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MHD Flow of Incompressible Fluid through Parallel Plates in Inclined Magnetic field having Porous Medium with Heat and Mass Transfer

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Abstract: *Flow of fluids through parallel plates with porous medium is very important in the study of oil extraction, geothermal energy, nuclear power, chemical industries etc. In view of its application, in the present paper the flow of visco-elastic electrically conducting fluid through two parallel plates filled with porous medium placed in an inclined magnetic field has been considered. Initially the flow is generated by a constant pressure gradient parallel to bounding fluid. After attaining the steady state, the pressure gradient is withdrawn and simultaneously inclined magnetic field is applied to find the velocity profile of the fluid. After forming the necessary equations and assuming suitable boundary conditions, the relation for velocity has been derived using Separation of variable technique. It has been found that the applied magnetic field has a significant role on the velocity profile.*

Keywords: *MHD, Porous plates, Unsteady, Heat Transfer.*

Nomenclature

Re	:	Reynolds number
Pr	:	Prandtl number
Gr	:	Grashof number
Gc	:	Solutal Grashof number
ρ	:	Density of the fluid
σ	:	Electrical conductivity of the fluid
k	:	Thermal conductivity
C_p	:	Specific heat at constant pressure
μ	:	Viscosity of the fluid
B_0	:	Magnetic field strength component
v	:	Kinematic Viscosity

1. INTRODUCTION

Magnetic fields are commonly applied in industry to pump, heat, levitate and stir liquid metals. There is the terrestrial magnetic field, which is maintained by fluid flow in the earth's core, the solar magnetic field, which originates sunspots and solar flares, and the galactic magnetic field, which is thought to control the configuration of stars from interstellar clouds. Recently, considerable attention has been focused on applications of MHD and heat transfer such as MHD generators, metallurgical processing, and geothermal energy extraction. The phenomenon concerning heat and mass transfer with MHD flow is important due to its numerous applications in science and technology as iot is used in many industries of food preservation, Geo-thermal energy, petroleum production, polymers technology and power generation engineering etc. Such flows occur in these areas as they transfer heat and mass which needs a deep knowledge of MHD flow.

Several researchers have contributed their work in this field. Al-Hadhrami (2003) discussed flow through horizontal channels of porous material and obtained velocity expressions in terms of Reynolds number. Rajput and Sahu (2011) studied the effect of a uniform transverse magnetic field on unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite parallel porous plates with constant temperature and variable mass diffusion. Manyonge et al (2012) studied steady MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field and discover that high magnetic field strength decreases the velocity.

In the present paper the effect of inclined magnetic field on two infinite parallel plates having porous medium with heat and mass transfer has been studied. Here the equation of continuity, momentum equation, and the equations which govern the flow field are solved by using non dimensional parameters and a graphical approach has been studied.

2. Formulation of the Problem

An electrically conducting, unsteady, viscous, incompressible, non-Newtonian fluid moving between two infinite parallel plates kept at a distance of 2h apart are placed in inclined magnetic field. Consider one dimensional flow so that the axis of the channel formed by two plates is *x-axis* and the flow is in this direction. The equation governing the flow field is:

The equation of continuity:

$$\nabla . V = 0 \tag{1}$$

The Navier –Stokes equation:

$$\rho\left[\left(\frac{\partial}{\partial t}+V.\nabla\right)\right]V = f_B - \nabla P + \mu\nabla^2 V \tag{2}$$

Where ρ = fluid density

f_B= body force per unit mass of the fluid

μ = fluid viscosity

P = pressure exerted on the fluid

V = velocity of electrically conducting fluid

It is observed that u*, v* and w* are the velocity components in x*, y* and z* directions respectively. Then this implies v*=w*=0and u*$\neq$ 0, then the continuity equation is satisfied. Here the body force is neglected and replaced with Lorentz force and from the assumptions that the flow is one dimensional.

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{F_x}{\rho} \tag{3}$$

Where F_x is the component of magnetic force in the direction of $x - axis$.

Therefore$\frac{F_x}{\rho} = \frac{\sigma}{\rho}B_0^2 u^*$; and when the angle of inclination is introduced then we have,

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma}{\rho}B_0^2 u^* sin^2\theta \tag{4}$$

Where θ is the angle between $V$and B. The characteristic velocity v_0 is taken due to the porosity of the lower plate which is constant to maintain the same pattern of flow. The momentum equation with heat and mass parameters is given as:

$$\frac{\partial u^*}{\partial t^*} = -v_0\frac{\partial u^*}{\partial y^*} - \frac{\partial p^*}{\partial x^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma}{\rho}B_0^2 u^* sin^2\theta + \rho g\beta(T^* - T_\infty^*) + \rho g\beta(C^* - C_\infty^*) \tag{5}$$

The energy equation is:

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial {y^*}^2} \tag{6}$$

The boundary conditions are:

$u^*(y,t) = 0, T^* = T_\infty$ at $t^* = 0,$

$$u^*(-L,t^*) = 0, u^*(L,t^*) = \frac{v}{L}, T^* = T_w \text{ at } t^* > 0 \tag{7}$$

Non-dimensional parameters are:

$$x^* = xL, y^* = yL, p^* = p\rho\frac{v^2}{L^2}, u^* = \frac{uv}{L}, t^* = \frac{tL^2}{v}, P_r = \frac{\mu c_p}{k}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, C = \frac{C^* - C_\infty^*}{C_w^* - T_\infty^*},$$

$$Ha^2 = \frac{\sigma L^2 B_0^2}{\mu} \tag{8}$$

The equations (5) and (6) are solved using the boundary condition and non-dimensional parameter and we obtain the following equations:

$$\frac{\partial u}{\partial t} = -\frac{Re}{\rho}\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} + M^2 u + Gr\theta + GcC \tag{9}$$

Where $M = M^* sin\theta$ and$M^* = LB_0\sqrt{\frac{\sigma}{\mu}}$. $Gr = \rho L^2 g\beta({T^*}_w - {T^*}_\infty)$, $Gc = \rho L^2 g\beta({C^*}_w - {C^*}_\infty)$, $\frac{\partial p}{\partial x} = 0$ (assuming the flow is couette flow).

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{10}$$

The dimensionless boundary conditions are:

$u(y,t) = 0, \theta(-1,t) = 0 \; at \; t = 0$

$$u(-1,t) = 0, u(1,t) = 1, \theta(1,t) = 1 \; at \; t > 0 \tag{11}$$

Hence solving the equation (9) and (10) with the help of separation of variable technique using boundary conditions (11) we get the equation of the form:

$$u(y,t) = u(y)u(t) \tag{12}$$

$$\theta(y,t) = \theta(y)\theta(t) \tag{13}$$

Therefore the solutions of velocity profile and temperature distribution are:

$$u(y,t) = e^{-\lambda^2 t}(C_1 e^{m_1 y} + C_2 e^{m_2 y}) + \frac{Gr\theta + Gc}{M^2 - \lambda^2} \tag{14}$$

$$\theta(y,t) = e^{-\frac{\lambda^2}{Pr}t}(C_3 \cos\lambda y + C_4 \sin\lambda y) \tag{15}$$

Where $B = M^2 + \lambda^2$

$$m_1 = \frac{A + \sqrt{A^2 + 4B}}{2}$$

$$m_2 = \frac{A - \sqrt{A^2 + 4B}}{2}$$

$C_1 = -C_2 e^{m_1 - m_2}$

$$C_2 = \frac{e^{-\lambda^2 t}}{e^{m_1 - m_2} - e^{m_2 - m_1}}$$

$$C_3 = \frac{e^{-\frac{\lambda^2}{Pr} t}}{2 \sin \lambda}$$

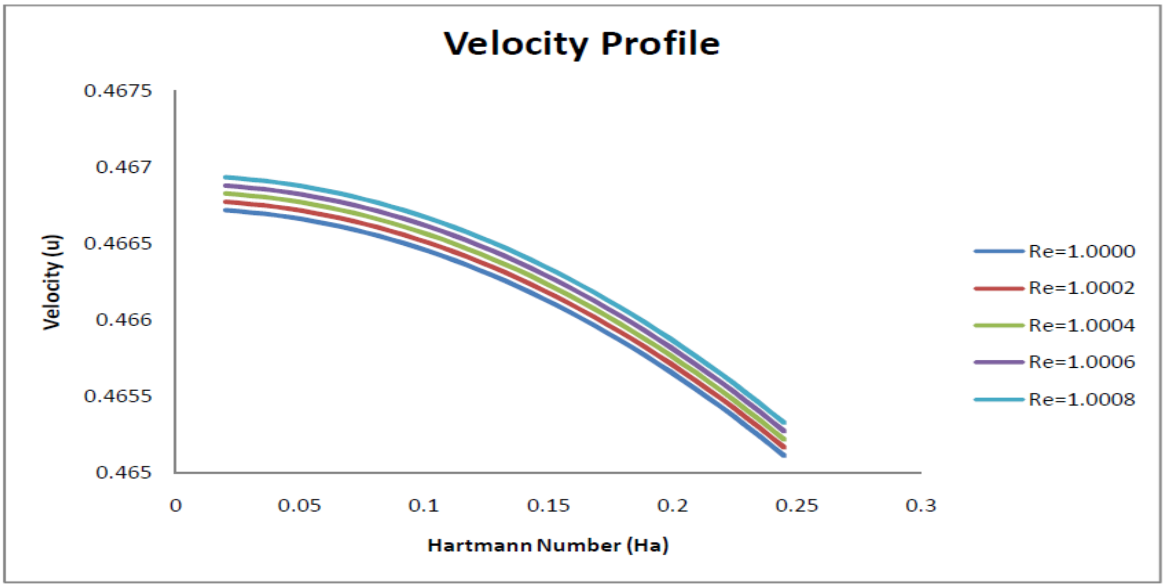
$C_4 = C_3 \tan \lambda$

3. Result and Discussion

Table . *Effect of Hartmann Number on Velocity profile*

Hartmann Number (Ha)	Reynolds Number (Re=1.0000)	Reynolds Number (Re=1.0002)	Reynolds Number (Re=1.0004)	Reynolds Number (Re=1.0006)	Reynolds Number (Re=1.0008)
0.020412415	0.466720395	0.466774068	0.466827745	0.466881423	0.466935104
0.040824829	0.466686701	0.466740376	0.466794053	0.466847732	0.466901413
0.061237244	0.466630536	0.466684211	0.466737889	0.466791569	0.466845252
0.081649658	0.466551883	0.466605560	0.466659239	0.466712921	0.466766605
0.102062073	0.466450722	0.466504401	0.466558082	0.466611765	0.466665451
0.122474487	0.466327027	0.466380708	0.466434391	0.466488077	0.466541765
0.142886902	0.466180763	0.466234447	0.466288133	0.466341821	0.466395512
0.163299316	0.466011894	0.466065580	0.466119269	0.466172961	0.466226654
0.183711731	0.465820372	0.465874063	0.465927755	0.465981450	0.466035147
0.204124145	0.465606149	0.465659843	0.465713539	0.465767238	0.465820939
0.224536560	0.465369166	0.465422865	0.465476565	0.465530268	0.465583973
0.244948974	0.465109362	0.465163064	0.465216770	0.465270477	0.465324187

The values of the parameters are: ($\theta = 15°, \lambda = 4, Gr = 2, Gc = 2, \sigma = 0.01, t = 0.2$)



The above graph shows velocity profile of a non-Newtonian fluid flowing between two infinite porous plates which are placed in an inclined magnetic field. It appears from the graph that velocity decreases with increasing magnetic field for all values ofReynolds number. For lower value of Reynolds number the velocity adopts lower values and vice-versa and thus the pattern remains the same.

It is clear that on increasing the magnetic field the force of attraction increases and which reduces the velocity of magnetic sensitive particles. Also from the derived relation the magnetic field term appears in the denominator which also supports the decrement in velocity and on further increasing the magnetic field the graph adopts hyperbolic tendency due to the presence of exponential terms in the derived relation. However the presence of other terms cannot be ignored in the velocity profile. The present study can be extended in this field and the results can be applied in various metallic industries.

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