497	-0.30718
	1.00	1.596479	0.997356	-1.06432	-0.49868	-1.11714	-0.30784
3	0.25	-0.84954	0.961726	-1.13272	-0.64115	-1.41965	-0.30353
	0.50	0	0.966596	-1.10749	-0.6444	-1.29993	-0.30428
	0.75	0.812573	0.971122	-1.08343	-0.64742	-1.19846	-0.30499
	1.00	1.590964	0.975415	-1.06064	-0.65028	-1.11151	-0.30566
4	0.25	-0.84788	0.953181	-1.1305	-0.71489	-1.41556	-0.30264
	0.50	0	0.957948	-1.10551	-0.71846	-1.29651	-0.30341
	0.75	0.811233	0.962374	-1.08164	-0.72178	-1.19556	-0.30412
	1.00	1.588522	0.966567	-1.05902	-0.72493	-1.10901	-0.30481
5	0.25	-0.84693	0.948535	-1.12924	-0.75883	-1.41323	-0.30216
	0.50	0	0.953243	-1.10439	-0.76259	-1.29457	-0.30293
	0.75	0.810473	0.957611	-1.08063	-0.76609	-1.19391	-0.30365
	1.00	1.587139	0.961747	-1.05809	-0.7694	-1.10759	-0.30434

Table 4. *Missing values and physical quantities with the variation of K and θ_r*

θ_r	K	c_f	Nu	$f''(0)$	$\theta'(0)$	$h'(0)$	$g'(0)$
2	0.25	-1.22889	0.972569	-0.98311	-0.72943	-1.18624	-0.23726
	0.50	-0.48457	0.977167	-0.96913	-0.73288	-1.09908	-0.23782
	0.75	0.238783	0.981313	-0.95513	-0.73599	-1.02391	-0.23833
	1.00	0.941322	0.985133	-0.94132	-0.73885	-0.95846	-0.2388
3	0.25	-0.84788	0.953181	-1.1305	-0.71489	-1.41556	-0.30264
	0.50	0	0.957948	-1.10551	-0.71846	-1.29651	-0.30341
	0.75	0.811233	0.962374	-1.08164	-0.72178	-1.19556	-0.30412
	1.00	1.588522	0.966567	-1.05902	-0.72493	-1.10901	-0.30481
4	0.25	-0.69542	0.945019	-1.19215	-0.70876	-1.51055	-0.33355
	0.50	0.193561	0.949878	-1.16136	-0.71241	-1.37646	-0.33442
	0.75	1.038152	0.954463	-1.13253	-0.71585	-1.26372	-0.33525
	1.00	1.842703	0.958862	-1.10562	-0.71915	-1.1678	-0.33606
5	0.25	-0.61312	0.940517	-1.22623	-0.70539	-1.56285	-0.35158
	0.50	0.297974	0.945431	-1.1919	-0.70907	-1.41993	-0.35252
	0.75	1.160086	0.950114	-1.16009	-0.71259	-1.30042	-0.35343
	1.00	1.97866	0.954641	-1.13066	-0.71598	-1.41556	-0.35432

Table 5. *Comparison of missing values with M.*

M	$f''(0)$		$\theta'(0)$		$h'(0)$	
	original	new	Original	new	original	new
0.25	-1.76534	-1.00498	-1.36638	-0.73038	-0.52405	-1.18383
0.5	-1.86036	-1.04454	-1.3471	-0.72547	-0.51891	-1.24028
0.75	-1.94925	-1.08118	-1.32952	-0.72106	-0.51438	-1.29269
1	-2.03303	-1.11539	-1.31336	-0.71707	-0.51034	-1.34172
1.25	-2.11248	-1.14752	-1.2984	-0.71345	-0.50671	-1.38787
1.5	-2.18821	-1.17789	-1.28448	-0.71015	-0.50341	-1.43154
1.75	-2.26069	-1.20672	-1.27145	-0.70714	-0.50039	-1.47305
2	-2.33031	-1.23422	-1.25921	-0.70437	-0.49762	-1.51268
2.25	-2.39737	-1.26053	-1.24765	-0.70183	-0.49506	-1.55063
2.5	-2.46214	-1.2858	-1.23671	-0.69948	-0.49268	-1.58709

Table 6. *Comparison of physical quantities with M.*

M	c_f		Nu	
	original	new	original	New
0.25	-1.05921	-0.30149	-1.36638	0.973833
0.5	-1.11622	-0.31336	-1.3471	0.967294
0.75	-1.16955	-0.32435	-1.32952	0.961409
1	-1.21982	-0.33462	-1.31336	0.956091
1.25	-1.26749	-0.34426	-1.2984	0.951265
1.5	-1.31293	-0.35337	-1.28448	0.94687
1.75	-1.35642	-0.36202	-1.27145	0.942852
2	-1.39818	-0.37026	-1.25921	0.939165
2.25	-1.43842	-0.37816	-1.24765	0.935772
2.5	-1.47729	-1.23671	-1.23671	0.93264

4. CONCLUSION

In this study the effects of variable viscosity and thermal conductivity with Joules dissipation on MHD micropolar fluid flow, heat and mass transfer past a stretching porous surface embedded in a porous medium is investigated. It is observed from the above discussions that the viscosity and thermal conductivity parameters along with the other parameters mentioned above have significant effects on velocity, temperature, concentration and micro-rotation distributions within the boundary layer. Also the presence of micro-constituents results significant change of behaviours in the flow problems in micropolar fluid in comparison with the ordinary fluids. The following conclusions can be drawn from this study:

1. Velocity decreases for the increase of both viscosity and magnetic parameter.

2. Microrotation increases with the increase of both viscosity and magnetic parameter.

3. Temperature increases for the increase of viscosity and magnetic parameter while it decreases with thermal conductivity.

4. Skin friction increases with the increase of both viscosity and thermal conductivity.

5. Nusselt number decreases with the increase of both viscosity and thermal conductivity.

This investigation must be helpful for further study of micropolar fluid in different branches of fluid dynamics.

Nomenclatures

β = Volumetric coefficient of thermal expansion

γ = Spin-gradient or micro rotation viscosity

η = Dimensionless co-ordinates

λ = Thermal conductivity

λ_∞ = Thermal conductivity of the ambient fluid

μ = Dynamic viscosity

μ_∞ = Dynamic viscosity of the ambient fluid

ν = Kinematic viscosity

κ = Vortex viscosity

ρ = Density

σ = Electrical conductivity

θ = Dimensionless temperature

θ_c = Dimensionless reference temperature corresponding to thermal conductivity parameter

θ_r = Dimensionless reference temperature corresponding to viscosity parameter

τ_w = Wall shear stress

q_w = Heat transfer from the plate

c_p =Specific heat at constant pressure

u = Velocity in the x −direction

v = Velocity in the y −direction

f' =Dimensionless velocity

h =Dimensionless micro-rotation

g =Dimensionless species concentration

C =Species concentration

T =Temperature

T_∞ = Ambient temperature

T_w = Wall temperature

C_w = Species concentration at the wall

C_∞ = Species concentration far from the wall

U_w = Stretching velocity

j =Micro rotation density

D = Mass diffusivity

B_0= Strength of the magnetic field

$S_c = \frac{\nu_\infty}{D}$, Schmidt number

$P_r = \frac{\nu_\infty \rho c_p}{\lambda_\infty}$, Prandtl number

$E_c = \frac{U_w^2}{c_p (T_w - T_\infty)}$, Eckert number.

$Re_x = \frac{U_w x}{\nu_\infty}$, Local Reynolds number.

$M = \frac{2\sigma B_0^2 x}{\rho(1+m)U_w}$, Magnetic parameter (Hartmann number)

$K = \frac{\kappa}{\rho\nu_\infty}$, coupling constant parameter

$K_1 = \frac{K}{j}$, coupling parameter.

K_p= Permeability of the medium.

R_1 = Permeability parameter.

Nu = Nusselt number

Nu_x = Local Nusselt number

c_f =Skin –friction coefficient

Subscripts

w, the condition at the wall

∞, the condition far away from the surface

Superscripts

′, Differentiation with respect to η

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