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Hamiltonian Equations of Kähler-Einstein Manifolds with Equal Kähler Angles

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Abstract: *The paper aims to introduce Hamiltonian formalism for mechanical systems using Kähler-Einstein manifolds with equal Kähler angles, which represent an interesting multidisciplinary field of research. Also, solutions of these equations will be made using the computer program Maple and the geometrical-physical results related to on Kähler-Einstein mechanical systems are also to be issued.*

Keywords: *Kähler-Einstein Manifold, Hamiltonian, Mechanical System.*

1. Introduction

There are lots of useful applications of differential geometry and mathematical physics that are used in many areas. One of the most important applications of differential geometry is on geodesics. A geodesic is the shortest route between given two points. Geodesics can be found with the help of the Hamilton equations. We can say that differential geometry provides a suitable field for studying Hamiltonians of classical mechanics, analytic mechanics and field theory. The dynamic equations for moving bodies are obtained according to Hamiltonian mechanics formulation by many authors and are illustrated as follows:

Especially since 1980 there have been many studies about Hamiltonian dynamics, mechanics, formalisms. There are real, complex, paracomplex and other analogues. As is known, it is possible to produce different analogous in different spaces. Now, we give some examples as follows: If a Hamiltonian function and the initial state of the atoms in the system are known, one can compute the instantaneous positions and momenta of the atoms at all successive times [1]. It is possible to write Hamiltonians as in Newton's second rule "F=ma" that lets you avoid having to deal with vector-valued force balances. They not only make hideous mechanics problems simple, but they also expose deep symmetries and conserved properties. Rigid-body dynamics is the study of the movement of the objects like baseballs, planets, tops, and snowflakes through space. Gravitation concerns the intricacies of the n-body problem: n masses pulling on one another in the standard $G((mM)/(r^2))$ way [2]. Dynamic optimization problems for systems governed by differential inclusions are considered. The main focus is on the structure of and inter-relations between necessary optimally conditions stated in terms of Hamiltonian formalisms [3]. Antoniou and Pronko proposed a Hamiltonian approach to fluid mechanics based on the dynamics formulated in terms of Lagrangian variables [4]. The gravitational two-body problem in given was generalized by Barker and O'Connell [5]. A Lagrangian had been developed for leading the equations of motion which are isomorphic to the full Navier-Stokes equation, including dissipation [6]. Tekkoyun and Yayli presented generalized-quaternionic Kählerian analogue of Lagrangian and Hamiltonian mechanical systems [7]. Spotti investigated how Fano manifolds equipped with a Kähler-Einstein metric can degenerate as metric spaces (in the Gromov-Hausdorff topology) and some of the relations of this question with Algebraic Geometry [8]. Kasap and Tekkoyun introduced Lagrangian and Hamiltonian formalism for mechanical systems using para/pseudo-Kähler manifolds [9]. In joint work with Chen and Weber [10], LeBrun has elsewhere showed that $\mathbb{C}P_2 2\mathbb{C}P_2$ admits an Einstein metric. In this study gave a new and rather different proof of this fact [11]. Heier carry out Nadel's method of multiplier ideal sheaves to show that every complex del Pezzo surface of degree at most six whose automorphism group acts without fixed points has a Kähler-Einstein metric [12]. Li solved a folklore conjecture, it is often referred as the Yau-Tian-Donaldson conjecture, on Fano manifolds without nontrivial holomorphic vector fields [13]. Coevering gave many examples of Kähler-Einstein strictly pseudoconvex manifolds on bundles and resolutions [14]. Roček had studied the relationship between the curvature of a Kähler-Einstein manifold with Kähler potential $K = A + C\theta\overline{\theta}$ and the curvature of the base manifold [15]. Nadel was used to establish the existence of Kähler-Einstein metrics of positive scalar curvature on a very large class of compact complex manifolds [16].

2. Preliminaries

In this study, all manifolds and geometric structures are supposed that diffrerentiable. The Einstein summation convention ($\sum a_j x_j = a_j x_j$) is in use. Also, TM is tangent manifold, T^*M is cotangent manifold of a manifold M and M is an n-dimensional differentiable manifold. Additionally, the set of para-complex numbers, the set of para-complex functions on TM, the set of para-complex vector fields on TM and the set of para-complex 1-forms on TM are represented by A, F(TM), χ(TM) and Λ^1(TM) , respectively. The definitions and geometric structures on the differentiable manifold M are given in [17] and they may be extended to TM as follows:

3. Pseudo-Riemannian Manifold

Definition 1. Let M be a smooth manifold of dimension n≥3. Let ∇ be its Levi-Civita connection, a torsion free connection on the tangent bundle TM of M and let g=<.,.> be a pseudo-Riemann metric on M of signature (p,q). (M,g) be called the **pseudo-Riemannian manifold** [18].

4. Einstein Manifold

In differential geometry and mathematical physics, an Einstein manifold is a Riemannian or pseudo-Riemannian manifold whose Ricci tensor is proportional to the metric. They are named after Albert Einstein because this condition is equivalent to saying that the metric is a solution of the vacuum Einstein field equations (with cosmological constant), although the dimension, as well as the signature, of the metric can be arbitrary, unlike the four-dimensional Lorentzian manifolds usually studied in general relativity.

Definition 2. The **Ricci curvature tensor** (or Ricci tensor) r of a pseudo-Riemannian manifold (M,g) is the 2-tensor

$$r(X,Y)=tr(Z\rightarrow R(X,Z)Y), \tag{1}$$

where tr donetes the trace of the linear map Z→R(X,Z)Y. Note that the Ricci tensor is symmetric.

Definition 3. A **Riemannian manifold** (M,g) consists of the following data: M is a compact C^∞ manifold and a metric tensor field g is a positive definite bilinear symmetric differential 2-form on M. In other words, we associate with every point x of M a Euclidean structure g_x on the tangent space T_xM of M at x and require the association $x\rightarrow g_x$ to be C^∞. We say that g is a Riemannian metric on M.

Definition 4. A pseudo-Riemannian manifold (M,g) is **Einstein** if a real constant λ exists such that

$$r(X,Y)=\lambda g(X,Y) \tag{2}$$

for each X,Y in T_xM and each x in M.

Theorem 1. Assume n≥3. Then an n-dimensional pseudo-Riemannian manifold is Einstein if and only if, for each x in M, there exists a constant λ_x such that

$$r_x=\lambda_x g_x \;\; [19]. \tag{3}$$

5. Complex Manifolds

Let M be a smooth manifold of real dimension 2n. We say that a smooth atlas A of M is holomorphic if for any two coordinate charts $z:U\rightarrow U'\subset\mathbb{C}^m$ and $w:V\rightarrow V'\subset\mathbb{C}^m$ in A, the coordinate transition map

$z\circ w^{-1}$ is holomorphic. Any holomorphic atlas uniquely determines a maximal holomorphic atlas, and a maximal holomorphic atlas is called a complex structure for M. We say that M is a Complex Manifold of complex dimension n if M comes equipped with a holomorphic atlas. Any coordinate chart of the corresponding complex structure will be called a holomorphic coordinate chart of M. A Riemann surface or complex curve is a complex manifold of complex dimension 1.

6. Kähler Manifolds

Definition 5. Let M be a complex manifold with complex simplectic structure J and compatible Riemannian metric $g=\langle .,. \rangle$ as in $\langle JX,JY\rangle=\langle X,Y\rangle$. The alternating 2-form

$$\omega(X,Y):=g(JX,Y) \tag{4}$$

is called the associated **Kähler form**. We can retrieve g from ω,

$$g(X,Y)=\omega(X,JY). \tag{5}$$

We say that g is a Kähler metric and that M (together with g) is a Kähler manifold if ω is closed and (M,g) is displayed in the form. Let M be a complex manifold. A Riemannian metric on M is called Hermitian if it is compatible with the complex (simplectic) structure J of M, $\langle JX,JY\rangle=\langle X,Y\rangle$. Then the associated differential 2-form ω defined by $\omega(X,Y)=\langle JX,Y\rangle$ is called the Kähler form. It turns out that ω is closed if and only if J is parallel. Then M is called a Kähler manifold and the metric on M a Kähler metric. Kähler manifolds are modelled on complex Euclidean space [20].

7. Kähler-Einstein Manifolds

Definition 6. A Kähler metric g on a complex manifold M is Einstein if and only if there exists $\lambda\in\mathbb{R}$

$$\rho=\lambda\omega, \tag{6}$$

where ω is the fundamental form associated to g and $\rho(X,Y)=Ric(X,JY)$ for $X,Y\in\chi(M)$. The pair (M,g), where M is a complex manifold and g a Kähler-Einstein metric is said a **Kähler-Einstein manifold**.

If M is a smooth manifold with real dimension 2n, then a smooth field $J=(J_x)$ of complex structures on TM is called an almost complex structure of M. An almost complex structure $J=J_x$ is called a complex structure if it comes from a complex structure on M as in $J_xX_\alpha(x)=Y_\alpha(x)$, $J_xY_\alpha(x)=-X_\alpha(x)$ for $X_\alpha=\partial/(\partial x_\alpha)$ and $Y_\alpha=\partial/(\partial y_\alpha)$, $1\leq\alpha\leq n$. Any almost complex structure on a surface is a complex structure (existence of isothermal coordinates). A celebrated theorem of Newlander and Nirenberg [21] says that an almost complex structure is a complex simplectic structure if and only if its Nijenhuis tensor or torsion N vanishes, where, for vector fields X and Y on M,

$$N(X,Y)=2\{[JX,JY]-[X,Y]-J[X,JY]-J[JX,Y]\} \quad [20]. \tag{7}$$

We say that the coordinate frame of a coordinate chart x consists of the ordered tuple of vector fields $X_\alpha=\partial/(\partial x_\alpha)$ and $Y_\alpha=\partial/(\partial y_\alpha)$, $1\leq\alpha\leq n$. Now, we will define of complex structures that use in our operations. Let $\{X_\alpha,Y_\alpha\}_{1\leq\alpha\leq n}$ be a g_M-orthonormal basis of T_xM and diagonalizes $F^*\omega$ at x, that is

$$F^*\omega=\bigoplus_{0\leq\alpha\leq n}\begin{bmatrix}0 & -\cos\theta_\alpha\\ \cos\theta_\alpha & 0\end{bmatrix}_{0\leq\alpha\leq n} \tag{8}$$

where $\cos\theta_1\geq\cos\theta_2\geq\cdots\geq\cos\theta_n\geq0$. The angles $\{\theta_\alpha\}_{1\leq\alpha\leq n}$ are the Kähler angles of F at x. Thus, $\forall\alpha$, $F^*\omega(X_\alpha)=\cos\theta_\alpha Y_\alpha$, $F^*\omega(Y_\alpha)=-\cos\theta_\alpha X_\alpha$ and if $k\geq1$, where 2k is the rank of $F^*\omega$ at x, $J\omega X_\alpha=Y_\alpha,\forall\alpha\leq k$. Also, $\mathbf{i}^2=-1$, $Z_\alpha=(X_\alpha-\mathbf{i}Y_\alpha)/2$ and $\overline{Z}_\alpha=(X_\alpha+\mathbf{i}Y_\alpha)/2$ are complex vectors of the complexified tangent space of M at x. It disappears if and only if $F^*\omega$, $F^*\omega$ taken instead of J, is an integrable almost (para)-

complex structure, i.e. given any point $x\in M$, there are local coordinates $(x_1,...,x_n)$ which are centered at x so

$$F\omega(Z_\alpha)=\mathbf{i}\cos\theta_\alpha Z_\alpha\ ,\ F\omega(\overline{Z}_\alpha)=-\mathbf{i}\cos\theta_\alpha \overline{Z}_\alpha\ \text{[22]}. \tag{9}$$

8. HAMILTON DYNAMICS EQUATIONS

Hamilton Dynamics Equations: Let M is the base manifold of dimension n and its cotangent manifold T^*M. By a symplectic form we mean a 2-form Φ on T^*M. Let (T^*M,Φ) be a symplectic manifold, there is a unique vector field Z_H on T^*M and $H: T^*M\to\mathbb{R}$ is called as Hamiltonian Function (H=T+V) such that Hamiltonian Dynamical Equation is determined by

$$i_{Z_H}\Phi=dH. \tag{10}$$

We say Z_H is locally Hamiltonian vector field. Φ is closed and also shows the canonical symplectic form so that $\Phi=-d\lambda$, $\lambda=J^*(\xi)$, J^* a dual of J, ξ a 1-form on T^*M. The triple (T^*M,Φ,Z_H) is named Hamiltonian System which is defined on the cotangent bundle T^*M. From the local expression for Z_H we see that $p_i=(\partial L)/(\partial\dot{q}_i)$, $\dot{p}_i=(\partial L)/(\partial q_i)$ is an integral curve of Z_H if Hamilton's Equations is expressed as follows:

$$\dot{q}_i=(\partial H)/(\partial p_i),\ \ \dot{p}_i=-(\partial H)/(\partial q_i)\ \text{[23]}. \tag{11}$$

9. HAMILTONIAN MECHANICAL SYSTEMS

Now, we will present Hamilton equations and Hamiltonian mechanical systems for quantum and classical mechanics constructed on Kähler-Einstein manifolds with equal Kähler angles.

Proposition : Let $(M,g,F\omega)$ be on Kähler-Einstein manifolds. Suppose that the complex structures, a Liouville form and a 1-form on Kähler-Einstein manifolds with equal Kähler angles are shown by $F\omega$, λ and ξ, respectively. Consider a 1-form ξ such that

$$\xi=(1/2)(\overline{Z}_\alpha dz_\alpha+Z_\alpha d\bar{z}_\alpha), \tag{12}$$

the following equations for (9) are obtained.

$$\text{dif 1.}\ \ (\partial H)/(\partial\bar{z}_\alpha)=(-\mathbf{i}\cos\theta_\alpha)((dZ_\alpha)/(dt)),$$

$$\text{dif 2.}\ \ (\partial H)/(\partial z_\alpha)=(\mathbf{i}\cos\theta_\alpha)((d\overline{Z}_\alpha)/(dt)). \tag{13}$$

Proof: We obtain the Liouville form as follows:

$$\lambda=F\omega(\xi)=(1/2)[-\mathbf{i}\cos\theta_\alpha\overline{Z}_\alpha dz_\alpha+\mathbf{i}\cos\theta_\alpha Z_\alpha d\bar{z}_\alpha]. \tag{14}$$

It is well known that if Φ is a closed on Kähler-Einstein manifolds with equal Kähler angles $(M,g,F\omega)$, then Φ is also a symplectic structure on $(M,g,F\omega)$. Therefore the 2-form $\Phi=-d\lambda$ indicates the canonical symplectic form and derived from the 1-form λ to find to mechanical equations. Then the 2-form Φ is calculated as below:

$$\Phi=-(1/2)d[-\mathbf{i}\cos\theta_\alpha\overline{Z}_\alpha dz_\alpha+\mathbf{i}\cos\theta_\alpha Z_\alpha d\bar{z}_\alpha]$$

$$=\mathbf{i}\cos\theta_\alpha d\bar{z}_\alpha\wedge dz_\alpha. \tag{15}$$

Take a vector field Z_{H} so that called to be Hamiltonian vector field associated with Hamiltonian energy H and determined by

$$Z_H=Z_\alpha(\partial/(\partial z_\alpha))+\overline{Z}_\alpha(\partial/(\bar{z}_\alpha)). \tag{16}$$

So, we have

$$i_{Z_H}\Phi=\Phi(Z_H)=-\mathbf{i}\cos\theta_\alpha Z_\alpha d\bar{z}_\alpha+\mathbf{i}\cos\theta_\alpha \overline{Z}_\alpha dz_\alpha. \tag{17}$$

Furthermore, the differential of Hamiltonian energy H is obtained by

$$dH=((\partial H)/(\partial z_\alpha))dz_\alpha+((\partial H)/(\partial\bar{z}_\alpha))d\bar{z}_\alpha. \tag{18}$$

From (10), the Hamiltonian vector field is found as follows:

$$(-1)/(\mathbf{i}\cos\theta_\alpha)((\partial H)/(\partial\bar{z}_\alpha))(\partial/(\partial z_\alpha))+(1/(\mathbf{i}\cos\theta_\alpha))((\partial H)/(\partial z_\alpha))(\partial/(\bar{z}_\alpha))$$

$$=((dZ_\alpha)/(dt))(\partial/(\partial z_\alpha))+((d\overline{Z}_\alpha)/(dt))(\partial/(\bar{z}_\alpha)) \tag{19}$$

and then

$$Z_H=(-1)/(\mathbf{i}\cos\theta_\alpha)((\partial H)/(\partial\bar{z}_\alpha))(\partial/(\partial z_\alpha))+1/(\mathbf{i}\cos\theta_\alpha)((\partial H)/(\partial z_\alpha))(\partial/(\bar{z}_\alpha)). \tag{20}$$

Consider the curve and its velocity vector

$$\alpha:I\subset\mathbb{R}\to M,\ \alpha(t)=(z_\alpha,\bar{z}_\alpha),$$

$$\dot{\alpha}(t)=(\partial\alpha)/(\partial t)=((dz_\alpha)/(dt),(d\bar{z}_\alpha)/(dt)), \tag{21}$$

such that an integral curve of the Hamiltonian vector field Z_H, i.e.,

$$Z_H(\alpha(t))=(\partial\alpha)/(\partial t),\quad t\in I. \tag{22}$$

Then, we choose i instead of j and find the following equations;

$$\text{dif 1.}\quad (\partial H)/(\partial z_\alpha)=(-\mathbf{i}\cos\theta_\alpha)((d\overline{Z}_\alpha)/(dt)),$$

$$\text{dif 2.}\quad (\partial H)/(\partial z_\alpha)=(\mathbf{i}\cos\theta_\alpha)((dZ_\alpha)/(dt)). \tag{23}$$

Hence, the equations introduced in (23) are named Kähler-Einstein Hamilton Equations on Kähler-Einstein manifolds with equal Kähler angles and then the triple (M,Φ,Z_H) is said to be a Hamiltonian Mechanical System on Kähler-Einstein manifolds.

10. Solving and Graph of Hamilton Equations System

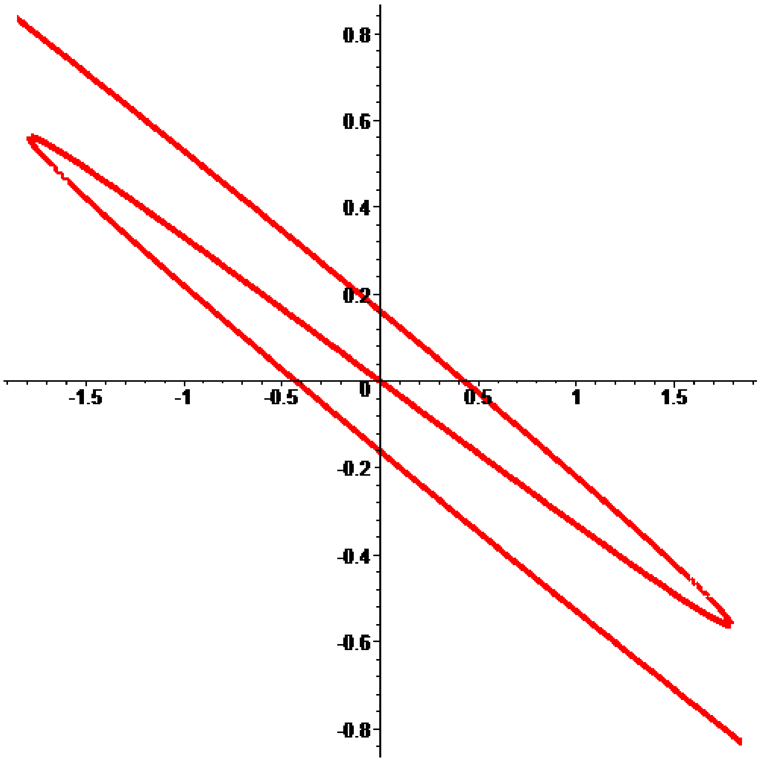
The location of each object in space is represented by three dimensions in physical space. These three dimensions can be labeled by a combination of three chosen from the terms time, length, width, height, depth, mass, density and breadth. (13) are partial differential equations and there are 4 independent variables on contact manifolds. Using Maple program, the solution of the equation system (13) is as follows.

$$H(Z_\alpha,Z_\alpha,t)=((-Z_\alpha+\overline{Z}_\alpha-F_1(t))*\cos^2(t)-2*\mathbf{i}*Z_\alpha*\cos(t)$$

$$+\overline{Z}_\alpha+Z_\alpha-F_1(t))/(\cos(t)*\mathbf{i}-1),$$

$$\text{for}\quad \theta_\alpha=0,\ Z_\alpha=\sin(t)+t.\mathbf{i},\ \overline{Z}_\alpha=\sin(t)-t.\ \mathbf{i},\ \mathbf{i}=\sqrt{-1}. \tag{24}$$

Example : We draw a graph based on specific selected function for system (13) for $F_1(t)=t$ of (24),



Graph 1 (25)

11. Discussion

In this study, the paths of Hamiltonian vector fields Z_H on Kähler-Einstein manifolds with equal Kähler angles are the solutions Kähler-Einstein Hamilton Equations raised in (13) on Kähler-Einstein manifolds particularly. Nowadays, well-known Hamiltonian models have emerged as a very important tool since they present a simple method to describe the model for mechanical systems. Furthermore, the metrics are interpreted as the gravitational potential, as in general relativity, and the corresponding forms are interpreted as the electromagnetic potentials. Therefore, the equations we have found are only considered to be a first step to realize how on Kähler-Einstein geometry has been used in solving problems in different physical areas. For further research, the conformal Hamilton mechanical systems based on Kähler-Einstein manifolds are recommended to deal with the matters in electromagnetically and gravitational fields of classical and quantum mechanics of physics [24].

In the literature, the equations, which explain the angle independent structures of the objects, were presented. None of these studies ignored the angle. This study explained the angle structures orbits of the objects in the space by the help of revised equations (13). Also, we used the Maple software to solve (24) and we were obtained of these equations solutions (25).

References

[1] Liu W.K. and Jun S., Computational nanomechanics of materials, Handbook of theoretical and computational nanotechnology, American Scientic Publishers, Stevenson Ranch, CA (2005).

[2] Bradley L., Classical mechanics notes for CSCI4446/6446, (1999), 1-13.

[3] Ioffe A., Euler-Lagrange and Hamiltonian formalisms in dynamic optimization, Trans. Amer. Math. Soc., **349(7)**, (1997), 2871-2900.

[4] Antoniou I. and Pronko G.P., The Hamiltonian Formalism In Fluid Dynamics, Theoretical and Mathematical Physics, **141(3)**, (2004), 1670-1685.

[5] Barker B. M. and O.Connell R. F., Lagrangian-Hamiltonian formalism for the gravitational two-body problem with spin and parametrimd post-Newtonian parameters, PR-D, **14(4)**, (1976), 861-869.

[6] Becker R. J. and Scherpen J.M.A., Lagrangian/Hamiltonian formalism for description of Navier-Stokes fluids, PRL, **58(14)**, (1987), 1419-1422.

[7] Tekkoyun M. and Yayli Y., Mechanical Systems on Generalized-Quaternionic Kähler Manifolds, IJGMMP, **8(7)**, (2011), 1.13.

[8] Spotti C., Degenerations of Kähler-Einstein Fano Manifolds, http://arxiv.org/pdf/1211.5334.pdf, (2012).

[9] Kasap Z.and Tekkoyun M., Mechanical Systems on Almost Para/Pseudo-Kähler.Weyl Manifolds, IJGMMP, **10(5)**, (2013), 1-8.

[10] X. Chen , C. LeBruny and B. Weber, On Conformally Käahler, Einstein Manifolds, J. Amer. Math. Soc., 21, (2008), 1137-1168.

[11] LeBrun C., Einstein manifolds and extremal Kähler metrics,Journal für die reine und angewandte Mathematik, ISSN (Print) 0075-4102, (2012).

[12] Heier G., Existence of Kähler.Einstein metrics and multiplier ideal sheaves on del Pezzo surfaces, Math. Z., **264**, (2010), 727-743.

[13] Li C., K-stability and Kähler-Einstein metrics, http://arxiv.org/pdf/1211.4669v2.pdf, (2013).

[14] van Coevering C., Kähler.Einstein metrics on strictly pseudoconvex domains, Annals of Global Analysis and Geometr, **42(3)**, (2012), 287-315.

[15] M. Rocek, Kähler-Einstein supermanifolds, ttp://www.eecs.berkeley.edu/~joschu/docs/superman.pdf.

[16] A.M. Nadel, Multiplier ideal sheaves and existence of Kahler-Einstein metrics of positive scalar curvature, Proc. Nati. Acad. Sci., **86**, (1989), 7299-7300.

[17] Cruceanu V., Gadea P.A. and Masqué J.M., Para-Hermitian and - Kähler manifolds, (http://digital.csic.es/bitstream/10261/15773/1/RockyCFG.PDF.

[18] Gilkey P. and Nikcevic S., Kähler and para-Kähler curvature Weyl manifolds, arXiv:1011.4844v1, (2010).

[19] Besse A.L., Einstein Manifolds , World Publishing Company, (1987).

[20] Ballmann W., Lectures on Kähler Manifolds, European Mathematical Society, (2006).

[21] Newlander A. and Nirenberg L., Complex analytic coordinates in almost complex manifolds. Ann. of Math., **65**, (1957), 391-404.

[22] Salavessa I.M.C. and Valli G., Minimal Submanifolds of Kähler-Einstein manifolds with Equal Kähler angles, Paci.c Journal of Mathematics, **205(1)**, (2002).

[23] Klein J., Escapes Variationnals et Mécanique, Ann. Inst. Fourier, Grenoble, **12**, (1962).

[24] Miron R., Hrimiuc D., Shimada H. And Sabau S.V., The geometry of Hamilton and Lagrange spaces, eBook ISBN: 0-306-47135-3, Kluwer Academic Publishers, New York, (2002).

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