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An Approach for Solving Multi-Objective Linear Fractional Programming Problem and It's Comparison with Other Techniques

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Abstract: *This paper presents Harmonic Average and Advanced Harmonic Average technique to solve multi-objective linear fractional programming problem (MOLFPP) to single objective linear fractional programming problem (SOLFPP) and suggested an algorithm for its solution. The proposed method can be illustrated with the help of numerical example. The numerical result in this paper indicates that Advanced Harmonic Average technique is better than other techniques (such as Chandra Sen., Mean, Median, Geometric Mean, New Geometric Average and Harmonic average).*

Keywords: *Linear Fractional Programming Problem (LFPP), Harmonic Average and Advanced Harmonic Average Techniques.*

1. Introduction

Linear Fractional Programming deals with that class of mathematical programming problems in which the relations among the variables are linear; the constraint relations must be in linear form and the objective function to be optimized must be a ratio of two linear functions such as profit/cost, actual cost/ standard cost, output/employee, etc and it is applied to different disciplines such as production planning, financial and corporative planning, health care and hospital planning. A study of multi-objective linear programming problem (MOLPP) is introduced in [2] which suggest an approach to set up multi-objective function under the limitation so that the optimum value of individual problem was greater than zero. Sulaiman and Sadiq studied the Multi-objective function by using mean and median technique [4]. Also Sulaiman and Salih studied the multi-objective fractional programming problem by using mean and median technique [5]. Nahar Samsun et al. suggested a new geometric average technique to optimize the objective function where a single objective function is developed from multi-objective functions [1]. In 2016 Sulaiman et al suggested a new technique by using Harmonic mean of the values of objective functions for solving Multi-objective linear programming problem [3].

In order to extend this work, we have defined MOLFPP and suggest an algorithm to solve linear factional programming problem for multi-objective functions by using Harmonic Average and Advanced Harmonic Average techniques. The result is compared with different techniques such as Chandra Sen., Mean & Median, Arithmetic Average Geometric Average and New Geometric Average. The Advanced Harmonic Average technique gives better result than all those techniques.

2. Mathematical Form of LFPP

The mathematical form of LFP problem is given as follows:

$$\text{Max. } Z = \frac{(\boldsymbol{c}^T\boldsymbol{X}+\alpha)}{(\boldsymbol{d}^T\boldsymbol{X}+\beta)}$$

Subject to:

$$AX\begin{pmatrix}\le\\ \ge\\ =\end{pmatrix}b$$

$$X \ge 0$$

Where X, c and d are n × 1 vectors, b is an m × 1 vector, c^T, d^T denote transpose of vectors, A is an m × n matrix and α , β are scalars.

3. Multi-Objective Linear Fractional Programming Problem

Multi-Objective functions that are the ratio of two linear objective functions are said to be MOLFPP which can be defined as:

$$\left.\begin{aligned}
\text{Max. } z_1 &= \frac{c_1{}^T X+\alpha_1}{d_1{}^T X+\beta_1}\\
\text{Max. } z_2 &= \frac{c_2{}^T X+\alpha_2}{d_2{}^T X+\beta_2}\\
&\vdots\\
\text{Max. } z_r &= \frac{c_r{}^T X+\alpha_r}{d_r{}^T X+\beta_r}\\
\text{Min. } z_{r+1} &= \frac{c_{r+1}{}^T X+\alpha_{r+1}}{d_{r+1}{}^T X+\beta_{r+1}}\\
&\vdots\\
\text{Min. } z_s &= \frac{c_s{}^T X+\alpha_s}{d_s{}^T X+\beta_s}
\end{aligned}\right] \tag{3.1}$$

Subject to:

$$AX = b \tag{3.2}$$
$$X \geq 0 \tag{3.3}$$

Where b is an m-dimensional vector of constants, X is an n-dimensional column vector of decision variables, r is number of objective functions to be maximized, s is the number of objective functions to be maximized and minimized and (s-r) is the number of objective functions that is minimized. A is an m× n matrix of constants, all vectors are assumed to be column vectors unless transposed(T). c_i, d_i (where i = 1,2,…,s) are n-dimensional vectors of constants, α_i, β_i(where i = 1,2,…,s) are scalars.

4. Solving MOLFPP by Using the Following Techniques

4.1 Harmonic Average Technique:-

Step1: Solve each objective function by using simplex technique.

Step2: Check the feasibility of the solution obtained in step1, if it is feasible then go to step3, otherwise use dual simplex technique to remove infeasibility.

Step3: Assign a name to the optimum value of each objective function Max z_i say φ_i, i = 1,2,…r and Min z_i say φ_i, i = r+1, r+2,…,s .

Step4: Calculate Harmonic Average $Hav_1 = Hav(|\varphi_i|), i$ =1,2,…r and $Hav_2 = Hav(|\varphi_i|), i =$ r+1, r+2,…,s.

Step5: Optimize the combined objective function under the same constraints (3.2) & (3.3) as follows:

$$\text{Max. Z} = \sum_{i=1}^{r} \frac{Max\ z_i}{Hav_1} - \sum_{i=r+1}^{s} \frac{Min\ z_i}{Hav_2} \tag{4.1.1}$$

4.2. Advanced Harmonic Average (AH_{av})Technique:-

Step1, Step2, Step3 are the same as given in algorithm (4.1).

Step4: Select $m_1 = min\{\varphi_i\}$, ∀ i = 1, 2,…,r and $m_2 = max\{\varphi_i\}$, ∀ i= r+1,…,s then calculate

$AH_{av} = \frac{2|m_1||m_2|}{|m_1|+|m_2|}$

Step5: Optimize the combined objective function under the same constraints (3.2) & (3.3) as:

$$\text{Max. Z} = \frac{(\sum_{i=1}^{r} Max\ z_i - \sum_{i=r+1}^{s} Min\ z_i)}{AH_{av}} \qquad (4.2.1)$$

5. NUMERICAL EXAMPLE

5.1. Example.

Max. $z_1 = \frac{3x_1-2x_2}{x_1+x_2+1}$

Max. $z_2 = \frac{9x_1+3x_2}{x_1+x_2+1}$

Max. $z_3 = \frac{3x_1-5x_2}{2x_1+2x_2+2}$

Min. $z_4 = \frac{-6x_1+2x_2}{2x_1+2x_2+2}$

Min. $z_5 = \frac{-3x_1-x_2}{x_1+x_2+1}$

Subject to:

$$x_1 + x_2 \leq 2, \quad 9x_1 + x_2 \leq 9, \quad x_1, x_2 \geq 0$$

Solution: After finding the value of each of individual objective functions, the results are given below:

Table 1

i	φ_i	x_i	Hav_1	Hav_2	AH_{av}
1	3/2	(1,0)	27/20		
2	9/2	(1,0)			1
3	3/4	(1,0)			
4	-3/2	(1,0)		3/2	
5	-3/2	(1,0)			

i) Harmonic Average Technique:-

$$\text{Max. Z} = \sum_{i=1}^{r} \frac{Max\ z_i}{Hav_1} - \sum_{i=r+1}^{s} \frac{Min\ z_i}{Hav_2}$$

$$\text{Max. } Z = \frac{126x_1-10x_2}{9x_1+9x_2+9}$$

Subject to:

$$x_1 + x_2 \leq 2, \quad 9x_1 + x_2 \leq 9, \quad x_1, x_2 \geq 0$$

Hence the optimal solution is:

Max. Z = 7, $x_1 = 1$, $x_2 = 0$.

ii) Advanced Harmonic Average (AH_{av}) Technique:-

$$\text{Max. Z} = \frac{(\sum_{i=1}^{r} Max\ z_i - \sum_{i=r+1}^{s} Min\ z_i)}{AH_{av}} \quad \text{where } AH_{av} = \frac{2|m_1||m_2|}{|m_1|+|m_2|}$$

$$\text{Max. } Z = \frac{39x_1-3x_2}{2x_1+2x_2+2}$$

Subject to:

$$x_1 + x_2 \leq 2, \quad 9x_1 + x_2 \leq 9, \quad x_1, x_2 \geq 0$$

Hence the optimal solution is:

Max. Z = 9.75, x_1 = 1, $x_2 = 0$.

6. Comparison of the Numerical Results

Comparison of the numerical results which are obtained from the example 5.1 is shown in the following table2:

Table 2

Techniques	**Example 6.1**
Chandra Sen. Technique	5
Mean Technique	5
Median Technique	6.5
Arithmetic Mean	5
New Arithmetic Average	8.665
Geometric Mean	5.931
New Geometric Average	9.1895
Harmonic Average	7
Advanced Harmonic Average	9.75

In the above table**,** it is clear that the results obtained in example 5.1 when using advanced harmonic average technique is better than other results.

7. Conclusion

In this paper, we have defined Harmonic Average and Advanced Harmonic Average techniques and then compare Advanced Harmonic Average technique with other techniques namely Chandra Sen., Mean & Median, Arithmetic Mean & New Arithmetic Average, Geometric Mean & Advanced Geometric Average and Harmonic Average techniques.

The comparisons of these techniques are based on the value of the objective functions. After solving the numerical example, we found that Max.Z which obtained by our technique(Advanced Harmonic average technique) is better than other techniques(Chandra Sen., mean & median, arithmetic mean & new arithmetic average, geometric mean & new geometric average and harmonic average techniques).

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