International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 4, Issue 6, June 2016, PP 6-10
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
DOI: http://dx.doi.org/10.20431/2347-3142.0406002
www.arcjournals.org

Square Multiplicative Labeling for Some Graphs

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Abstract: *Let G be (p, q) graph. G is said to be a Square multiplicative labeling if there exists a bijection $f: V(G) \to \{1,2, \dots, p\}$ such that the induced function $f^* : E(G) \to N$ given by $f^*(uv) = f(u)^2 . f(v)^2$ for every $uv \in E(G)$ are all distinct. A graph which admits Square multiplicative labeling is called Square multiplicative graph. In this paper, we prove cycle with one chord, cycle with twin chords, quadrilateral triangles, triangular snakes, 2mΔ_k− snake, double triangular snakes, Bistar$B_{n,n}$ are Square multiplicative graphs.*

Keywords: *Strongly multiplicative graphs, Square multiplicative graphs, Double triangular snake, Chord, Two chords, Bistar$B_{n,n}$.*

1. Introduction

Graph labelings were first introduced in the mid sixties. A labeling of a graph G is an assignment of labels to vertices or edges or both following certain rules [4]. In this paper we deal only finite, simple, connected and undirected graphs obtained through graph operations. Labeling of graphs plays an important role in application of graph theory in Neural Networks, Coding theory, Circuit Analysis etc. A useful survey on graph labeling by J.A.Gallian (2015) can be found in [1]. In most applications labels are positive or nonnegative integers. Beineke and Hedge [3] call a graph with p vertices strongly multiplicative if the vertices of G can be labelled with distinct integers $1,2, \dots, p$ such that the labels induced on the edges by the product of the end vertices are distinct. Triangular snakes [8] was proved by Murugesan, J.Shiama in Square Difference 3-Equitable Labeling of Some Graphs. Quadrilateral snakes [2] was proved by J.Shiama in Square Difference Labeling for Some Graphs. G is said to be a Square multiplicative labeling if there exists a bijection $f: V(G) \to \{1,2, \dots, p\}$ such that the induced function $f^* : E(G) \to N$ given by $f^*(uv) = f(u)^2 . f(v)^2$ for every $uv \in E(G)$ are all distinct.

Definition.1.1: G is said to be a Square multiplicative labeling if there exists a bijection $f: V(G) \to \{1,2, \dots, p\}$ such that the induced function $f^* : E(G) \to N$ given by $f^*(uv) = f(u)^2 . f(v)^2$ for every $uv \in E(G)$ are all distinct. A graph which admits Square multiplicative labeling is called Square multiplicative graph.

Definition.1.2: A double triangular snake [5] is a graph formed by two triangular snakes have a same path.

Definition.1.3: A chord [6] of a cycle C_n is an edge joining two non-adjacent vertices of cycle C_n.

Definition.1.4: Two chords of a cycle are twin chords (consecutive chords) if they form a triangle with an edge of the cycle C_n.

Definition.1.5: Bistar $B_{n,n}$ [7] is the graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge.

2. 2. Square Multiplicative Labeling

Theorem.2.1

Every Cycle with one chord is a Square multiplicative graph.

Proof:

Let G be the cycle C_n with one chord and $v_{1,}\ v_{2,} \dots \dots v_n$ be the consecutive of C_n arranged in the clockwise direction. Let $e = v_1 v_i$ be the chord vertices of cycle.

Let us define labeling $f: V(G) \rightarrow \{1,2,\dots,n\}$ as follows
$f(v_1) = 1;$

$f(v_i) = p_1;$ Where p_1 is the highest prime number such that $p_1 \leq n.$

$f(v_n) = p_2;$ Where p_2 is the second highest prime number such that $1 < p_2 < p_1 \leq n.$

Now label the remaining vertices starting from v_2 consecutively in clock wise direction from the set $\{1,2,\dots.,n\}$ except 1, p_1 & p_2 as these numbers are already used as labels.

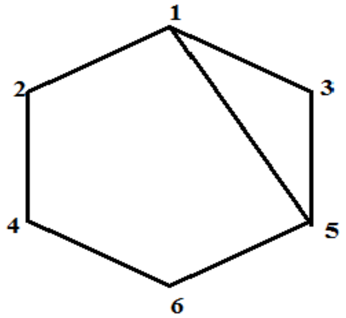

Figure.1

Theorem.2.2

Every Cycle with twin chords are square multiplicative graphs.

Proof: Let G be the cycle C_n with twin chords. Let $v_{1,}\ v_{2,} \dots \dots v_n$ be the consecutive vertices of C_n arranged in the clockwise direction. Let $e_1 = v_1 v_i,\ e_2 = v_1 v_{i+1}$ be two chords of C_n. Let us define labeling $f: V(G) \rightarrow \{1,2,\dots,n\}$ as follows

$f(v_1) = 1;$
$f(v_i) = p_1;$ Where p_1 is the highest prime number such that $p_1 \leq n.$

$f(v_{i+1}) = p_2;$ Where p_2 is the second highest prime number such that $1 < p_2 < p_1 \leq n.$

$f(v_n) = p_3;$ Where p_3 is the third highest prime number such that $1 < p_3 < p_2 < p_1 \leq n.$

Now label the remaining vertices starting from v_2 consecutively in clock wise direction from the set $\{1,2,\dots.,n\}$-$\{1, p_1, p_{2,} p_3\}$.

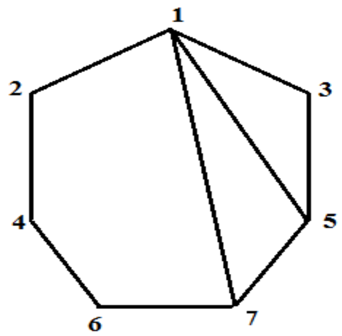

Figure.2

Theorem.2.3

Quadrilateral snakes are square multiplicative graphs.

Proof:
Consider $S_{4,n}$ with vertices labeled.

We define a function $f: V(G) \rightarrow \{1,2,3,\dots,3n+1\}$ by

$f(u_i) = 3i - 2; i = \{1,2,3, \dots, n+1\}$

$f(v_i) = 3i; i = \{1,2, \dots, n\}$

$f(w_i) = 3i - 1; i = \{1,2, \dots, n\}$

And the induced function $f^*: E(G) \to N$ defined by $f^*(uv) = f(u)^2 f(v)^2$ for every $uv \in E(G)$are all distinct. Hence $S_{4,n}$ are square multiplicative graphs.

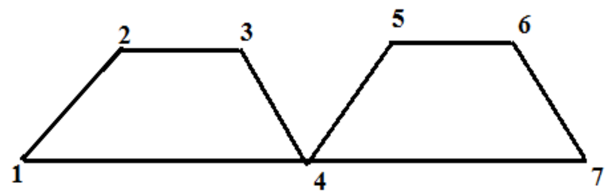

Figure.3

Theorem.2.4

Triangular snakes are square multiplicative graph.

Proof:

Consider $T_2$with vertices labeled.

We define a function $f: V(G) \to \{1,2, \dots, 2n+1\}$ by

s$f(u_i) = 2i - 1; i = \{1,2, \dots, n+1\}$

$f(v_i) = 2i; i = \{1,2,3, \dots, n\}$

And the induced function $f^*: E(G) \to N$ defined by $f^*(uv) = f(u)^2 f(v)^2$ for every $uv \in E(G)$ are all distinct. Hence T_2 are square multiplicative graphs.

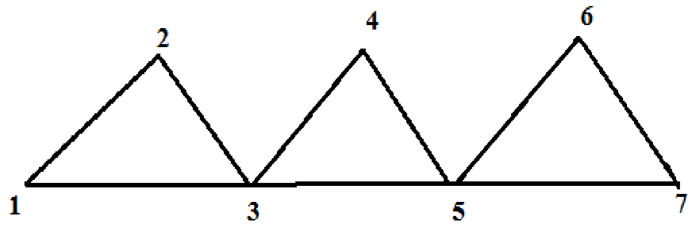

Figure.4

Theorem.2.5

All double triangular snake ($2\Delta_k - snake$) are square multiplicative graphs.

Proof:

Let G denote the $2\Delta_k - snake$ with p vertices. Let the vertices of G be

$V(G) = \{u_i /1 \le i \le k\} \cup \{v_i^j /1 \le i \le k, 1 \le j \le 2\}$ and
$E(G) = \{u_i \ u_{i+1} / 1 \le i \le k\} \cup \{u_i v_i^j /1 \le i \le k, 1 \le j \le 2\} \cup \{u_{i+1} v_i^j / 1 \le i \le k, 1 \le j \le 2\}$

Here $|V(G)| = 3k + 1$.

Define $f: V(G) \to \{1,2, \dots, 3k+1\}$ by

$f(u_1) = 1,$

$f(u_i) = f(u_{i-1}) + 3;\ 2 \le i \le k+1$

$f(v_i^j) = f(u_i) + j; 1 \le i \le k, 1 \le j \le 2$

One can easily verify that f so defined is injective. With the above defined vertex label, induced edge labels can be arranged in increasing order.

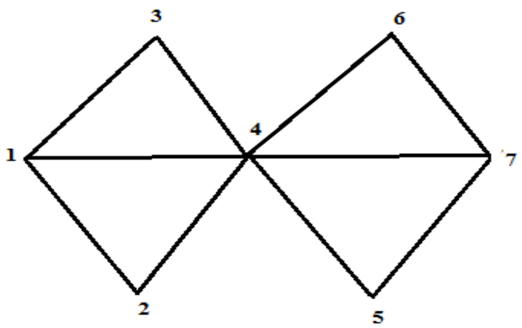

Figure.5

Theorem.2.6

$2m\Delta_k$ $- snake$ with k-blocks are square multiplicative graphs.

Proof:

Let G denote the $2m\Delta_k - snake$ with p vertices. Let the vertices of G be

$V(G) = \{u_i /1 \leq i \leq k+1\} \cup \{v_i^j /1 \leq i \leq k, 1 \leq j \leq 2m\}$ and

Here $|V(G)| = k(2m+1)+1$.

Define $f: V(G) \to \{1,2, \dots, k(2m+1)+1\}$ by

$f(u_1) = 1,$

$f(u_i) = f(u_{i-1}) + 2m + 1;\ 2 \leq i \leq k+1$

$f(v_i^j) = f(u_i) + j; 1 \leq i \leq k, 1 \leq j \leq 2m$

One can easily verify that f so defined is injective. With the above defined vertex label, induced edge labels can be arranged in increasing order

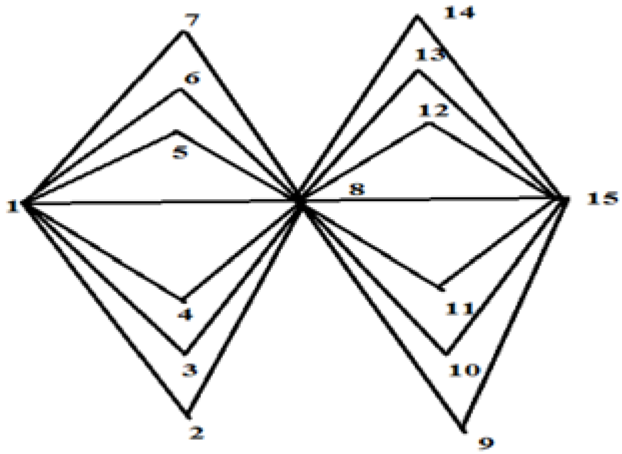

Figure.6

Theorem.2.7

Bistar $B_{n,n}$ is a square multiplicative graph.

Proof:

Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i, 1 \leq i \leq n\}$ where u_i , v_i pendant vertices are. If G=$B_{n,n}$ then $|V(G)| = 2n+2$ and$|E(G)| = 2n+1$. We define vertex labeling $f: V(G) \to \{1,2, \dots, 2n+2\}$ as follows $f(u) = 1,$

$f(v) = 2n + 1,$

$f(u_i) = 1 + i;\ 1 \leq i \leq n$

$f(v_i) = n + 1 + i;\ 1 \leq i \leq n - 1$

$f(v_n) = 2n + 2$

The induced function $f^*: E(G) \rightarrow N$ defined by $f^*(uv) = f(u)^2 . f(v)^2$ for every $uv \in E(G)$ are all distinct. Hence Bistar $B_{n,n}$ is a square multiplicative graph.

Square multiplicative labelling of the graph $B_{5,5}$ is shown in the Figure.7

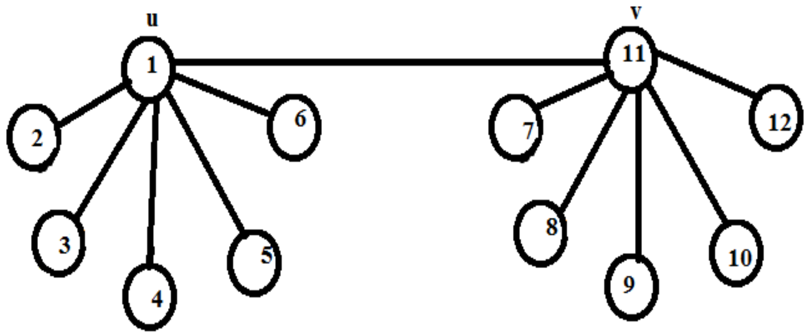

Figure.7

3. CONCLUSION

Square multiplicative labeling for cycle with one chord, cycle with twin chords, quadrilateral triangles, triangular snakes, $2m\Delta_k$− snake, double triangular snakes, Bistar $B_{n,n}$ have been discussed in this paper.

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