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Approximation of Alternating Series using Correction Function and Error Function

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Abstract: *In this paper we give a rational approximation of an alternating series using remainder term of the series. For that we shall introduce a correction function to the series. The correction function plays a vital role in series approximation. Using correction function we shall deduce an error function to the series.*

Keywords: *Correction function, error function, remainder term, alternating series, Madhava series, rational approximation.*

1. Introduction

In 14^{th} century, the Indian mathematician Madhava gave an approximation of the pi series using remainder term of the series.

Madhava series is,

C $= \frac{4d}{1} - \frac{4d}{3} + \frac{4d}{5} - \dots + (-1)^{n-1}\frac{4d}{2n-1} + (-1)^n \frac{4d(2n)/2}{(2n)^2+1}$, where C is the circumference of a circle of diameter d. Here the remainder term is $(-1)^n$ 4d G_n where $G_n = \frac{(2n)/2}{(2n)^2+1}$ is the correction function. The introduction of the correction term gives a better approximation of the series.

2. Method

APPROXIMATION OF THE ALTERNATING SERIES $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)(n+2)}$

The alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)(n+2)}$ satisfies the conditions of alternating series test and so it is convergent.

If R_n denotes the **remainder term** after n terms of the series, then

$R_n = (-1)^n G_n$ where G_n is the **correction function** after n terms of the series

Theorem:

The correction function for the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)(n+2)}$ is

$$G_n = \frac{1}{2n^3+9n^2+\frac{29}{2}n+\frac{33}{4}}$$

<u>Proof:</u>

If G_n denotes the correction function after n terms of the series, then

we have $G_n + G_{n+1} = \frac{1}{(n+1)(n+2)(n+3)}$

The error function is $E_n = G_n + G_{n+1} - \frac{1}{(n+1)(n+2)(n+3)}$

For $r_1, r_2\ \ r_3 \in R$ and for any fixed n ,

Let $G_n\ (r_1, r_2, r_3) = \frac{1}{2n^3+12n^2+22n+12-(r_1n^2+r_2n+r_3)}$

Then the error function is

$E_n\ (r_1, r_2, r_3) = G_n\ (r_1, r_2, r_3) + G_{n+1}(r_1, r_2, r_3) - \frac{1}{(n+1)(n+2)(n+3)}$ is a rational function of r_1, r_2 and r_3.

ie $\quad E_n(r_1, r_2, r_3) = \frac{N_n(r_1,r_2,r_3)}{D_n(r_1,r_2,r_3)}$

$D_n(r_1, r_2, r_3) \approx 4n^9$ is a maximum for large n.

$|N_n(r_1, r_2, r_3)|$ is minimum for $r_1=3,\ r_2=\frac{15}{2},\ r_3=\frac{15}{4}$

So $|E_n(r_1, r_2, r_3)|$ is minimum for $r_1=3,\ r_2=\frac{15}{2},\ r_3=\frac{15}{4}$

Thus for $r_1=3,\ r_2=\frac{15}{2},\ r_3=\frac{15}{4}$, we have both G_n and E_n are functions of a single variable n.

That is the correction function for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)(n+2)}$ is

$G_n = \frac{1}{2n^3+9n^2+\frac{29}{2}n+\frac{33}{4}}$

The absolute value of the error function is

$$|E_n| = \frac{|\frac{27}{4}n^2+27n+\frac{423}{16}|}{\left(2n^3+9n^2+\frac{29}{2}n+\frac{33}{4}\right)\left(2n^3+15n^2+\frac{77}{2}n+\frac{135}{4}\right)\{(n+1)(n+2)(n+3)\}}$$

Hence the theorem.

3. Results and Discussions

For the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)(n+2)}$,

(1) The correction function is $G_n = \frac{1}{2n^3+9n^2+\frac{29}{2}n+\frac{33}{4}}$

(2) The magnitude of error function is

$$|E_n| = \frac{|\frac{27}{4}n^2+27n+\frac{423}{16}|}{\left(2n^3+9n^2+\frac{29}{2}n+\frac{33}{4}\right)\left(2n^3+15n^2+\frac{77}{2}n+\frac{135}{4}\right)\{(n+1)(n+2)(n+3)\}}$$

(3) Clearly $G_n < \frac{1}{(n+1)(n+2)(n+3)}$, the absolute value of the $(n+1)^{th}$ term.

4. Conclusion

The correction function and error function play a vital role in series approximation. We can improve the accuracy of the sum of the series using these functions.

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