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Peristaltic Transport of a Third-Order Nano-Fluid in a Circular Cylindrical Tube with Radiation and Chemical Reaction

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Abstract: *Explicit Finite-Difference method was used to obtain the solution of the system of the non-linear ordinary differential equations which obtained from the non-linear partial differential equations. These equations describe the two- dimensional flow of a MHD third-order Nano-fluid with heat and mass transfer in a circular cylindrical tube having two walls that are transversely displaced by an infinite, harmonic traveling wave of large wave length. Accordingly, the solutions of momentum, energy, concentration, and Nano-particles concentration equations were obtained. The numerical formula of the stream function, the velocity, the temperature, the concentration, and the Nano-particles distributions of the problem were illustrated graphically. Effects of some parameters of this problem such as, local nanoparticle Grashofnumber Br, local temperature Grashof number Gr, Darcy number Da, magnetic field parameter M, Eckert number Ec, Dufour number Nd, Brownian motion parameter Nb, Thermophoresis parameter Nt, Prandtl number Pr, radiation parameter Rn, Lewis number Le, Sort number Sr,and Chemical reaction parameter Rc on those formula were discussed. Also, an estimation of the global error for the numerical values of the solutions is calculated by using Zadunaisky technique.*

Keywords: *Third-order nano-Fluid, peristatic Motion, MHD flows, Porous medium, Radiation, Chemical reaction.*

Nomenclature

A	Chemical Reaction rate constant	Re	Reynolds number, defined by Eq. (38)
B_r	Local nanoparticle Grashof number, defined by Eq. (38)	Rn	Radiation parameter, defined by Eq. (38)
C	The concentration of the fluid	S	The dimensionless nanoparticles
C	The concentration at the centerline $(y = 0)$	Sr	Sort number, defined by Eq. (38)
C	The concentration at the wall $(y = h)$	t	The time
C	Nanoparticle susceptibility	T	The fluid temperature
Da	Darcy number, defined by Eq. (38)	T_1	The temperature at the centerline $(y = 0)$
D_B	Brownian diffusion coefficient	T_2	The temperature at the wall $(y = h)$
D_T	Thermophoretic diffusion coefficient	$\underline{V}$	The velocity vector
			Greek symbols
$\underline{E}$	Electrical field		
Ec	Eckert number, defined by Eq. (38)	$\hat{\phi}$	The nanoparticles phenomena
$\underline{F}$	The external force	Φ	The dissipation function
G	Gravitational acceleration	ϕ	The dimensionless concentration
G	Local temperature Grashof number,	θ	The dimensionless temperature

$_r$	defined by Eq. (38)		
$\underline{H}$	The magnetic field	∇	Gradient operator
$\underline{J}$	The currentdensity,$\underline{J} = \sigma(\underline{E} + \mu_e \underline{V} \wedge \underline{H})$	∇^2	Laplacian operator
k	Thermal conductivity	Γ	Deborah number, defined by Eq. (38)
k	Permeability constant	δ	Wave number, defined by Eq. (38)
$\kappa_($	The mean absorption coefficient	μ	the dynamic viscosity of fluid
k_T	Thermal diffusion ratio	μ_e	the magnetic permeability
Le	Lewis number, defined by Eq. (38)	ν	The kinematic viscosity($^{\mu}/_{\rho_f}$)
m	Chemical Reaction order	c_p	The specific heat capacity at constant pressure
M	Magnetic field parameter, defined by Eq. (38)	ρ_f	The density of the fluid
N_1	Viscosity parameter, defined by Eq. (38)	ρ_p	The density of the particle
Nb	Brownian motion parameter, defined by Eq. (38)	(ρc)	heat capacity of the fluid
Nd	Dufournumbe, defined by Eq. (38)	(ρc)	effective heat capacity of the nanoparticle material
Nt	The thermophoresis parameter, defined by Eq. (38)	σ	Electrical conductivity of the fluid
P	The fluid pressure	σ_0	Stefan Boltizman constant
Pr	Prandtl number, defined by Eq. (38)	τ	The Cauchy Stress tensor
q	The radiative heat flux	λ_1, λ	The material coefficients, defined by Eq. (38)
R	Chemical reaction parameter, defined by Eq. (38)	α_T , c	Volumetric thermal and solute expansion coefficients of the base fluid

1. INTRODUCTION

The analysis of flow dynamics of a fluid in a circular tube induced by a travelling wave on its wall has numerous applications in various branches of science. The word peristaltic stems from the Greek word peristaltikos which means clasping and compressing. The peristaltic transport is a physical mechanism that occurs due to the action of a progressive wave which propagates along thelength of a distensible tube containing fluid. The peristaltic mechanism is nature’s way of moving the content within hollow structures by successive contraction on their muscular fibers. This mechanism is responsible for transport of biological fluids such as urine in the ureter, chime in gastro-intestinal tract, semen in the vas deferens and ovum in the female fallopian tube [1]. The application of peristaltic motion as a mean of transporting fluid has aroused interested in engineering fields. Latham [2] was probably the first to study the mechanism of peristaltic pumping in his M. S. Thesis. Several researches have analyzed the phenomenon of peristaltic transport under various assumptions. Haroun [3] studied the effect of a third-order fluid on the peristaltic transport in an asymmetric channel. Eldabe et al. [4] analyzed the incompressible flow of electrically conducting biviscosity fluid through an axisymmetric non-uniform tube with a sinusoidal wave under the considerations of long wavelength and low Reynolds number.

Due to complexity of fluids, several models mainly based on empirical observations have been proposed for non-Newtonian fluids. Amongst these, there is a particular class of fluids, called viscoelastic fluid, which has specially attracted the attention of numerous researchers for varying reasons. The rheologists have been able to provide a theoretical foundation in the form of a constitutive equation which can in principle, have any ‘order’. For applied mathematicians and computer scientists the challenge comes from a different quarter. The constitutive equations of even the simplest viscoelastic fluids, namely second grade fluids are such that the differential equations describing the motion have, in general, their order higher than those describing the motion of the Newtonian fluids but apparently there is no corresponding increase in the number of boundary conditions. Applied mathematicians and computer scientists are thus forced with the so-called ill-posed boundary value problems which, in theory, would have a family of infinitely many solutions [5].

Non-Newtonian nanofluids are widely encountered in many industrial and technology applications, for example, melts of polymers, biological solutions, paints, asphalts and glues etc. Nanofluids appear to have the potential to significantly increase heat transfer rates in a variety of areas such as industrial cooling applications, nuclear reactors, transportation industry, micro-electromechanical systems, heat exchangers, chemical catalytic reactors, fiber and granular insulations, packed beds, petroleum reservoirs and nuclear waste repositories and biomedical applications [6].Choi [7] is the first to use the term nano-fluid, who proposed that nanometer sized metallic particles can be suspended in industrial heat transfer fluids. Therefore, a nano-fluid is a suspension of nano-particles (metallic, non-metallic, or polymeric) in a conventional base fluid which enhances its heat transfer characteristics. Enhanced thermal properties of nano-fluids enable them to use in automotive industry, power plants, cooling systems, computers, etc [8].

In recent years, biomagnetic fluid dynamic is an interesting area of research. This is due to its extensive applications in bioengineering and medical sciences. Examples include the development of magnetic devices for cell separation, targeted transport of drugs using magnetic particles as drug carriers, magnetic wound or cancer tumor treatment causing magnetic hyperthermia, reduction of bleeding during surgeries or provocation of occlusion of the feeding vessels of cancer tumors and development of magnetic tracers [9].

Thermal radiation has a significant role in the overall surface heat transfer when the convection heat transfer coefficient is small. Thermal radiation effect on mixed convection from vertical surface in a porous medium was studied by Bakier[10]. He applied fourth-order Runge-Kutta scheme to solve the governing equations. Effect of MHD flow with mixed convection from radiative vertical isothermal surface embedded in a porous medium was numerically analyzed by Damseh [11]. He used an implicit iterative tri-diagonal finite difference method in order to solve the dimensionless boundary-layer equations. It is well known that the effect of thermal radiation is important in space technology and high temperature processes. Thermal radiation also plays an important role in controlling heat transfer process in polymer processing industry. The effect of radiation on heat transfer problems have been studied by Hossain and Takhar [12]. Zahmatkesh [13] has found that the presence of thermal radiation makes temperature distribution nearly uniform in the vertical sections inside the enclosure and causes the streamlines to be nearly parallel with the vertical walls. Mohsen et al. [14] studied the effect of thermal radiation on magneto-hydrodynamics nanofluid flow and heat transfer by means of two phase model.

Heat and mass transfer problems with chemical reaction are significant in many processes such as drying, evaporations at the surface of a water body, geothermal reservoirs, thermal insulation, enhanced oil recovery, cooling of nuclear reactors and the flow in a desert cooler. This type of flows have many applications in industries. Many practical diffusive operations involve the molecular diffusion of species in the presence of chemical reaction within or at the boundary. Heat and mass transfer effects are also encountered in the chemical industry, in reservoirs, in thermal recovery processes and in the study of hot salty springs in the sea [15]. Vajravelu et al. [16] investigated heat and mass transfer properties of three-layer fluid flow in which nano-fluid layer is squeezed between two clear viscous fluid. Farooq et al. [17] studied heat and mass transfer of two-layer flows of third-grade nano-fluids in a vertical channel. Abou-zeid et al. [18] obtained numerical solutions and Global error estimation of natural convection effects on gliding motion of bacteria on a power-law nano-slime through a non-Darcy porous medium. El-Dabe et al. [19] investigated magneto-hydrodynamic non-Newtonian nano-fluid flow over a stretching sheet through a non-Darcy porous medium with radiation and chemical reaction.Shaaban et al. [20] studied the effects of heat and mass transfer on MHD peristaltic flow of a non-Newtonian fluid through a porous medium between two coaxial cylinders.

The objective of this work is to investigate the numerical solution by using Explicit Finite Difference method [21] for the system of non-linear differential equations which arises from the two-dimensional flow of a magneto-hydrodynamic third-order Nano-fluid with heat and mass transfer in a circular cylindrical tube having two walls that are transversely displaced by an infinite, harmonic traveling wave of large wave length.We obtained the distributions of the stream function, the velocity, the temperature, the concentration, and the Nanoparticles. Numerical results are found for different values of various non-dimensional parameters. The results are shown graphically and discussed in detail. Also, the global error estimation for the error propagation is obtained by Zadunaisky technique [22].

2. Fluid Model

By using arguments of modern continuum thermodynamics, the model for a fluid of third grade has been derived by Fosdick and Rajagopal [23]. They show that the constitutive relation for the stress tensor for an incompressible, third-order, homogeneous fluid has the form,

$\overline{\tau} = -\overline{P}I + \overline{S}$,(1)

$$\overline{S} = \mu\,\overline{A}_1 + \alpha_1\overline{A}_2 + \alpha_2\overline{A}_1^2 + \beta\left(tr\,\overline{A}_1^2\right)\overline{A}_1 \quad . \tag{2}$$

Here, $-\overline{P}I$ is the indeterminate part of the stress due to the constraint of incompressibility, $\overline{A}_n$ are the Rivlin-Ericksen tensors, defined by

$$\overline{A}_1 = \left(\nabla\overline{V}\right) + \left(\nabla\overline{V}\right)^T \quad ,$$

$$\overline{A}_n = \frac{d\overline{A}_{n-1}}{d\,\overline{t}} + \overline{A}_{n-1}\left(\nabla\overline{V}\right) + \left(\nabla\overline{V}\right)^T\overline{A}_{n-1} \quad , \quad n > 1 \ . \tag{3}$$

Where, $\overline{V}$is the velocity and $\frac{d}{d\,\overline{t}}$ is the material time derivative. The clausius-Duhem inequality and the requirement that the free energy be a minimum in equilibrium imposes the following constraints on the dynamic viscosity μ, the normal stress coefficients α_1 and α_2, and the coefficient β,

$$\mu \geq 0 \quad , \ \alpha_1 \geq 0 \ , \ \beta \geq 0 \ , \ |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta} \quad . \tag{4}$$

Further remarks of third-order fluid models may be found in Dunn and Rajagopal [24].

3. Mathematical Formulation

Consider a two-dimensional infinite tube of uniform width $2n$ filled with an incompressible non-Newtonian nano-fluid obeying third-order model through a porous medium with heat and mass transfer of solar radiation with chemical reaction. A uniform magnetic field intensity H_0 is imposed and acting along the $\overline{Y}$ −axis. By using cartesian coordinates, the tube walls are parallel to the $\overline{X}$ −axis and located at $\overline{Y} = \pm n$.

We assume that an infinite train of sinusoidal waves progresses with velocity c along the walls in the $\overline{X} -$ direction.

The geometry of the wall surface is defined as

$$\overline{h}\left(\overline{x}\ ,\ \overline{t}\right) = n + b\sin\left(\frac{2\pi}{\lambda}\left(\overline{x} - c\,\overline{t}\right)\right) \quad . \tag{5}$$

Where, b is the amplitude and λ is the wavelength. We also assume that there is no motion of the wall in the longitudinal direction.

4. Basic Equations

The basic equations governing the flow of an incompressible Nano-fluid are the following equations [25, 26],

The continuity equation

$$\nabla.\underline{V} = 0, \tag{6}$$

The momentum equation

$\rho_f \frac{d\overline{V}}{d\overline{t}} = \nabla.\overline{\tau} + \mu_e\left(\underline{J} \wedge \underline{H}\right) - \frac{\mu}{k^*}\underline{\overline{V}} + \underline{F}$,(7) The energy equation

$$\left(\rho c_p\right)_f\left[\frac{dT}{d\bar{t}}\right]=\kappa\,\nabla^2 T+\Phi+\nabla.q+\left(\rho c_p\right)_p\left[D_B\left(\nabla T.\nabla\hat{\phi}\right)+\frac{D_T}{T_1}(\nabla T)^2\right]+\frac{\rho_f D_B \kappa_T}{C_s}\,\nabla^2 C,\quad (8)$$

The concentration equation

$$\frac{dC}{d\bar{t}}=D_B\,\nabla^2 C+\frac{D_T\kappa_T}{T_1}\,\nabla^2 T-A(C-C_2)^m,\qquad (9)$$

The nanoparticles concentration equation

$$\frac{d\hat{\phi}}{d\bar{t}}=D_B\nabla^2\hat{\phi}+\frac{D_T}{T_2}\nabla^2 T\qquad (10)$$

For unsteady two-dimensional flows, velocity components can be written as follows,

$$\underline{\bar{V}}=\left(\bar{U}\left(\bar{X},\bar{Y},\bar{t}\right),\ \bar{V}\left(\bar{X},\bar{Y},\bar{t}\right),0\right)\qquad (11)$$

Also, the temperature, the concentration, and the nanoparticles functions can be written as follows,

$$T=T\left(\bar{X},\bar{Y},\bar{t}\right)\quad,\quad C=C\left(\bar{X},\bar{Y},\bar{t}\right)\quad,\quad\hat{\phi}\left(\bar{X},\bar{Y},\bar{t}\right)\ .\qquad (12)$$

The equations (6), (7) and the constitutive relations (1), (2), and (3) take the following form:-

$$\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\partial\bar{V}}{\partial\bar{Y}}=0\qquad ,(13)$$

$$\rho_f\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right)\bar{U}=-\frac{\partial\bar{P}}{\partial\bar{X}}+\mu\left(\frac{\partial^2}{\partial\bar{X}^2}+\frac{\partial^2}{\partial\bar{Y}^2}\right)\bar{U}-\frac{\mu}{k^*}\bar{U}-\sigma\mu_e^2H_0^2\bar{U}+\frac{\partial\bar{S}_{\overline{XX}}}{\partial\bar{X}}+\frac{\partial\bar{S}_{\overline{XY}}}{\partial\bar{Y}}+\rho_f g\alpha_T(T-T_2)+\rho_f g\alpha_C(C-C_2)\qquad ,\qquad (14)$$

$$\rho_f\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{X}}+\bar{V}\frac{\partial}{\partial\bar{Y}}\right)\bar{V}=-\frac{\partial\bar{P}}{\partial\bar{Y}}+\mu\left(\frac{\partial^2}{\partial\bar{X}^2}+\frac{\partial^2}{\partial\bar{Y}^2}\right)\bar{V}-\frac{\mu}{k^*}\bar{V}+\frac{\partial\bar{S}_{\overline{XY}}}{\partial\bar{X}}+\frac{\partial\bar{S}_{\overline{YY}}}{\partial\bar{Y}}\quad ,\qquad (15)$$

$$\bar{S}_{\overline{XX}}=2\mu\frac{\partial\bar{U}}{\partial\bar{X}}+4(\alpha_1+\alpha_2)\left(\frac{\partial\bar{U}}{\partial\bar{X}}\right)^2+2\alpha_1\frac{\partial\bar{U}}{\partial\bar{Y}}\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)+\alpha_2\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)^2+2\beta\frac{\partial\bar{U}}{\partial\bar{X}}\left(4\left(\frac{\partial\bar{U}}{\partial\bar{X}}\right)^2+4\left(\frac{\partial\bar{V}}{\partial\bar{Y}}\right)^2+2\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)^2\right)\quad ,\qquad (16)$$

$$\bar{S}_{\overline{XY}}=\bar{S}_{\overline{YX}}=\mu\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)+2\alpha_1\left(\frac{\partial\bar{U}}{\partial\bar{X}}\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\frac{\partial\bar{V}}{\partial\bar{Y}}\right)+(\alpha_1+2\alpha_2)\left(\frac{\partial\bar{U}}{\partial\bar{X}}+\frac{\partial\bar{V}}{\partial\bar{Y}}\right)\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)+\beta\left(4\left(\frac{\partial\bar{U}}{\partial\bar{X}}\right)^2+4\left(\frac{\partial\bar{V}}{\partial\bar{Y}}\right)^2+2\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)^2\right)\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right),\qquad (17)$$

$$\bar{S}_{\overline{YY}}=2\mu\frac{\partial\bar{V}}{\partial\bar{Y}}+2\alpha_1\frac{\partial\bar{V}}{\partial\bar{X}}\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)+4(\alpha_1+\alpha_2)\left(\frac{\partial\bar{V}}{\partial\bar{Y}}\right)^2+\alpha_2\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)^2+2\beta\frac{\partial\bar{V}}{\partial\bar{Y}}\left(4\left(\frac{\partial\bar{U}}{\partial\bar{X}}\right)^2+4\left(\frac{\partial\bar{V}}{\partial\bar{Y}}\right)^2+2\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)^2\right)\quad .\qquad (18)$$

The dissipation function Φ can be written as follows

$$\Phi=\bar{\tau}_{ij}\frac{\partial\bar{V}_i}{\partial\bar{X}_j}\qquad ,\qquad (19)$$

$$\Phi=\bar{S}_{\overline{XX}}\frac{\partial\bar{U}}{\partial\bar{X}}+\bar{S}_{\overline{XY}}\left(\frac{\partial\bar{V}}{\partial\bar{X}}+\frac{\partial\bar{U}}{\partial\bar{Y}}\right)+\bar{S}_{\overline{YY}}\frac{\partial\bar{V}}{\partial\bar{Y}}\qquad ,\qquad (20)$$

Also, by using Rosselant approximation [27] we have,

$$q=\frac{4\sigma_0}{3\kappa_0}\frac{\partial T^4}{\partial Y}\qquad ,\qquad (21)$$

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_2, and neglecting higher-order terms [28], one gets,

$$T^4 \cong 4T_2^3 T - 3T_2^4 \quad . \tag{22}$$

Then, equations (8), (9), and (10) can be written as follows:-

$$(\rho c_p)_f \left[\frac{\partial T}{\partial \bar{t}} + \bar{U}\frac{\partial T}{\partial \bar{X}} + \bar{V}\frac{\partial T}{\partial \bar{Y}}\right] = \kappa\left(\frac{\partial^2 T}{\partial \bar{X}^2} + \frac{\partial^2 T}{\partial \bar{Y}^2}\right) + \frac{16\sigma_0}{3\kappa_0}T_2^3\frac{\partial^2 T}{\partial \bar{Y}^2} + \bar{S}_{XX}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{S}_{XY}\left(\frac{\partial \bar{V}}{\partial \bar{X}} + \frac{\partial \bar{U}}{\partial \bar{Y}}\right) + \bar{S}_{YY}\frac{\partial \bar{V}}{\partial \bar{Y}} + (\rho c_p)_p\left[D_B\left(\frac{\partial T}{\partial \bar{X}}.\frac{\partial \hat{\phi}}{\partial \bar{X}} + \frac{\partial T}{\partial \bar{Y}}\frac{\partial \hat{\phi}}{\partial \bar{Y}}\right) + \frac{D_T}{T_1}\left(\left(\frac{\partial T}{\partial \bar{X}}\right)^2 + \left(\frac{\partial T}{\partial \bar{Y}}\right)^2\right)\right] + \frac{\rho_f D_B \kappa_T}{C_s}\left(\frac{\partial^2 C}{\partial \bar{X}^2} + \frac{\partial^2 C}{\partial \bar{Y}^2}\right) \quad , \tag{23}$$

$$\left[\frac{\partial C}{\partial \bar{t}} + \bar{U}\frac{\partial C}{\partial \bar{X}} + \bar{V}\frac{\partial C}{\partial \bar{Y}}\right] = D_B\left(\frac{\partial^2 C}{\partial \bar{X}^2} + \frac{\partial^2 C}{\partial \bar{Y}^2}\right) + \frac{D_T \kappa_T}{T_1}\left(\frac{\partial^2 T}{\partial \bar{X}^2} + \frac{\partial^2 T}{\partial \bar{Y}^2}\right) - A(C - C_2)^m \quad , \tag{24}$$

$$\left[\frac{\partial \hat{\phi}}{\partial \bar{t}} + \bar{U}\frac{\partial \hat{\phi}}{\partial \bar{X}} + \bar{V}\frac{\partial \hat{\phi}}{\partial \bar{Y}}\right] = D_B\left(\frac{\partial^2 \hat{\phi}}{\partial \bar{X}^2} + \frac{\partial^2 \hat{\phi}}{\partial \bar{Y}^2}\right) + \frac{D_T}{T_2}\left(\frac{\partial^2 T}{\partial \bar{X}^2} + \frac{\partial^2 T}{\partial \bar{Y}^2}\right) \quad . \tag{25}$$

In the fixed coordinate system $(\bar{X},\ \bar{Y})$, the motion is unsteady because of the moving boundary. However, if observed in a coordinate system $(\bar{x},\ \bar{y})$ moving with the speed c, it can be treated as steady because the boundary shape appears to be stationary.

The transformation between the two frames is given by

$$\bar{x} = \bar{X} - c\,\bar{t} \quad , \qquad \bar{y} = \bar{Y} \quad . \tag{26}$$

The velocities in the fixed and moving frames are related by

$$\bar{u} = \bar{U} - c \quad , \qquad \bar{v} = \bar{V} \quad . \tag{27}$$

Where, $\bar{u}, \bar{v}$ are components of the velocity in the moving coordinate system.

And, in order to simplify the governing equations of the motion, we may introduce the following dimensionless transformations:-

$$x = \frac{2\pi}{\lambda}\bar{x} \quad , \quad y = \frac{\bar{y}}{n} \quad , \quad u = \frac{\bar{u}}{c} \quad , \quad v = \frac{\bar{v}}{c} \quad ,$$

$$s = \frac{n}{\pi c}\bar{s} \quad , \quad p = \frac{2\,\pi\, n^2}{\lambda(\mu + \eta)c}\bar{p} \quad , \quad h = \frac{\bar{h}}{n} \; ,$$

$$\theta = \frac{T - T_2}{T_1 - T_2} \quad , \quad \phi = \frac{C - C_2}{C_1 - C_2} \quad , \quad S = \frac{\hat{\phi} - \hat{\phi}_2}{\hat{\phi}_1 - \hat{\phi}_2} \quad . \tag{28}$$

Substituting by equations (26), (27), and (28) into equations (13) – (18), and (23) – (25) we obtain the following non-dimensionless equations:-

$$\delta\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad , \tag{29}$$

$$Re\left(\delta\, u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right) = -(1 + N_1)\frac{\partial p}{\partial x} + \left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{Da}\,u - M\,u + \delta\frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{xy}}{\partial y} + Gr\,\theta + Br\,\phi \quad , \tag{30}$$

$$\delta\,Re\,\left(\delta\, u\,\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -(1 + N_1)\frac{\partial p}{\partial y} + \delta\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\delta}{Da}\,v + \delta^2\frac{\partial s_{xy}}{\partial x} + \delta\frac{\partial s_{yy}}{\partial y} \; , \tag{31}$$

$$s_{xx} = 2\delta\frac{\partial u}{\partial x} + 4(\lambda_1 + \lambda_2)\delta^2\left(\frac{\partial u}{\partial x}\right)^2 + 2\lambda_1\frac{\partial u}{\partial y}\left(\delta\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + \lambda_2\left(\delta\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + 2\Gamma\delta\frac{\partial u}{\partial x}\left(4\delta^2\left(\frac{\partial u}{\partial x}\right)^2 + 4\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\delta\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right) \quad , \tag{32}$$

$$s_{xy} = s_{yx} = \left(\delta\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + 2\lambda_1\left(\delta^2\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\right) + (\lambda_1 + 2\lambda_2)\left(\delta\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$\left(\delta\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+\Gamma\left(4\delta^2\left(\frac{\partial u}{\partial x}\right)^2+4\left(\frac{\partial v}{\partial y}\right)^2+2\left(\delta\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right)\left(\delta\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)$,(33)

$$s_{yy}=2\frac{\partial v}{\partial y}+2\lambda_1\delta\frac{\partial v}{\partial x}\left(\delta\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+4(\lambda_1+\lambda_2)\left(\frac{\partial v}{\partial y}\right)^2+$$

$$\lambda_2\left(\delta\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\ 2\Gamma\frac{\partial v}{\partial y}\left(4\delta^2\left(\frac{\partial u}{\partial x}\right)^2+4\left(\frac{\partial v}{\partial y}\right)^2+2\left(\delta\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right). \quad (34)$$

$$Re\left[\delta u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\frac{1}{Pr}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+\frac{4}{3Rn}\frac{\partial^2\theta}{\partial y^2}+Ec\,\delta\,s_{xx}\frac{\partial u}{\partial x}+Ec\,s_{xy}\left(\delta\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)+Ec\,s_{yy}\frac{\partial v}{\partial y}+$$
$$\left[Nb\ \left(\delta^2\frac{\partial\theta}{\partial x}\cdot\frac{\partial S}{\partial x}+\frac{\partial\theta}{\partial y}\cdot\frac{\partial S}{\partial y}\right)+Nt\left(\delta^2\left(\frac{\partial\theta}{\partial x}\right)^2+\left(\frac{\partial\theta}{\partial y}\right)^2\right)\right]+Nd\ \left(\delta^2\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)\quad , \quad (35)$$

$$Re\left[\delta u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}\right]=\frac{1}{Le}\left(\delta^2\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+Sr\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-Rc(\phi)^m\quad , \quad (36)$$

$$Re\ \left[\delta u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}\right]=Nb\left(\delta^2\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}\right)+Nt\ \left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)\quad . \quad (37)$$

Where, the dimensionless parameters are defined by:-

$\delta=\frac{2\pi n}{\lambda},\quad Re=\frac{\rho c n}{\mu}, N_1=\frac{\eta}{\mu}, Da=\frac{k^*}{n^2}, M=\frac{\sigma\mu_e^2H_0^2n^2}{\mu}$

$\lambda_1=\frac{\alpha_1 c}{\mu\ n}\quad ,\quad \lambda_2=\frac{\alpha_2 c}{\mu\ n}\ ,\ \Gamma=\frac{\beta c^2}{\mu n^2},\ Gr=\frac{gn\alpha_T(T_1-T_2)}{c^2}, Br=\frac{gn\alpha_C(C_1-C_2)}{c^2}$

$Pr=\frac{\nu(\rho c_p)_f}{\kappa},\ Rn=\frac{(\rho c_p)_f\kappa_0\nu}{4\sigma_0T_2^3},\ Ec=\frac{c^2}{c_p(T_1-T_2)}, Nd=\frac{D_B\kappa_T(C_1-C_2)}{C_sc_p\ \nu\ (T_1-T_2)}\quad ,Nb=\frac{D_B(\hat{\phi}_1-\hat{\phi}_2)}{\nu}$

$Nt=\frac{D_T\ (T_1-T_2)}{T_1\ \nu}\quad ,\quad Le=\frac{\nu}{D_B}\quad ,\quad Sr=\frac{D_T\ \kappa_T(T_1-T_2)}{T_1\ \nu\ (C_1-C_2)}\quad ,\quad R_c=\frac{n^2A(C_1-C_2)^{m-1}}{\nu}$ (38)

Equation (29) allows the introducing of the dimensionless stream function $\psi\ (x,y)$ in terms of

$u=\frac{\partial\psi}{\partial y}\quad ,\quad v=-\delta\frac{\partial\psi}{\partial x}\quad .$ (39)

Then, we carry out our investigation on the basis that the dimensionless wave number is small, that is,

$\delta<<1,$ (40)

which corresponds to the long-wavelength approximation [29]. Thus, to lowest order in δ, equations (30 – 37) give,

$(1+N_1)\frac{\partial p}{\partial x}=\left(\frac{\partial^3\psi}{\partial y^3}\right)-\left(\frac{1}{Da}+M\right)\frac{\partial\psi}{\partial y}+\frac{\partial s_{xy}}{\partial y}+Gr\,\theta+Br\,\phi\quad ,$ (41)

$\frac{\partial p}{\partial y}=0\ ,$ (42)

$s_{xx}=(2\lambda_1+\lambda_2)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2\quad ,$ (43)

$s_{xy}=s_{yx}=\left(\frac{\partial^2\psi}{\partial y^2}\right)+2\Gamma\left(\frac{\partial^2\psi}{\partial y^2}\right)^3\quad ,$ (44)

$s_{yy}=\ \lambda_2\left(\frac{\partial^2\psi}{\partial y^2}\right)^2$,(45)

$\left(\frac{1}{Pr}+\frac{4}{3Rn}\right)\frac{\partial^2\theta}{\partial y^2}+Ec\,s_{xy}\left(\frac{\partial^2\psi}{\partial y^2}\right)+Nb\ \left(\frac{\partial\theta}{\partial y}\cdot\frac{\partial S}{\partial y}\right)+Nt\left(\frac{\partial\theta}{\partial y}\right)^2\ +Nd\ \left(\frac{\partial^2\phi}{\partial y^2}\right)=0\quad ,$ (46)

$\frac{1}{Le}\left(\frac{\partial^2\phi}{\partial y^2}\right)+Sr\left(\frac{\partial^2\theta}{\partial y^2}\right)-Rc(\phi)^m=0\quad ,$ (47)

$$Nb\left(\frac{\partial^2 S}{\partial y^2}\right)+Nt\left(\frac{\partial^2 \theta}{\partial y^2}\right)=0 \quad . \tag{48}$$

Boundary conditions in dimensionless form are:-

For the stream function in the moving frame are,

$$\left.\begin{aligned} \psi &= 0 \quad , \quad (by\ convection) \\ \frac{\partial^2\psi}{\partial y^2} &= 0 \quad , \quad (by\ symmetry) \end{aligned}\right\} \text{on the centerline } y=0 ,$$

$$\left.\begin{aligned} \frac{\partial\psi}{\partial y} &= -1 \quad , \quad (no\ slip\ condition) \\ \psi &= F \quad . \end{aligned}\right\} \quad \text{at the wall } y=h \ .$$

Where, F is the total flux number. We also note that $h$represents the dimensionless form of the surface of the peristaltic wall.

$$h(x)=1+\chi \sin x \quad . \tag{49}$$

Where, $\chi=\frac{b}{n}$ (is the amplitude ratio or the occlusion)

For the temperature, the concentration, and the nanoparticles are,

$$\theta=1 \ , \quad \phi=1 \ , \quad S=1 \quad , \quad at\ \ y=0 \ ,$$

$$\theta=0 \ , \quad \phi=0 \ , \quad S=0 \quad , \quad at\ \ y=h \ . \tag{50}$$

Then the dimensionless boundary conditions can be written as

$$\psi=0 \ , \ \frac{\partial^2\psi}{\partial y^2}=0 \ , \theta=1 \ , \quad \phi=1 \ , \quad S=1 \quad , \quad at\ \ y=0 \ ,$$

$$\psi=F \ , \ \frac{\partial\psi}{\partial y}=-1 \ , \ \theta=0 \ , \quad \phi=0 \ , \quad S=0 \quad , \quad at\ \ y=h \ . \tag{51}$$

Substituting (44) into (41) and (46) and using (42) we finally have,

$$\psi^{(4)}=\frac{\left(\frac{1}{Da}+M\right)\psi''-Gr\,\theta'-Br\,\phi'-12\,\Gamma\psi''\,\psi'''^{\,2}}{\left(2+6\Gamma\,\psi''^{\,2}\right)} \quad ,$$

$$\theta''=\frac{-Ec\psi''^{\,2}-2Ec\,\Gamma\,\psi''^{\,4}-Nd\,\phi''-Nb\,\theta' S'-Nt\,\theta'^{\,2}}{\left(\frac{1}{Pr}+\frac{4}{3Rn}\right)} ,$$

$$\phi''=-Le\ Sr\ \theta''+Le\ Rc\ \phi^{m} ,$$

$$S''=-\frac{Nt}{Nb}\theta'' . \tag{52}$$

The system of non-linear ordinary differential (52) together with the boundary conditions (51), will be solved numerically by using the explicit finite-difference method. And, we computed the global error for the solutions of the problem.

5. NUMERICAL SOLUTION

The equations (52) can be written after applied explicit finite difference schemes [21] as:

$\left(\frac{2\psi[i+2]-9\psi[i+1]+16\psi[i]-14\psi[i-1]+6\psi[i-2]-\psi[i-3]}{h^4}\right)\left(2+6\,\Gamma\left(\frac{\psi[i+1]-2\psi[i]+\psi[i-1]}{h^2}\right)^2\right)-\left[\left(M+\right.\right.$
1Daψi+1−2ψi+ψi−1h2−Grθi+1−θi−12h−Brϕi+1−ϕi−12h−12
Γψi+1−2ψi+ψi−1h2ψi+2−3ψi+1+3ψi−ψi−1h32=0,

$$\left(\frac{1}{Pr}+\frac{4}{3Rn}\right)\left(\frac{\theta[i+1]-2\theta[i]+\theta[i-1]}{h^2}\right)+Ec\left(\frac{\psi[i+1]-2\psi[i]+\psi[i-1]}{h^2}\right)^2+2Ec\,\Gamma\left(\frac{\psi[i+1]-2\psi[i]+\psi[i-1]}{h^2}\right)^4+ Nd\left(\frac{\phi[i+1]-2\phi[i]+\phi[i-1]}{h^2}\right)+Nb\left(\frac{\theta[i+1]-\theta[i-1]}{2h}\right)\left(\frac{S[i+1]-S[i-1]}{2h}\right)+Nt\left(\frac{\theta[i+1]-\theta[i-1]}{2h}\right)^2=0 \quad ,$$

$$\left(\frac{\phi[i+1]-2\phi[i]+\phi[i-1]}{h^2}\right)+Le\,Sr\left(\frac{\theta[i+1]-2\theta[i]+\theta[i-1]}{h^2}\right)-Le\,Rc\,(\phi[i])^m=0 \quad ,$$

$$\left(\frac{S[i+1]-2S[i]+S[i-1]}{h^2}\right)+\frac{Nt}{Nb}\left(\frac{\theta[i+1]-2\theta[i]+\theta[i-1]}{h^2}\right)=0 \quad , \qquad (53)$$

Where the index i refers to y and the $\Delta\, y = h = 0.04$. According to the boundary conditions (51) we can solved equations (53) numerically, then a Newtonian iteration method continues until either of goals specified by accuracy goal or precision goal is achieved.

6. Estimation of the Global Error

We used Zadunaisky technique [22] for calculating the global error for the solutions of the problem. We calculate an estimation of the global error from the formulas,

$$\underline{e}_n = \underline{f}_n - \underline{f}(x_n) = \underline{f}_n - \underline{P}(x_n) \quad , \quad (n = 0,1,2,\dots\dots\dots,25) \, . \qquad (54)$$

In this relation, $\underline{f}_n$ is the approximate solutions of the new problem (the pseudo-problem) at the point x_n , and $\underline{f}(x_n)$ is the exact solutions of pseudo-problem at x_n .

The values of global error for the solutions of the problem which solved by the explicit finite difference method are shown in (tab.1). The error in (tab. 1) based on using 26 points to find interpolating polynomials $P_i(x)$ of degree 25. In order to achieve the above task, we used the Mathematica package.

Table 1. *The global error by using finite difference technique*

y	u =Y2	error (e2n)	θ =Y5	error (e5n)	ϕ= Y7	error (e_{7n})	S=Y9	error (e9n)
0	-2.37728	7.29673E-04	1.00E+00	0.00E+00	1.00E+00	0.00E+00	1.00E+00	0.00E+00
0.08	-2.37381	2.89037E-04	0.942545	2.13292E-04	0.916916	2.47102E-05	0.915491	2.34465E-04
0.16	-2.36041	6.39575E-04	0.883307	4.12111E-04	0.834279	4.59696E-05	0.831339	1.37629E-03
0.24	-2.33457	1.41259E-03	0.822206	5.87384E-04	0.752052	6.27746E-05	0.747559	3.26827E-03
0.32	-2.29477	1.74905E-03	0.759054	7.26556E-04	0.670211	7.38514E-05	0.664189	5.73609E-03
0.4	-2.2394	1.62674E-03	0.69352	8.12272E-04	0.588752	7.75766E-05	0.581296	8.58304E-03
0.48	-2.16637	1.12886E-03	0.625087	8.24749E-04	0.507695	7.22617E-05	0.498983	1.15732E-02
0.56	-2.07304	3.78352E-04	0.552965	7.45725E-04	0.427089	5.66182E-05	0.417407	1.4408E-02
0.64	-1.95602	4.79557E-04	0.475984	5.64493E-04	0.347028	3.04824E-05	0.336803	1.6695E-02
0.72	-1.81115	1.27505E-03	0.392423	2.88096E-04	0.267663	3.95057E-06	0.257515	1.79035E-02
0.8	-1.6333	1.80269E-03	0.299767	4.04455E-05	0.189231	3.99397E-05	0.180047	1.73019E-02
0.88	-1.41622	1.80209E-03	0.194348	3.0874E-04	0.112087	6.23851E-05	0.10513	1.38654E-02
0.96	-1.15231	9.21908E-04	0.0708377	2.72493E-04	0.0367567	4.16893E-05	0.0338325	6.12791E-03
1	-1.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Note: this table contains the values of the dimensionless physical quantities, the velocity u, the temperature θ, the concentration ϕ, and the nano-particle concentration S at the values of the dimensionless distance y, Also, this table contains the values of the global error e2n of u, the global error e5n of θ, the global error e7n of ϕ, and the global error e9n of S, by using finite difference technique.

7. Numerical Results and Discussion

In this paper we generalized the problem ofa magneto-hydrodynamic third-order Nano-fluid with heat and mass transfer in a circular cylindrical tube having two walls that are transversely displaced by an infinite, harmonic traveling wave of large wave length. The system of non-linear ordinary differential equations (52) with the boundary conditions (51) was solved numerically by using an explicit finite difference method. The functionsψ, u , θ, ϕ, and $S$are obtained and illustrated graphically as shown in figures (1-17) for different values of the parameters of the problem. We studied all parameters of the problem, but we selected some of them to save space.

Figure (1) shows the distribution of the velocity profile u at different values of local nanoparticle Grashofnumber Br. It is clear that, the velocity increases by increasing local nanoparticle Grashof number Br in the region $0 \le y \le 0.45$ and, it returns decrease at $0.45 \le y \le 1$. And, we have led to the local temperature Grashof number Greffects on the velocity the same effect of local nanoparticle Grashofnumber Br on the velocity.

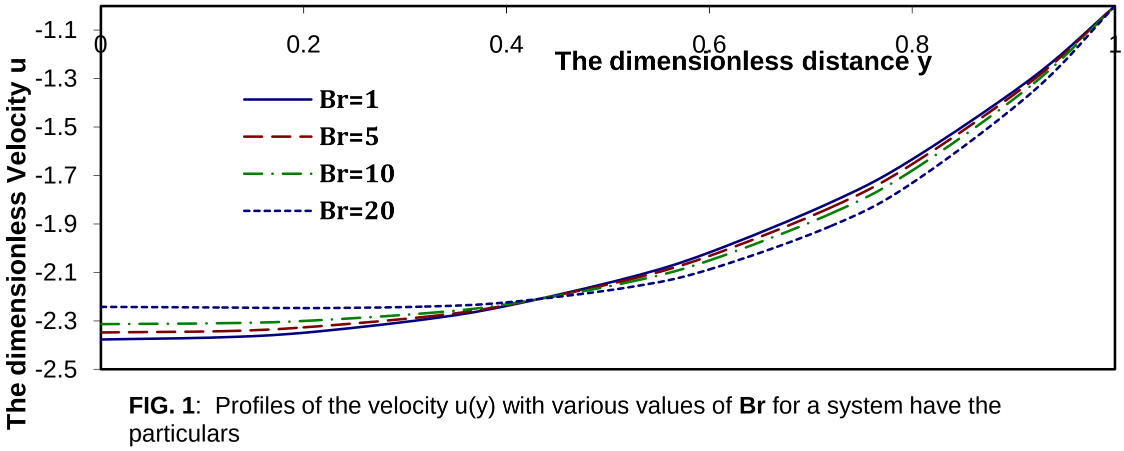

FIG. 1: Profiles of the velocity u(y) with various values of **Br** for a system have the particulars
M=1.5, Da=0.1, Gr=0.1, Ec=0.5, Pr=0.7, Rn=2, Nd=2, Nb=0.5, Nt=0.1, Le=0.5, Sr=0.2, R c=0.1, m=2.

Figure (2) describes the effect of the Darcy number Da on the stream function. It is noted that, by increasing of the Darcy number Da, the stream function decreases. And, we have led to the Darcy number Da effects on the temperature the opposite effect of the Darcy number Da on the stream function. Figure (3) illustrates the distributions of the velocity profile at different values of the Darcy number Da. It is seen that, the velocity decreases by increasing the Darcy number Da in the region $0 \le y \le 0.55$ and, it returns increase at $0.55 \le y \le 1$.

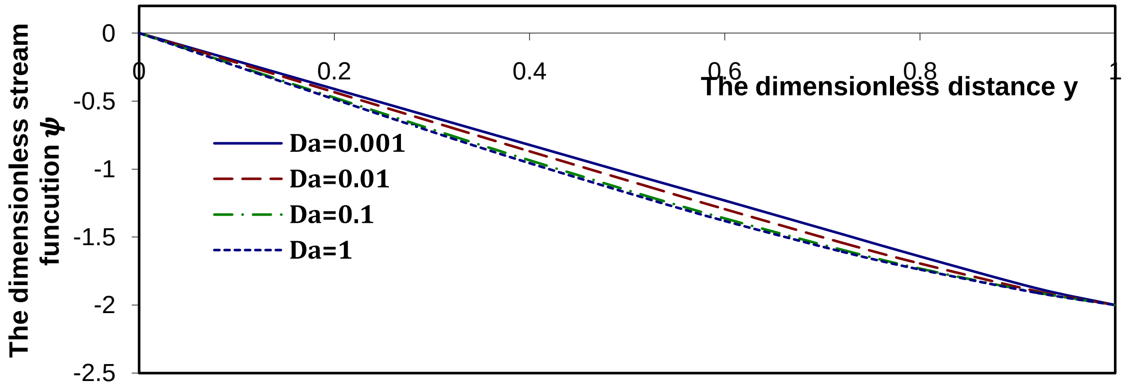

FIG. 2: Profiles of the stream function ψ(y) with various values of **Da** for a system have the particulars
M=1.5, Gr=0.1, Br=0.1, Ec=0.5, Pr=0.7, Rn=2, Nd=2, Nb=0.5, Nt=0.1, Le=0.5, Sr=0.2, Rc =0.1, m=2.

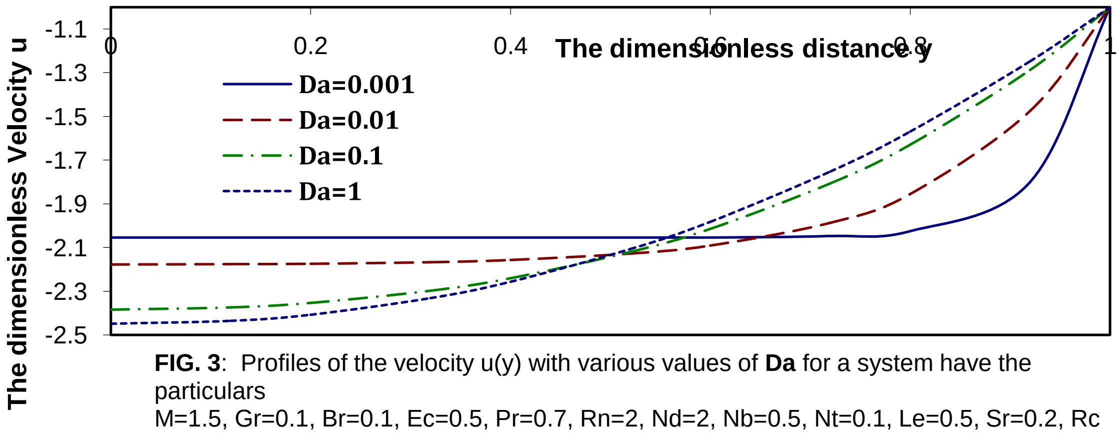

FIG. 3: Profiles of the velocity u(y) with various values of **Da** for a system have the particulars
M=1.5, Gr=0.1, Br=0.1, Ec=0.5, Pr=0.7, Rn=2, Nd=2, Nb=0.5, Nt=0.1, Le=0.5, Sr=0.2, Rc =0.1, m=2.

Figure (4) shows the effect of magnetic field parameter M on the stream function. It is clear that, the stream function increases by increasing of magnetic field parameter M. And, we have led to magnetic field parameter M effects on the temperature the opposite effect of magnetic field parameter M on the stream function. Figure (5) illustrates the effect of magnetic field parameter M on the velocity profiles. It is clear that, the velocity increases by increasing magnetic field parameter M in the region $0 \leq y \leq 0.5$ and, it returns decrease at $0.5 \leq y \leq 1$.

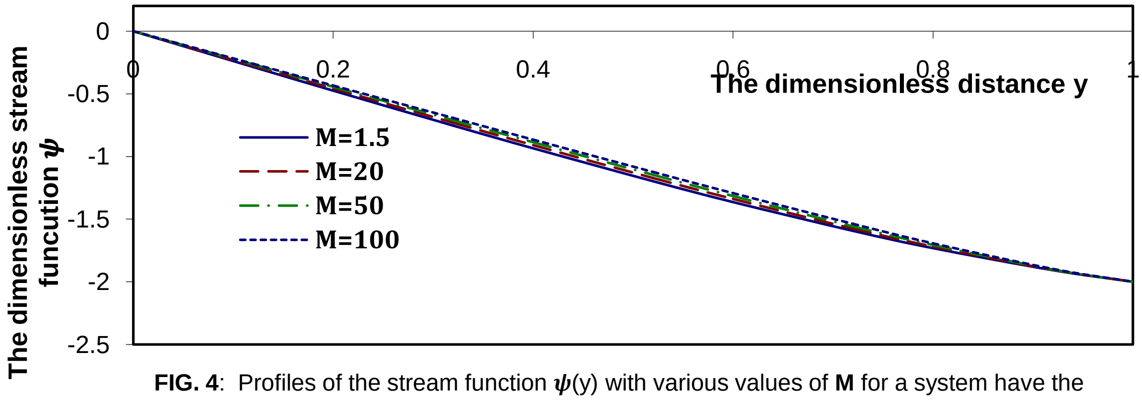

FIG. 4: Profiles of the stream function ψ(y) with various values of **M** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, Ec=0.5, Pr=0.7, Rn=2, Nd=2, Nb=0.5, Nt=0.1, Le=0.5, Sr=0.2, Rc= 0.1, m=2.

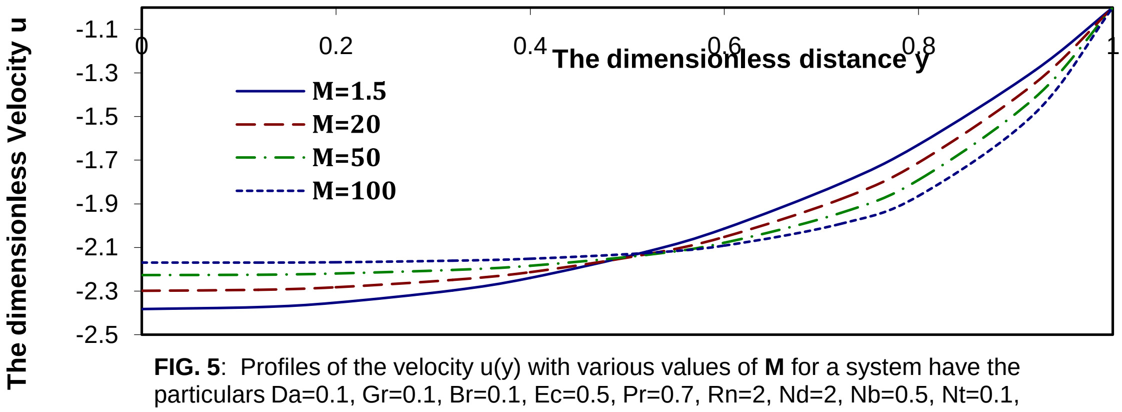

FIG. 5: Profiles of the velocity u(y) with various values of **M** for a system have the particulars Da=0.1, Gr=0.1, Br=0.1, Ec=0.5, Pr=0.7, Rn=2, Nd=2, Nb=0.5, Nt=0.1, Le=0.5, Sr=0.2, Rc=0.1, m=2.

Figure (6, 7) show the effect of Eckert number Ec on the temperature, and concentration profiles, respectively. It is seen that, the temperature increases by increasing of Eckert number Ec, but the concentration decreases by increasing of Eckert number Ec.And, we have led to Eckert number Ec effects on the nanoparticles the same effect of Eckert number Ec on the concentration.

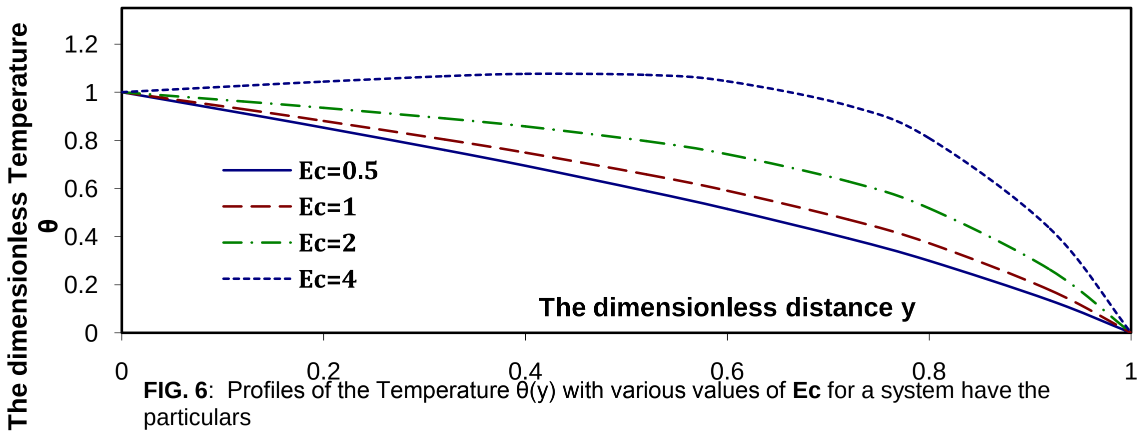

FIG. 6: Profiles of the Temperature θ(y) with various values of **Ec** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Pr=0.7, Rn=2, Nd=2, Nb=0.5, Nt=0.1, Le=0.5, Sr=0.2, Rc= 0.1, m=2.

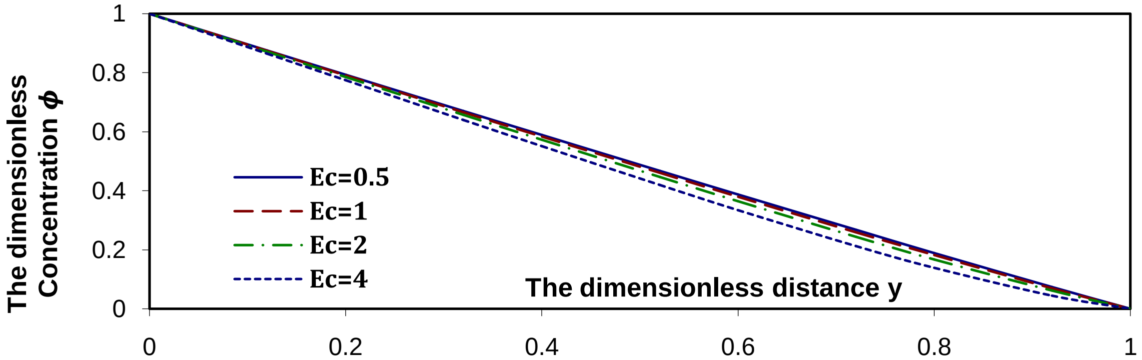

FIG. 7: Profiles of the Concentration ϕ(y) with various values of **Ec** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Pr=0.7, Rn=2, Nd=2, Nb=0.5, Nt=0.1, Le=0.5, Sr=0.2, R c=0.1, m=2.

Figure (8) illustrates the effect of Dufour number Nd on the nanoparticles profiles. It is seen that, the nanoparticles decreases by increasing of Dufour number Nd. And, we have led to Dufournumber Nd effects on the concentration the same effect of Dufour number Nd on the nanoparticles, and the opposite effect on the temperature.

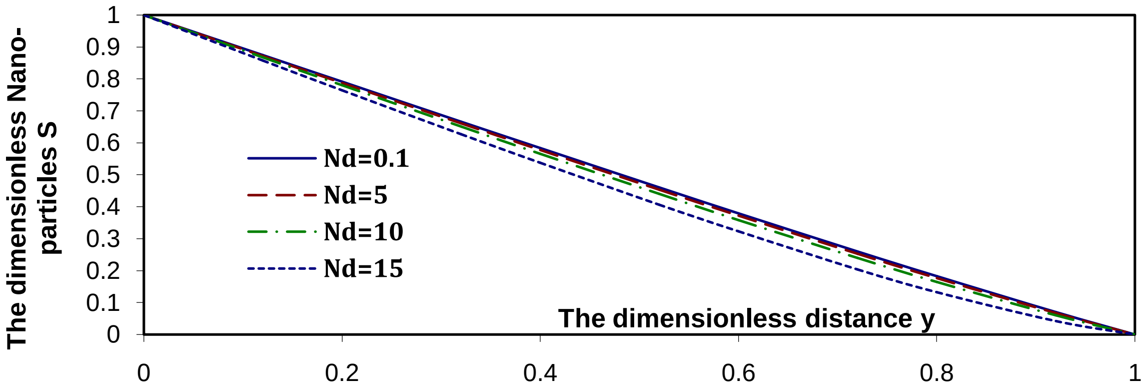

FIG. 8: Profiles of the Nano-particles S(y) with various values of **Nd** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Pr=0.7, Rn=2, Ec=0.5, Nb=0.5, Nt=0.1, Le=0.5, Sr=0.2, Rc=0.1, m=2.

Figure (9) obtains the effect of Brownian motion parameter Nb on the concentration profiles. It is clear that, the concentration decreases by increasing of Brownian motion parameter Nb. And, we have led to Brownian motion parameter Nb effects on the temperature and the nanoparticles the same effect of Brownian motion parameter Nb on the concentration.

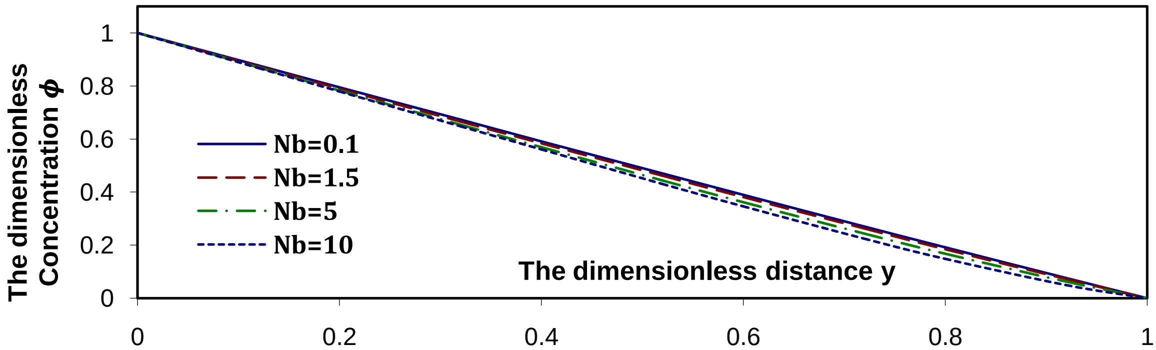

FIG. 9: Profiles of the Concentration ϕ(y) with various values of **Nb** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Pr=0.7, Rn=2, Ec=0.5, Nd=2, Nt=0.1, Le=0.5, Sr=0.2, Rc =0.1, m=2.

Figure (10, 11) shows the effect of Thermophoresis parameter Nt on the temperature and the concentration, respectively. It is clear that, the temperature increases by increasing of Thermophoresis parameter Nt, but the concentration decreases by increasing of Thermophoresis parameter Nt. And, we have led to Thermophoresis parameter Nt effects on the nanoparticles the same effect of Thermophoresis parameter Nt on the concentration.

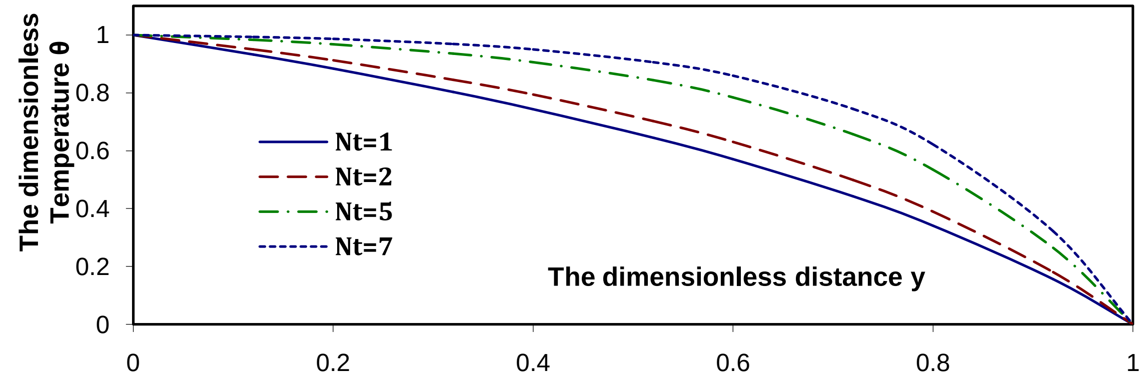

FIG. 10: Profiles of the Temperature θ(y) with various values of **Nt** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Pr=0.7, Rn=2, Ec=0.5, Nd=2, Nb=0.5, Le=0.5, Sr=0.2, R c=0.1, m=2.

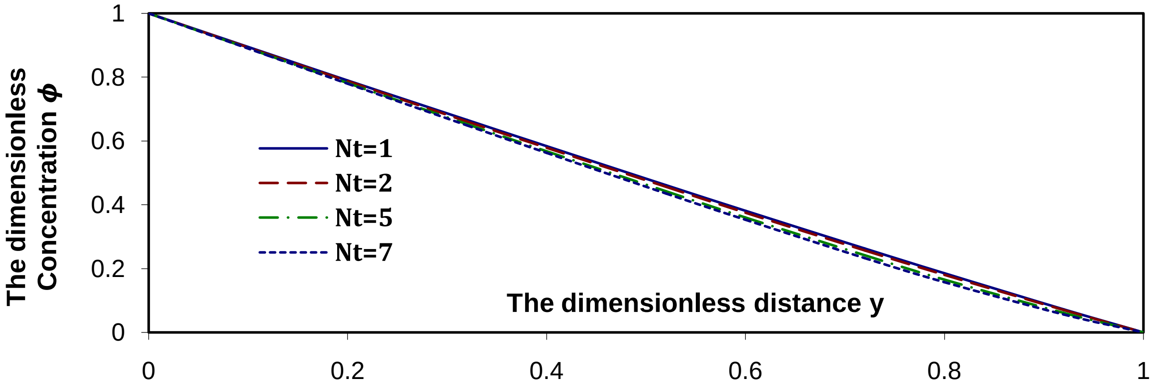

FIG. 11: Profiles of the Concentration ϕ(y) with various values of **Nt** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Pr=0.7, Rn=2, Ec=0.5, Nd=2, Nb=0.5, Le=0.5, Sr=0.2, R c=0.1, m=2.

Figure (12) shows the effect of Prandtl number Pr on the nanoparticles. It is seen that, the nanoparticles decreases by increasing of Prandtl number Pr. And, we have led to Prandtl number Pr effects on the concentration the same effect of Prandtl number Pr on the nanoparticles, and the opposite effect on the temperature.

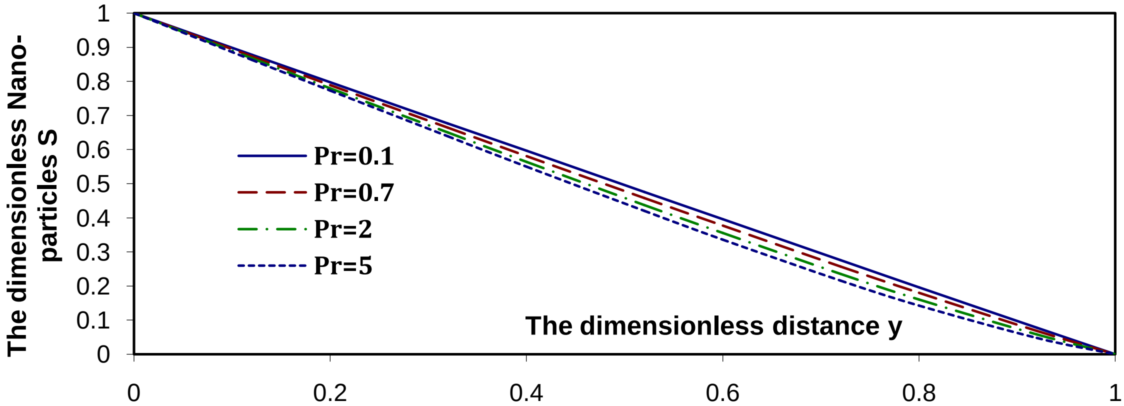

FIG. 12: Profiles of the Nano-particles **S**(y) with various values of **Pr** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Nt=0.1, Rn=2, Ec=0.5, Nd=2, Nb=0.5, Le=0.5, Sr=0.2, R c=0.1, m=2.

Figure (13, 14) illustrate the effect of radiation parameter Rnon the temperature and nanoparticles, respectively. It is shown that, the temperature increases by increasing of radiation parameter Rn, but the nanoparticles decreases by increasing of radiation parameter Rn. And, we have led to radiation parameter Rn effects on the concentration the same effect of radiation parameter Rn on the nanoparticles.

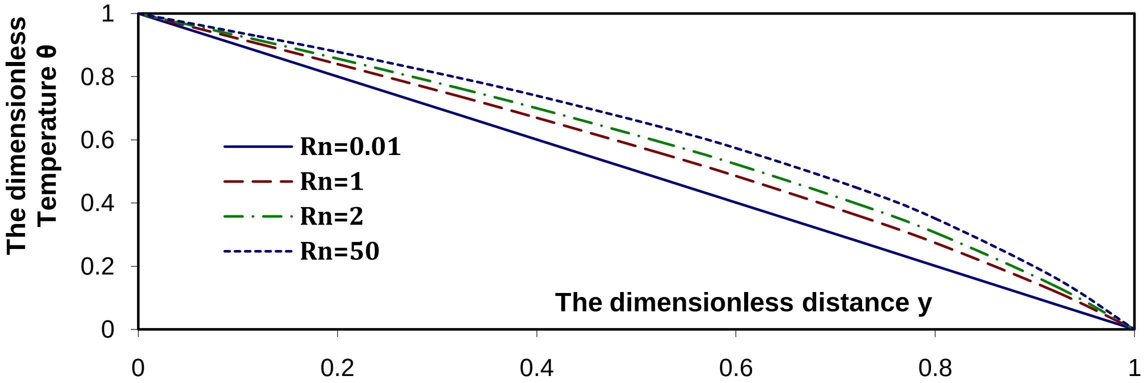

FIG. 13: Profiles of the Temperature θ(y) with various values of **Rn** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Nt=0.1, Pr=0.7, Ec=0.5, Nd=2, Nb=0.5, Le=0.5, Sr=0.2, Rc=0.1, m=2.

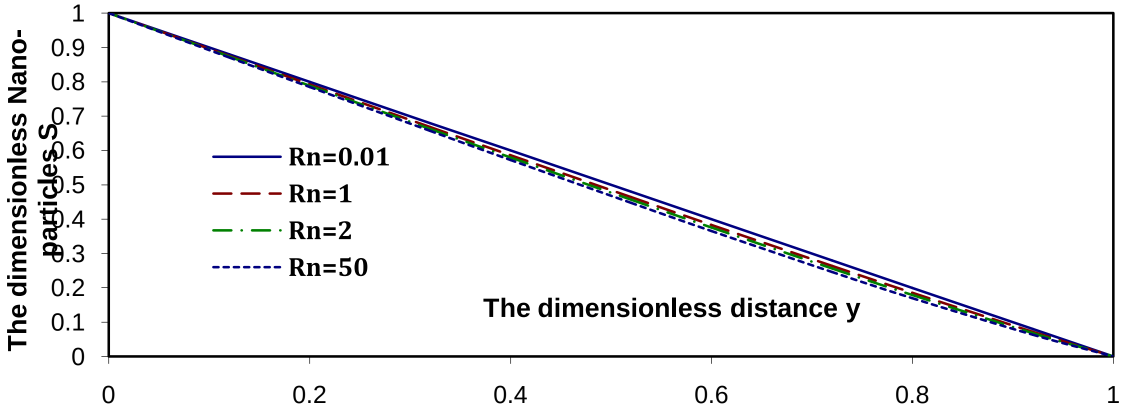

FIG. 14: Profiles of the Nano-particles **S**(y) with various values of **Rn** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Nt=0.1, Pr=0.7, Ec=0.5, Nd=2, Nb=0.5, Le=0.5, Sr=0.2, Rc=0.1, m=2.

Figure (15) shows the effect of Lewis number Le on the concentration. It is clear that, the concentration decreases by increasing of Lewis number Le. And, we have led to Lewis number Le effects on the nanoparticles the same effect of Lewis number Le on the concentration, and the opposite effect on the temperature.

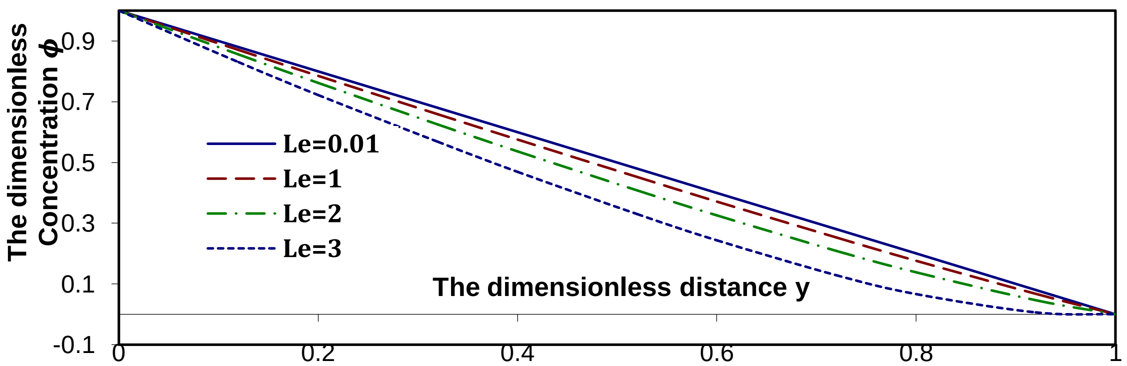

FIG. 15: Profiles of the Concentration ϕ(y) with various values of **Le** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Nt=0.1, Pr=0.7, Ec=0.5, Nd=2, Nb=0.5, Rn=2, Sr=0.2, R c=0.1, m=2.

Figure (16) shows the effect of Sort number Sr on the nanoparticles. It is seen that, the nanoparticles decreases by increasing of Sort number Sr. And, we have led to Sort number Sr effects on the concentration the same effect Sort number Sr on the nanoparticles, and the opposite effect on the temperature.

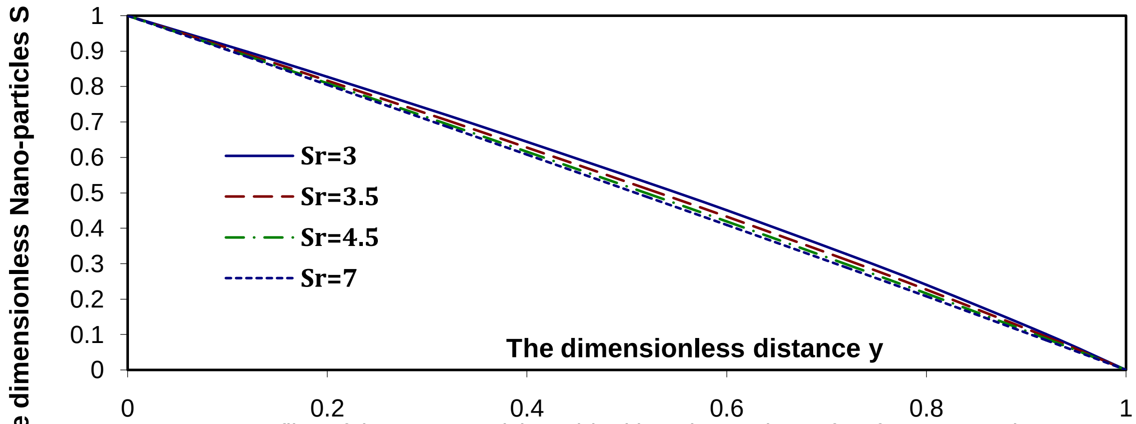

FIG. 16: Profiles of the Nano-particles **S**(y) with various values of **Sr** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Nt=0.1, Pr=0.7, Ec=0.5, Nd=2, Nb=0.5, Rn=2, Le=0.5, Rc=0.1, m=2.

Figure (17) shows the effect of Chemical reaction parameter Rc on the concentration. It is clear that, the concentration decreases by increasing of Chemical reaction parameter Rc. And, we have led to Chemical reaction parameter Rc effects on the nanoparticles the same effect of Chemical reaction parameter Rc on the concentration, and the opposite effect on the temperature.

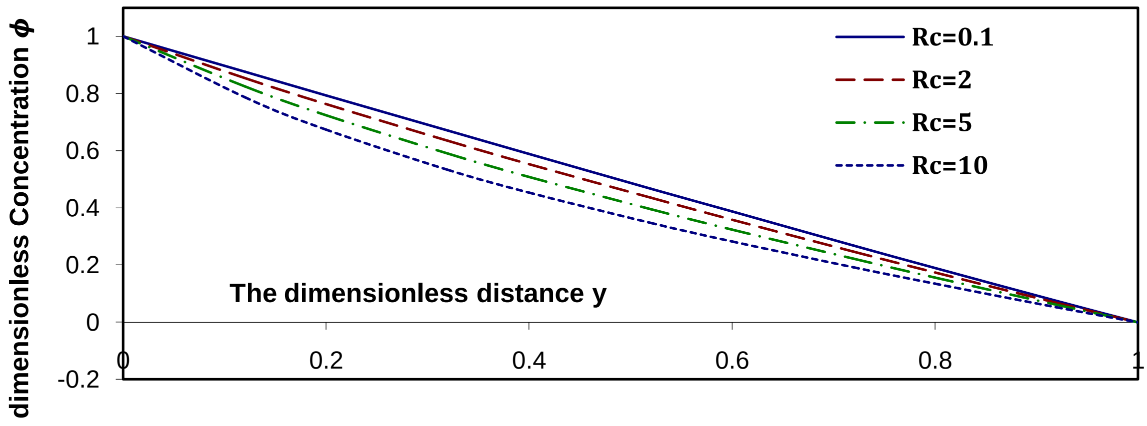

FIG. 17: Profiles of the Concentration ϕ(y) with various values of **Rc** for a system have the particulars
Da=0.1, Gr=0.1, Br=0.1, M=1.5, Nt=0.1, Pr=0.7, Ec=0.5, Nd=2, Nb=0.5, Rn=2, Le=0.5, S r=0.2, m=2.

8. Conclusion

In this work, we have studied a magneto-hydrodynamic third-order Nano-fluid with heat and mass transfer in a circular cylindrical tube having two walls that are transversely displaced by an infinite, harmonic traveling wave of large wave length. The governing boundary value problem was solved numerically by an Explicit Finite-Difference method. We concentrated our work on obtaining the stream function, the velocity, the temperature, the concentration, and the nanoparticlesdistributions which are illustrated graphically at different values of the parameters of the problem. Global error estimation is also obtained using Zadunaisky technique. We used 26 points to find the interpolating polynomial of degree 25 in interval [0,1] and the results are shown in (tab.1). We notice that, the error in (tab.1) is good enough to justify the use of resulting numerical values.

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