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# Improvement of New Eight and Sixteenth Order Iterative Methods for Solving Nonlinear Algebraic Equations by Using Least Square Method

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**Abstract:** *Finding the roots of nonlinear algebraic equations is an important problem in science and engineering, later many methods developed for solving nonlinear equations. These methods are given [1-24], in this work, we proposed two new higher order iterative methods. These methods based on the method given by Rafiullah [1], 2016, which is Eighth and Sixteenth-order convergence. The Least square method is used to find the present methods. We verified on a number of examples and numerical results obtained show that the present method is faster than the other methods.*

**Keywords:** *Nonlinear equations · Iterative methods · Newton's Method· Lagrange interpolation, Least square method.*

## 1. Introduction

Solving nonlinear equations f (x)=0, is one of the most important problem in scientific and engineering applications. There are several well-known methods for solving nonlinear algebraic equations of the form:

$$f(x)=0 \tag{1}$$

Where f denote a continuously differentiable function on [a, b]$\subset\mathcal{R}$, and has at least one root α, in [a, b] Such as Newton's Method, Bisection method, Regula Falsi method, Nonlinear Regression Method and several another methods see for example [2-24].Here we describe a new method by using Least square method as a polynomial form of degree two, then we find that, this procedure lead us to the root α of equation (1).Some test examples are given to show the efficiency of the proposed methods and we compare the results of these examples of present methods with the famous methods of classical Newton's method (NM) [4], Hou [12], Zheng et al method (QM) [13], Hu [14],and new Eighth higher and Sixteenth-order iterative methods given by Rafiullah (R1) and (R2) [1], the numerical results obtained show that thepresent method is faster than the other methods.

## 2. The Present Method

Consider a nonlinear equation (1),

Consider the following iterative method proposed by M. Rafiullah [1]. This method involve six functions evaluations at each step and the order of convergence is improved up to the sixteen:

$$
\begin{aligned}
y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\
z_n &= y_n - \frac{f(y_n)}{f'(y_n)} - \frac{2f(y_n)^2(f'(x_n)-f'(y_n))}{2(f(x_n)-f(y_n))f'(x_n)^2} \\
v_n &= z_n - \frac{f(z_n)}{f'(z_n)} \\
&= z_n - \frac{2f(z_n)^2((x_n-y_n)(x_n-z_n)(y_n-z_n))}{\left(-f(z_n)(x_n-y_n)(x_n-2z_n+y_n)+f(y_n)(x_n-z_n)^2-f(x_n)((y_n-z_n)^2\right)} \\
x_{n+1} &= v_n - \frac{f(v_n)}{f'(v_n)}
\end{aligned}
\tag{2}
$$

Which was used Lagrange interpolation (to reduce the number of functions)toapproximatef'(zn) with using three known points ((xn), f (xn)), ((yn), f (yn)) and ((zn), f(zn)).

Here, in present work, we used the least square method of degree twoto approximate f'(zn)in the form given by equation (3)

$$a+bx+cx^2=0 \tag{3}$$

Where a, b and c are the unknown constant.

We used three points ((xn), f(xn)), ((yn), f(yn)) and ((zn), f(zn)), then we find that, this procedure lead us to the root α of equation (1), let $e_i$is the error or the different value between the true value $y_i$ and the estimated value $\widehat{y_i}$ , therefore,

$$e_i=y_i-\widehat{y_i} \tag{4}$$

And the sum of square error,

$$\sum_{i=1}^{3} e_i{}^2=\sum_{i=1}^{3}(y_i-\widehat{y_i})^2 \tag{5}$$

Or, $$\sum_{i=1}^{3} e_i{}^2=\sum_{i=1}^{3}(y_i-[(a+bx_i+cx_i{}^2)]^2 \tag{6}$$

To find a, b, c we will minimize this function, taking the derivative of (6) equal to zero, we find the three normal equations:

$$\begin{aligned}
&3.a+(x_n+y_n+z_n).b+(x_n{}^2+y_n{}^2+z_n{}^2).c=\\
&f(x_n)+f(y_n)+f(z_n)\\
&(x_n+y_n+z_n).a+(x_n{}^2+y_n{}^2+z_n{}^2).b+(x_n{}^3+y_n{}^3+z_n{}^3).c=\\
&x_n.f(x_n)+y_n f(y_n)+z_n f(z_n)\\
&(x_n{}^2+y_n{}^2+z_n{}^2).a+(x_n{}^3+y_n{}^3+z_n{}^3).b+(x_n{}^4+y_n{}^4+z_n{}^4).c\\
&=x_n{}^2.f(x_n)+y_n{}^2 f(y_n)+z_n{}^2 f(z_n)
\end{aligned} \tag{7}$$

Hence, we find a, b and c, then we have the new iteration:

$$\begin{aligned}
y_n&=x_n-\frac{f(x_n)}{f'(x_n)}\\
z_n&=y_n-\frac{f(y_n)}{f'(y_n)}-\frac{2f(y_n)^2(f'(x_n)-f'(y_n))}{2(f(x_n)-f(y_n))f'(x_n)^2}\\
v_n&=z_n-\frac{f(z_n)}{b+2c\,z_n};\\
x_{n+1}&=v_n-\frac{f(v_n)}{f'(v_n)}
\end{aligned} \tag{8}$$

## 3. Algorithm

The present method has 6 steps:

Take $[a,b]$ is an initial interval, which has at least a root in this interval.

Compute ((xn), f (xn)), ((yn), f (yn)) and ((zn), f(zn)),

Determine the constants a, b and c by solving the system of three linear algebraic equations (7).

Find iteration (xn+1) from (8).

Return to step (2) until the absolute error$|f(x)|<\varepsilon$.

## 4. Examples

In this section, we shall check the effectiveness of present method. First we compare presentmethod (8) (PM) with the method of M. Rafiullah (R1) [1] with the classical Newton's method (NM) [4] Hou [12] and Hu [14] which areeighth, second, twelfth and ninth order methods respectively.

**Example** 1 [1, 2]:

| Function | $x_0$ | α (exact root) |
|---|---|---|
| $f_1(x) = sin(x)^2 + x$ | 1 | 0 |
| $f_2(x) = e-^{x} + \cos x - 1$ | 0 | 0.923632658955135 |
| $f_3(x) = 4x^5 - 3x^4 + 2x^3 - 3$ | 2 | 1 |
| $f_4(x) = xe-^{x} - x$ | 1 | 0 |
| $f_5(x) = x^3 - x^2 + \log x$ | 1.5 | 1 |
| $f_6(x) = e^x \sin x + log(1 + x^2)$ | 1.5 | 0 |
| $f_7(x)= x^9 - 45x^8 + 870x^7 - 9450x^6 + 63{,}273x^5 - 269{,}325x^4 + 723{,}680x - 15$ | 10 | 0.591897115013801022 |
| $f_8(x) = 3\tan x - x$ | 1 | 0 |

The following table shows the number of iterations to achieve α.

| Function | **NM** | **HOU** | **HU** | **R1** | **PM** |
|---|---|---|---|---|---|
| $f_1(x)$ | 7 | 2 | 2 | 2 | 2 |
| $f_2(x)$ | 5 | 2 | 2 | 2 | 1 |
| $f_3(x)$ | 9 | 3 | 3 | 3 | 2 |
| $f_4(x)$ | 9 | 3 | 3 | 13 | 9 |
| $f_5(x)$ | 7 | 2 | 2 | 2 | 2 |
| $f_6(x)$ | 8 | 2 | 2 | 3 | 2 |
| $f_7(x)$ | 12 | slow | 4 | 4 | 2 |
| $f_8(x)$ | 6 | slow | 3 | 3 | 2 |

Now, consider some test problems to illustrate the efficiency of our method (PM) with the second proposed method of M. Rafiullah (R2) [1], which is sixteenth order. We compare our results (PM) with the results of (R2) [1] and with the results method of Zheng et al. [13] (QM), which is sixteenth order as well.

**Example** 2 [2,13]

Consider the following functions

| Function | $x_0$ | α (exact root) |
|---|---|---|
| $f_1(x) = 21(ex-2 - 1)$ | 2.5 | 2 |
| $f_2(x) = x2 - 2\ e-x + 1$ | 0.5 | 0 |
| $f_3(x) = = e^{-x} + x + 2 - 1$ | -0.7 | -1 |
| $f_4(x) = e^x - arctan(x) - 1$ | 0.5 | 0 |

The following table shows the first three iteration of (R2) method.

| Function | $x_1$ | $x_2$ | $x_3$ | $\lvert\alpha - x_3\rvert$ |
|---|---|---|---|---|
| $f_1(x)$ | 2.5000*e*+000 | 2.0000*e*+000 | 2.0000*e*+000 | 0 |
| $f_2(x)$ | 5.0000*e*−001 | 1.9956*e*−010 | 2.0589*e*−159 | 2.0589*e*−159 |
| $f_3(x)$ | −7.0000*e*−001 | −1.0000*e*+000 | −1.0000*e*+000 | 0 |
| $f_4(x)$ | 5.0000*e*−001 | 5.0000*e*−002 | 5.0000*e*−003 | 5.0003*e*−001 |

The following table shows the first three iteration of (QM) method.

| Function | $x_1$ | $x_2$ | $x_3$ | $\lvert\alpha - x_3\rvert$ |
|---|---|---|---|---|
| $f_1(x)$ | 2.5000*e*+000 | 2.0000*e*+000 | 2.0000*e*+000 | 0 |
| $f_2(x)$ | 5.0000*e*−001 | 5.8971*e*−010 | 1.7520*e*−161 | 1.7520*e*−161 |
| $f_3(x)$ | −7.0000*e*−001 | −1.0000*e*+000 | −1.0000*e*+000 | 0 |
| $f_4(x)$ | 5.0000*e*−001 | 4.7826*e*−002 | 3.1265*e*−003 | 3.1265*e*−003 |

The following table shows the first three iteration of present method (PM) method.

| Function | $x_1$ | $x_2$ | $x_3$ | $\lvert\alpha - x_3\rvert$ |
|---|---|---|---|---|
| $f_1(x)$ | 2.5000*e*+000 | 2.0000*e*+000 | 2.0000*e*+000 | 0 |
| $f_2(x)$ | 5.0000*e*−001 | 119.5587e-12 | 173.8982*e*−204 | 173.8982*e*−204 |
| $f_3(x)$ | −7.0000*e*−001 | −999.9999*e*+030 | −1.0000*e*+000 | 0 |
| $f_4(x)$ | 5.0000*e*−001 | 36.1505*e*−003 | 2.2040*e*−003 | 2.2040*e*−003 |

If we compare the results of present method (PM) with (R2)method and with (QM)method, we see that the performance of (PM) method is better than of both(R2) and(QM) methods.

## 5. CONCLUSIONS

In this work, we have proposed a new iterative method by using least square method. The efficiency of this method is shown for some test problems, comparison of the obtained result is given with the existing methods such as the Newton–Raphson [7], Hou [12] and Hu [14] the M. Rafiullah (R2) method [1] and, Zheng et al. [13], it is shown that this new method is more efficient than these existing methods.

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