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L-Cyclic Magma versus R-Cyclic Magma

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Abstract: *A mapping* $*: X \times X \rightarrow X$ *is a (binary) operation, and the pair* $(X,*)$ *is named as a Magma [1]. Magma with the property:* $x * (y * z) = z * (x * y) = y * (z * x)$ *for all* x, y, z *in Magma is named as L-cyclic magma or with the property* $(x * y) * z = (z * x) * y = (y * z) * x$ *for all* x, y, z *in Magma is named as R-cyclic magma. In this paper, every result show only identical gaol, that is , to prove when a L-cyclic magma becomes R-cyclic magma and vice versa .*

Keywords: *Magma, L-cyclic magma, R-cyclic magma, L-R-cyclic magma, L-identity, R-identity and cross cancellation law.*

1. Introduction

The algebraic objects encountered in this chapter are sets with a binary operation defined on them. Andreas[1] introduced a term "magma" in his Ph.D., theses with entitle "Classification and Enumeration of finite semigroups". Magma nothing but an algebraic structure with one binary operation on a nonempty set. Throughout this paper , we consider the magma with atleast any one of the property

i. $x * (y * z) = z * (x * y) = y * (z * x)$ for all x, y, z in Magma

or

ii. $(x * y) * z = (z * x) * y = (y * z) * x$ for all x, y, z in Magma

The magma with first property is named as L-cyclic magma, with second property is named as R-cyclic magma. If it has both properties, then it is named as L-R-cyclic magma.

This paper contains two sections: In section 1, it contains the introduction and in section 2 shows the results when a L-cyclic magma becomes R-cyclic magma and vice versa.

2. L-cyclic Magma Becomes R-cyclic Magma and Vice Versa

In this section contains all necessary and sufficient conditions of L-cyclic magma becomes R-cyclic magma by using additional property: "commutative, left cancellation, right cancellation right identity, left identity or idempotent" on it.

Result 2.1: Let $(S,*)$ be a commutative magma. Then $(S,*)$ is L-cyclic magma if and only if it is R-cyclic magma.

Proof:

Let $(S,*)$ be a L-cyclic magma $(S,*)$.

Consider, $(x * y) * z$

By using commutative property on $(x * y) * z$, we get $(x * y) * z = z * (x * y)$

By using cyclic property on $z * (x * y)$, we get $z * (x * y) = y * (z * x)$

By using commutative property on $y * (z * x)$, so $y * (z * x) = (z * x) * y$

Once again by using cyclic property on $* (z * x)$, so $y * (z * x) = x * (y * z)$

By using commutative property on $x * (y * z)$, so $x * (y * z) = (y * z) * x$

Thus $(x * y) * z = (z * x) * y = (y * z) * x$ for all x, y, z in S.

Conversely, Consider $x * (y * z)$ in R-cyclic magma $(S,*)$.

Similarly by applying commutative and R-cyclic properties on $x * (y * z)$, we have

$x * (y * z) = z * (x * y) = y * (z * x)$ for any x, y, z in S.

Result 2.2: let $(S,*)$ be a magma with L-cancellation property. Then $(S,*)$ is L-cyclic magma if and only if it is R-cyclic magma.

Proof:

Since magma $(S,*)$ is L-cyclic magma, so $x * (x * y) = y * (x * x) = x * (y * x)$, for any x, y in S.

That is $x * (x * y) = x * (y * x)$

Since magma $(S,*)$ has L-cancellation property, so $x * (x * y) = x * (y * x) \Rightarrow x * y = y * x$

Thus, $x * y = y * x$ for any x, y in S.

Hence $(S,*)$ is a commutative magma.

By using result 2.1, $(S,*)$ is R-cyclic magma.

Conversely,

Since magma $(S,*)$ is R-cyclic magma,

for any x, y in S,
$$
\begin{aligned}
(x * x) * (x * y) &= \big((x * y) * x\big) * x \;. \\
&= \big((x * x) * y\big) * x \\
&= \big((y * x) * x\big) * x \\
&= \big(x * (y * x)\big) * x \\
&= (x * x) * (y * x)
\end{aligned}
$$

Since magma $(S,*)$ has L-cancellation property,

so $(x * x) * (x * y) = (x * x) * (y * x) \Rightarrow x * y = y * x$

Thus, $x * y = y * x$ for any x, y in S.

Hence $(S,*)$ is a commutative magma.

By using result 2.1, $(S,*)$ is R-cyclic magma.

Result 2.3: let $(S,*)$ be a magma with R-cancellation property. Then $(S,*)$ is L-cyclic magma if and only if it is R-cyclic magma.

Proof:

Necessary Condition:

Since magma $(S,*)$ is L-cyclic magma, for any x, y in S ,

$$
\begin{aligned}
(x * y) * (x * x) &= x * \big((x * y) * x\big) \\
&= x * \big(x * (x * y)\big) \\
&= x * \big(y * (x * x)\big) \\
&= x * \big(x * (y * x)\big) \\
&= (y * x) * (x * x)
\end{aligned}
$$

Since magma $(S,*)$ has R-cancellation property,

So $(x * y) * (x * x) = (y * x) * (x * x) \Rightarrow x * y = y * x$

Thus, $x * y = y * x$ for any x, y in S.

Hence $(S,*)$ is a commutative magma.

By using result 2.1, $(S,*)$ is R-cyclic magma.

Sufficient condition:

Since magma $(S,*)$ is R-cyclic magma,

for any x, y in S, $(x * y) * x = (x * x) * y = (y * x) * x$

Since magma $(S,*)$ has R-cancellation property,

so $(x * y) * x = (y * x) * x \Rightarrow x * y = y * x$

Thus, $x * y = y * x$ for any x, y in S.

Hence $(S,*)$ is a commutative magma.

By using result 2.1, $(S,*)$ is L-cyclic magma.

Note: From above two results, it is understood that every L-cyclic magma is R-cyclic magma and vice versa if magma with cancellation property.

Result 2.4: let $(S,*)$ be a magma with R-identity. Then $(S,*)$ is L-cyclic magma if and only if it is R-cyclic magma.

Proof:

Since $(S,*)$ magma has R-identity, so there exist an element e in S, such that $x * e = x$ for all x in S.

Necessary Condition:

Since the magma $(S,*)$ has L-cyclic magma,

so it has L-cyclic property $x * (y * z) = z * (x * y) = y * (z * x)$ for all x, y, z in S.

Thus $x * y = (x * e) * (y * e)$

$$= e * \big((x * e) * y\big)$$

$$= y * \big(e * (x * e)\big)$$

$$= y * \big(e * (e * x)\big)$$

$$= y * \big(x * (e * e)\big)$$

$$= y * (x * e)$$

$$= y * x$$

Thus the magma $(S,*)$ is commutative magma.

By using result 2.1, $(S,*)$ is R-cyclic magma.

Sufficient Condition:

Since the magma $(S,*)$ has R-cyclic magma,

so it has $(x * y) * z = (z * x) * y = (y * z) * x$ for all x, y, z in S.

Thus $x * y = (x * e) * (y * e)$

$$= \big((y * e) * x\big) * e$$

$$= (y * x) * e$$

$$= y * x$$

Thus the magma $(S,*)$ is commutative magma.

By using result 2.1, $(S,*)$ is L-cyclic magma.

Result 2.5: let $(S,*)$ be a magma with L-identity. Then $(S,*)$ is L-cyclic magma if and only if it is R-cyclic magma.

Proof:

Since $(S,*)$ magma has L-identity, so there exist an element e in S, such that $e * x = x$ for all x in S.

Necessary Condition:

Since the magma $(S,*)$ has L-cyclic magma,

so it has L-cyclic property $x * (y * z) = z * (x * y) = y * (z * x)$ for all x, y, z in S.

Thus $x * y = (e * x) * (e * y)$

$$= y * \big((e * x) * e\big)$$

$$= y * \big((x * e)\big)$$

$$= e * (y * x)$$

$$= y * x$$

Thus the magma $(S,*)$ is commutative magma.

By using result 2.1, $(S,*)$ is R-cyclic magma.

Sufficient Condition:

Since the magma $(S,*)$ has R-cyclic magma,

so it has R-cyclic property $(x * y) * z = (z * x) * y = (y * z) * x$ for all x, y, z in S.

Thus $x * y = (e * x) * (e * y)$

$$= \big((e * y) * e\big) * x$$

$$= (y * e) * x$$

$$= (y * x) * e$$

$$= (e * y) * x$$

$$= y * x$$

Thus the magma $(S,*)$ is commutative magma.

By using result 2.1, $(S,*)$ is L-cyclic magma.

Note : from above two results ,it is easily show that L-cyclic magma is R-cyclic magma and vice versa if magma with identity.

Result 2.6: A cross cancelation magma $(S,*)$ with idempotent element e.Then $(S,*)$ is L-cyclic magma if and only if it is R-Cyclic magma.

Proof:

Since the magma $(S,*)$ has a cross cancelation property, so $x * y = y * z \Rightarrow x = z$ for any x, y, z in S

Necessary Condition:

Let e be an idempotent element of magma $(S,*)$.

So, $e * e = e$

Case 1:

since S has a cyclic property ,so $x * (e * e) = x * e$ for all x in S .

$\Rightarrow e * (x * e) = x * e$

$\Rightarrow$ since S has a cross cancelation property ,s o $x * e = x$, for all x in S .

Thus idempotent element e is a right identity in magma $(S,*)$.

Case 2:

Consider and element $e * x$

By using left identity of case 1, this is equal to $e * (e * x)$

By using L-cyclic property on $e * (e * x)$, we have $e * (e * x) = x * (e * e)$

Using idempotent property of e on $x * (e * e)$, $x * (e * e) = x * e$

Thus, $e * (e * x) = x * e$

$\Rightarrow$ since S has a cross cancelation property ,s o $e * x = x$, for all x in S .

Thus idempotent element e is a left identity in magma $(S,*)$.

Hence, the idempotent element e is identity in $(S,*)$.

Next to show that commutative property of $(S,*)$.

Let x, y are any two elements in magma $(S,*)$.

$x * y = (e * x) * (e * y)$

$= y * \big((e * x) * e\big)$

$= e * \big(y * (e * x)\big)$

$= e * (y * x)$ (since e is an left identity of S)

$= y * x$ (since e is an left identity of S)

Thus the magma $(S,*)$ is commutative magma

By using result 2.1, $(S,*)$ is R-cyclic magma.

Sufficient Condition:

Next to show that it is L-cyclic magma.

Consider $x * (y * z)$

By using commutative property on $x * (y * z)$, so we have $x * (y * z) = (y * z) * x$ Let e be an idempotent element of magma$(S,*)$.

So, $e * e = e$

Case 1:

since S has a cyclic property , so $(e * e) * x = e * x$ for all x in S .

$\Rightarrow (x * e) * e = e * x$

$\Rightarrow (e * x) * e = e * x$, for all x in.

$\Rightarrow$ since S has a cross cancelation property ,s o $(e * x) = x$, for all x in S .

Thus idempotent element e is a left identity in magma $(S,*)$.

Case 2:

Consider and element $x * e$

By using left identity of case 1, this is equal to $(e * x) * e$.

By using R-cyclic property on $(e * x) * e$, we have $(e * x) * e = (e * e) * x$

Again twice by using left identity of case 1, we have $(e * e) * x = e * x$ & $e * x = x$

Thus, $x * e = x$

Next to show that $(S,*)$ is commutative magma.

<u>Commutative property:</u>

Let x, y are any two elements in magma $(S,*)$.

$$x * y = (e * x) * (e * y)$$
$$= \big((e * y) * e\big) * x$$
$$= \big((e * e) * y\big) * x$$
$$= (e * y) * x$$
$$= y * x$$

Thus the magma $(S,*)$ is commutative magma.

By using result 2.1, $(S,*)$ is L-cyclic magma.

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