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Production Inventory System under Weibull Amelioration, Pareto Deterioration and Exponentially Time Based Demand under Fully Backlogged Shortages

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Abstract: *In this research paper we have considered production inventory model for the items whose demand starts at the same time with its production. In the initial phase of production, we have considered time to ameliorate following two parameter Weibull distribution and demand is an exponential function of time. After some period effect of deterioration can be observed, so in the second phase, we have assumed deterioration of items following Pareto type – I distribution with the same amelioration rate and demand rate as in previous time interval. Here shortages are allowed to occur and unsatisfied demand is fully backlogged. The model so derived is illustrated with a numerical example with its sensitivity analysis.*

Keywords: *Weibull distribution, Pareto type – I distribution, Fully backlogged shortage*

1. INTRODUCTION

Various inventory models have been developed by researchers in last few decades. A note on the EPQ model with shortages and variable lead time was given by Chang [3]. Lan, Yu, Lin, Tung, Yen and Deng [4] had presented a note on the improved algebic method for the EPQ model with stochastic lead time. Shamsi, Haji, Shadrokh and Nourbakhsh [13] published their work on economic production quantity in reworkable production systems with inspection errors, scraps and backlogging. Wee, Wang and Yang [5] had formulated a production quantity model for imperfect quality items with shortage and screening constraint. Moreover Gothi and Chatterji [22] have developed EPQ model for imperfect quality items under constant demand rate and varying IHC. Li, Lan and Mawhinney [15] reviewed on deteriorating inventory study. Economic production quantity models with shortage, price and stock-dependent demand for deteriorating items were developed by Jain, Sharma and Rathore [11]. Bansal and Ahalawat [8] have given integrated inventory models for decaying items with exponential demand under inflation. An EPQ model using Weibull distributed deterioration item with time varying holding cost was formulated by Kawale and Bansode [9]. Bhojak and Gothi [1] have developed EPQ model with time dependent IHC and Weibull distributed deterioration under shortages.

In addition to this, in order to make inventory models more appropriate for the current market scenario, amelioration of items is also considered along with its deterioration. A stochastic set-covering location model for both ameliorating and deteriorating items was given by Hwang [6]. Law and Wee [16] had also presented an integrated production inventory for ameliorating and deteriorating items taking account of time discounting. Optimal control of an inventory system with variable demand and ameliorating / deteriorating items was considered by Srichandan Mishra, Raju, U.K.Misra and G.Misra [19]. Bhojak and Gothi [2] have formulated two inventory models for ameliorating and deteriorating items with time dependent demand and IHC. Further an integrated inventory model with exponential amelioration and two parameter Weibull deterioration was developed by Gothi, Chatterji and Parmar [20].

Rao, Begum and Murty [14] had given optimal ordering policies of inventory model for deteriorating items having generalized Pareto lifetime. Singh, Vaish and Singh, S. R. [18] had presented an EOQ model with Pareto distribution for deterioration, trapezoidal type demand and backlogging under trade credit policy. Inventory model for deteriorating items having two component mixture of Pareto lifetime and selling price dependent demand was formulated by Vijayalakshmi. G, Srinivasa Rao. K and Nirupama Devi [23]. Gothi and Bhojak [21] .

In this paper, we have developed a production inventory model by considering two parameter Weibull distributed amelioration till stock exists. The distribution of the time to deteriorate is a random variable following two parameter Pareto type – I distribution. The probability density function of two parameter Pareto type – I distribution, given by $f(t) = \frac{\theta}{\mu}\left(\frac{t}{\mu}\right)^{-\theta-1}$; $t \geq \mu$, where θ and μ are parameters with positive real value. The instantaneous rate of deterioration θ(t) of the non-deteriorated inventory at time t, can be obtained from $\theta(t) = \frac{f(t)}{1-F(t)}$, where $F(t) = 1 - \left(\frac{t}{\mu}\right)^{-\theta}$ is the cumulative distribution function for the two parameter Pareto type – I distribution. Thus, the instantaneous rate of deterioration of the on-hand inventory is $\theta(t) = \frac{\theta}{t}$ and demand rate is an exponential function of time. In this model shortages are allowed to occur and unsatisfied demand is fully backlogged.

2. ASSUMPTIONS

The following assumptions are considered to develop this model

1. The inventory system involves only one item and one stocking point.
2. Replenishment rate is infinite.
3. Lead-time is zero.
4. The amelioration occurs when the item is effectively in stock but the deterioration starts at time t = μ
5. The deteriorated items are not replaced during the given cycle.
6. Infinite time horizon period is considered.
7. Shortages are allowed and the unsatisfied demand is fully backlogged.
8. Holding cost $C_h = h + r\,t$ (h, r > 0) is a linear function of time.
9. Time to deterioration follows Pareto type – I distribution in the time interval $[\mu,\ t_1]$ and $\theta(t) = \frac{\theta}{t}$ is a deterioration rate.
10. Demand rate is an exponential type and it is $R(t) = \lambda\, t^{-p}$ over a time period [0, T], where λ is a positive constant.
11. The amelioration rate is derived from Weibull distribution with two parameters and it is

    $$A(t) = \alpha\,\beta\, t^{\beta-1} \qquad ; \quad 0 \leq t \leq t_1$$

    where α and β are the positive parameters.
12. Unit amelioration cost, deterioration cost, production cost, ordering cost and shortage cost per unit are known and constants.
13. Total inventory cost is a continuous real function which is convex to the origin.

3. NOTATIONS

The following notations are used to develop the mathematical model:

1. Q (t) : Inventory level of the product at time t
2. R (t) : Demand rate varying over time.

3.	A (t)	:	Amelioration rate at any time t.
4.	θ (t)	:	Deterioration rate.
5.	A	:	Ordering cost per order during the cycle period.
6.	K	:	Production rate.
7.	C_h	:	Inventory holding cost per unit per unit time.
8.	C_a	:	Amelioration cost per unit.
9.	C_d	:	Deterioration cost per unit.
10.	C_p	:	Production cost per unit. ($C_p > C_a$)
11.	C_s	:	Shortage cost per unit.
12.	S_1	:	Inventory level at time t = μ.
13.	S_2	:	The maximum inventory level during shortage period.
14.	T	:	Duration of a cycle.
15.	TC	:	Total cost per unit time.

4. Mathematical Formulation and Solution

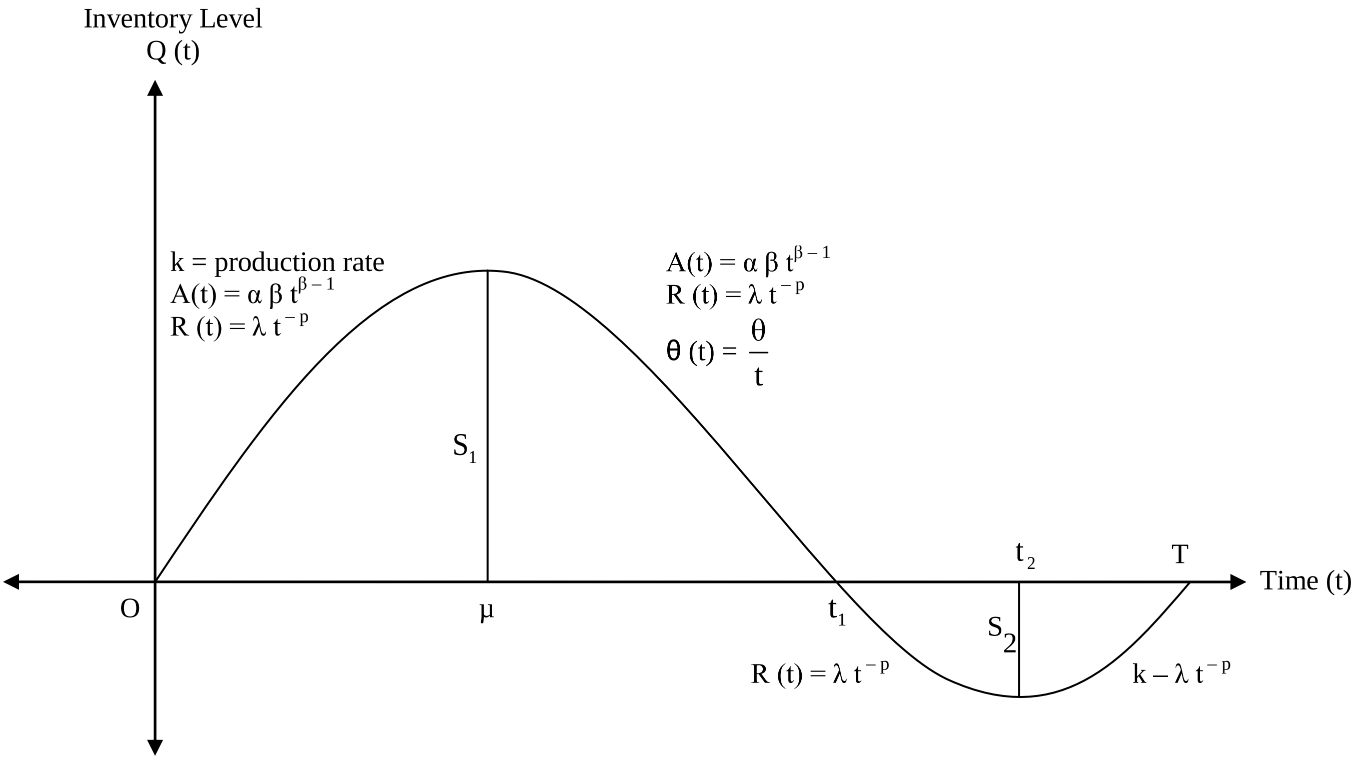

Fig. 1. *Graphical presentation of inventory system*

At the initial stage, inventory is zero. At time t = 0, the production and supply both start simultaneously. Then the production stops at t = μ where the maximum inventory level S_1 is reached. During the interval [0, μ] the inventory is built up at a rate $k - \lambda\, t^{-p} + \alpha\, \beta\, t^{\beta - 1}$. Even though two parameter Weibull amelioration rate exists in time interval [μ, t_1], the stock reaches to zero level at time t = t_1 due to demand rate $-\lambda\, t^{-p}$ along with the Pareto type – I deterioration rate $\frac{\theta}{t}$. Thereafter, shortages are allowed to occur during the time interval $[t_1, t_2]$ at a rate $-\lambda\, t^{-p}$. At time t = t_2 shortage reaches at the maximum level S_2 and then production starts at the same rate and the backlog is fulfilled at a rate of $k - \lambda\, t^{-p}$ in the time interval [t_2, T]. The stock level becomes zero at time t = T. The same cycle is repeated for the further time period T.

Differential Equations pertaining to the situations as explained above are given by

$$\frac{dQ(t)}{dt} = \alpha\,\beta\, t^{\beta-1} Q(t) + k - \lambda\, t^{-p} \qquad 0 \le t \le \mu \qquad \dots (1)$$

$$\frac{dQ(t)}{dt} = \alpha\,\beta\, t^{\beta-1} Q(t) - \frac{\theta}{t} Q(t) - \lambda\, t^{-p} \qquad \mu \le t \le t_1 \qquad \dots (2)$$

$$\frac{dQ(t)}{dt} = -\lambda\, t^{-p} \qquad t_1 \le t \le t_2 \qquad \dots (3)$$

$$\frac{dQ(t)}{dt} = k - \lambda\, t^{-p} \qquad t_2 \le t \le T \qquad \dots (4)$$

Using boundary conditions

$$Q(0) = 0\ , Q(t_1) = 0 \text{ and } Q(T) = 0$$

the solutions of the above differential equations (1), (2), (3) and (4) are given as

$$Q(t) = k\,t + \frac{k\,\alpha\,\beta\, t^{\beta+1}}{\beta+1} - \frac{\alpha\,\beta\,\lambda\, t^{\beta-p+1}}{(\beta-p+1)(1-p)} - \frac{\lambda\, t^{1-p}}{1-p} \qquad \dots (5)$$

$$Q(t) = \lambda \left[\begin{array}{l} \left(\dfrac{t_1^{\theta-p+1}}{\theta-p+1} - \dfrac{\alpha\, t_1^{\theta-p+\beta+1}}{\theta-p+\beta+1} \right) t^{-\theta} - \dfrac{t^{1-p}}{\theta-p+1} \\ + \dfrac{\alpha\, t^{\beta-p+1}}{\theta+\beta-p+1} + \dfrac{\alpha\, t_1^{\theta-p+1}}{\theta-p+1}\, t^{\beta-\theta} - \dfrac{\alpha\, t^{\beta-p+1}}{\theta-p+1} \end{array} \right] \qquad \dots (6)$$

$$Q(t) = \frac{\lambda}{1-p}\left(t_1^{1-p} - t^{1-p}\right) \qquad \dots (7)$$

$$Q(t) = \left(\frac{\lambda\, T^{1-p}}{1-p} - k\,T\right) + k\,t - \frac{\lambda\, t^{1-p}}{1-p} \qquad \dots (8)$$

Substituting $Q(t_2) = -S_2$ in equations (7) and (8), we get

$$-S_2 = \frac{\lambda}{1-p}\left(t_1^{1-p} - t_2^{1-p}\right) \qquad \dots (9)$$

$$-S_2 = \left(\frac{\lambda\, T^{1-p}}{1-p} - k\,T\right) + k\,t_2 - \frac{\lambda\, t_2^{1-p}}{1-p} \qquad \dots (10)$$

Eliminating S_2 from (9) and (10), we get

$$t_2 = T + \frac{1}{k}\left(\frac{\lambda}{1-p}\left(t_1^{1-p} - T^{1-p}\right)\right) \qquad \dots (11)$$

Thus t_2 can be expressed in terms of t_1 and T.

5. COST COMPONENTS

On the basis of the assumptions and description of the model, the total cost consists of the following cost components:

5.1 Operating Cost (OC)

The operating cost over the period [0, T] is

$$OC = A \qquad \dots (12)$$

5.2 Inventory Holding Cost (IHC)

The holding cost for carrying inventory over the period $[0, t_1]$ is

$$\mathrm{IHC} = \int_0^{\mu} (h + r t)\, Q(t)\, dt + \int_{\mu}^{t_1} (h + r t)\, Q(t)\, dt$$

$$\Rightarrow \mathrm{IHC} = \left\{ \begin{aligned} & h\left[\frac{k\mu^2}{2} - \frac{\lambda \mu^{2-p}}{(1-p)(2-p)} + \frac{k\alpha\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta\lambda\mu^{\beta-p+2}}{(1-p)(\beta-p+1)(\beta-p+2)}\right] \\ & + r\left[\frac{k\mu^3}{3} - \frac{\lambda \mu^{3-p}}{(1-p)(3-p)} + \frac{k\alpha\beta\mu^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{\alpha\beta\lambda\mu^{\beta-p+3}}{(1-p)(\beta-p+1)(\beta-p+3)}\right] \\ & + h\lambda \left[\begin{aligned} & \left(\frac{t_1^{\theta-p+1}}{\theta-p+1} - \frac{\alpha t_1^{\theta-p+\beta+1}}{\theta-p+\beta+1}\right)\left(\frac{t_1^{1-\theta} - \mu^{1-\theta}}{1-\theta}\right) - \frac{t_1^{2-p} - \mu^{2-p}}{(\theta-p+1)(2-p)} \\ & - \frac{\alpha\beta\left(t_1^{\beta-p+2} - \mu^{\beta-p+2}\right)}{(\theta+\beta-p+1)(\beta-p+2)(\theta-p+1)} + \frac{\alpha t_1^{\theta-p+1}}{(\theta-p+1)}\left(\frac{t_1^{\beta-\theta+1} - \mu^{\beta-\theta+1}}{\beta-\theta+1}\right) \end{aligned}\right] \\ & + r\lambda \left[\begin{aligned} & \left(\frac{t_1^{\theta-p+1}}{\theta-p+1} - \frac{\alpha t_1^{\theta-p+\beta+1}}{\theta-p+\beta+1}\right)\left(\frac{t_1^{2-\theta} - \mu^{2-\theta}}{2-\theta}\right) - \frac{t_1^{3-p} - \mu^{3-p}}{(\theta-p+1)(3-p)} \\ & - \frac{\alpha\beta\left(t_1^{\beta-p+3} - \mu^{\beta-p+3}\right)}{(\theta+\beta-p+1)(\beta-p+3)(\theta-p+1)} + \frac{\alpha t_1^{\theta-p+1}}{(\theta-p+1)}\left(\frac{t_1^{\beta-\theta+2} - \mu^{\beta-\theta+2}}{\beta-\theta+2}\right) \end{aligned}\right] \end{aligned} \right\} \quad \ldots (13)$$

5.3 Deterioration Cost (DC)

The deterioration cost during the period $[\mu, t_1]$ is

$$\mathrm{DC} = C_d \int_{\mu}^{t_1} \frac{\theta}{t}\, Q(t)\, dt$$

$$\Rightarrow \mathrm{DC} = \lambda\,\theta \left[\begin{aligned} & \left(\frac{t_1^{\theta-p+1}}{\theta-p+1} - \frac{\alpha t_1^{\theta-p+\beta+1}}{\theta-p+\beta+1}\right)\left(\frac{t_1^{-\theta} - \mu^{-\theta}}{-\theta}\right) - \frac{t_1^{1-p} - \mu^{1-p}}{(\theta-p+1)(1-p)} \\ & - \frac{\alpha\beta\left(t_1^{\beta-p+1} - \mu^{\beta-p+1}\right)}{(\theta+\beta-p+1)(\beta-p+1)(\theta-p+1)} + \frac{\alpha t_1^{\theta-p+1}}{(\theta-p+1)}\left(\frac{t_1^{\beta-\theta} - \mu^{\beta-\theta}}{\beta-\theta}\right) \end{aligned}\right] \ldots (14)$$

5.4 Production Cost (PC)

The production cost per cycle is

$$\mathrm{PC} = C_p\, k\,(\mu + T - t_2) \quad \ldots (15)$$

5.5 Shortage Cost (SC)

The shortage cost during the period $[t_1, T]$ is

$$\mathrm{SC} = -C_s \left[\int_{t_1}^{t_2} Q(t)\, dt + \int_{t_2}^{T} Q(t)\, dt\right]$$

$$\Rightarrow SC = -\,C_s \left[\begin{array}{l} \dfrac{\lambda}{1-p}\left(t_1^{1-p}\left(t_2 - t_1\right) - \left(\dfrac{t_2^{2-p} - t_1^{2-p}}{2-p} \right) \right) + \left(\dfrac{\lambda\, T^{1-p}}{1-p} - k\,T \right)\left(T - t_2\right) \\ + k \left(\dfrac{T^2 - t_2^2}{2} \right) - \dfrac{\lambda \left(T^{2-p} - t_2^{2-p}\right)}{(1-p)(2-p)} \end{array} \right] \quad \ldots (16)$$

5.6 Amelioration Cost (AMC)

The amelioration cost over the period $[0, t_1]$ is

$$AMC = C_a \left[\int_0^{\mu} \alpha\,\beta\, t^{\beta-1}\, Q(t)\, dt + \int_{\mu}^{t_1} \alpha\,\beta\, t^{\beta-1}\, Q(t)\, dt \right]$$

$$\Rightarrow AMC = C_a\, \alpha\, \beta \left[\begin{array}{l} \dfrac{k\,\mu^{\beta+1}}{\beta+1} - \dfrac{\lambda\,\mu^{\beta-p+1}}{(1-p)(\beta-p+1)} + \dfrac{\lambda\, t_1^{\theta-p+1}\left(t_1^{\beta-\theta} - \mu^{\beta-\theta}\right)}{(\theta-p+1)(\beta-\theta)} \\ - \dfrac{\lambda\left(t_1^{\beta-p+1} - \mu^{\beta-p+1}\right)}{(\theta-p+1)(\beta-p+1)} \end{array} \right] \quad \ldots (17)$$

6. Total Cost (TC)

Taking the relevant costs mentioned above, the total average cost of the system is given by

$$TC = \frac{1}{T}\left[OC + IHC + DC + PC + SC + AMC \right]$$

$$\Rightarrow TC = \frac{1}{T} \left[\begin{array}{l} A + \left[\begin{array}{l} h\left[\dfrac{k\,\mu^2}{2} - \dfrac{\lambda\,\mu^{2-p}}{(1-p)(2-p)} + \dfrac{k\,\alpha\,\beta\,\mu^{\beta+2}}{(\beta+1)(\beta+2)} - \dfrac{\alpha\,\beta\,\lambda\,\mu^{\beta-p+2}}{(1-p)(\beta-p+1)(\beta-p+2)} \right] + \\ r\left[\dfrac{k\,\mu^3}{3} - \dfrac{\lambda\,\mu^{3-p}}{(1-p)(3-p)} + \dfrac{k\,\alpha\,\beta\,\mu^{\beta+3}}{(\beta+1)(\beta+3)} - \dfrac{\alpha\,\beta\,\lambda\,\mu^{\beta-p+3}}{(1-p)(\beta-p+1)(\beta-p+3)} \right] + \\ h\,\lambda \left[\begin{array}{l} \left(\dfrac{t_1^{\theta-p+1}}{\theta-p+1} - \dfrac{\alpha\, t_1^{\theta-p+\beta+1}}{\theta-p+\beta+1} \right)\left(\dfrac{t_1^{1-\theta} - \mu^{1-\theta}}{1-\theta} \right) - \dfrac{t_1^{2-p} - \mu^{2-p}}{(\theta-p+1)\,(2-p)} \\ - \dfrac{\alpha\,\beta\,\left(t_1^{\beta-p+2} - \mu^{\beta-p+2}\right)}{(\theta+\beta-p+1)(\beta-p+2)(\theta-p+1)} + \dfrac{\alpha\, t_1^{\theta-p+1}}{(\theta-p+1)}\left(\dfrac{t_1^{\beta-\theta+1} - \mu^{\beta-\theta+1}}{\beta-\theta+1} \right) \end{array} \right] + \\ r\,\lambda \left[\begin{array}{l} \left(\dfrac{t_1^{\theta-p+1}}{\theta-p+1} - \dfrac{\alpha\, t_1^{\theta-p+\beta+1}}{\theta-p+\beta+1} \right)\left(\dfrac{t_1^{2-\theta} - \mu^{2-\theta}}{2-\theta} \right) - \dfrac{t_1^{3-p} - \mu^{3-p}}{(\theta-p+1)\,(3-p)} \\ - \dfrac{\alpha\,\beta\,\left(t_1^{\beta-p+3} - \mu^{\beta-p+3}\right)}{(\theta+\beta-p+1)(\beta-p+3)(\theta-p+1)} + \dfrac{\alpha\, t_1^{\theta-p+1}}{(\theta-p+1)}\left(\dfrac{t_1^{\beta-\theta+2} - \mu^{\beta-\theta+2}}{\beta-\theta+2} \right) \end{array} \right] \end{array} \right] \\ + \lambda\,\theta \left[\begin{array}{l} \left(\dfrac{t_1^{\theta-p+1}}{\theta-p+1} - \dfrac{\alpha\, t_1^{\theta-p+\beta+1}}{\theta-p+\beta+1} \right)\left(\dfrac{t_1^{-\theta} - \mu^{-\theta}}{-\theta} \right) - \dfrac{t_1^{1-p} - \mu^{1-p}}{(\theta-p+1)\,(1-p)} \\ - \dfrac{\alpha\,\beta\,\left(t_1^{\beta-p+1} - \mu^{\beta-p+1}\right)}{(\theta+\beta-p+1)(\beta-p+1)(\theta-p+1)} + \dfrac{\alpha\, t_1^{\theta-p+1}}{(\theta-p+1)}\left(\dfrac{t_1^{\beta-\theta} - \mu^{\beta-\theta}}{\beta-\theta} \right) \end{array} \right] \\ - C_s \left[\begin{array}{l} \dfrac{\lambda}{1-p}\left(t_1^{1-p}\left(t_2 - t_1\right) - \left(\dfrac{t_2^{2-p} - t_1^{2-p}}{2-p} \right) \right) + \left(\dfrac{\lambda\, T^{1-p}}{1-p} - k\,T \right)\left(T - t_2\right) \\ + k \left(\dfrac{T^2 - t_2^2}{2} \right) - \dfrac{\lambda \left(T^{2-p} - t_2^{2-p}\right)}{(1-p)(2-p)} \end{array} \right] \\ + C_p\, k \left(\mu + T - t_2\right) + C_a\, \alpha\, \beta \left[\begin{array}{l} \dfrac{k\,\mu^{\beta+1}}{\beta+1} - \dfrac{\lambda\,\mu^{\beta-p+1}}{(1-p)(\beta-p+1)} \\ + \dfrac{\lambda\, t_1^{\theta-p+1}\left(t_1^{\beta-\theta} - \mu^{\beta-\theta}\right)}{(\theta-p+1)(\beta-\theta)} - \dfrac{\lambda\left(t_1^{\beta-p+1} - \mu^{\beta-p+1}\right)}{(\theta-p+1)(\beta-p+1)} \end{array} \right] \end{array} \right] \quad (18)$$

Our objective is to determine optimum values μ^*, t_1^* and T^* of μ, t_1 and T respectively so that TC is minimum. Note that values μ^*, t_1^* and T^* can be obtained by solving the equations

$$\frac{\partial(TC)}{\partial \mu} = 0\,,\ \frac{\partial(TC)}{\partial t_1} = 0\ \&\ \frac{\partial(TC)}{\partial T} = 0 \qquad \text{... (19)}$$

with sufficient conditions

$$\left.\begin{array}{l} \left|\begin{matrix} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu\, \partial t_1} & \frac{\partial^2 TC}{\partial \mu\, \partial T} \\ \frac{\partial^2 TC}{\partial t_1\, \partial \mu} & \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1\, \partial T} \\ \frac{\partial^2 TC}{\partial T\, \partial \mu} & \frac{\partial^2 TC}{\partial T\, \partial t_1} & \frac{\partial^2 TC}{\partial T^2} \end{matrix}\right|_{\mu=\mu^*,\, t_1 = t_1^*,\, T=T^*} > 0, \quad \left|\begin{matrix} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu\, \partial t_1} \\ \frac{\partial^2 TC}{\partial t_1\, \partial \mu} & \frac{\partial^2 TC}{\partial t_1^2} \end{matrix}\right|_{\mu=\mu^*,\, t_1 = t_1^*,\, T=T^*} > 0 \\ \&\ \left.\frac{\partial^2 TC}{\partial \mu^2}\right|_{\mu=\mu^*,\, t_1 = t_1^*,\, T=T^*} > 0 \end{array}\right\} \qquad \text{... (20)}$$

The optimal solution of the equations in (19) can be obtained by using appropriate software.

7. Numerical Example

We consider the following numerical example to illustrate the above inventory model. We take the values of the parameters A = 350, p = 0.2, h = 7, r = 3, C_p = 30, α = 0.0001, C_d = 13, β = 7, λ = 10, k = 25, C_s = 24, θ = 4 and C_a = 12 (with appropriate units of measurement). We obtain the values μ = 0.9463 units, t_1 = 1.4480 units, T = 4.0082 units and total cost TC = 581.00 units by using appropriate software.

8. Sensitivity Analysis

Sensitivity analysis is very important technique to identify the effect on optimal solution of the model by changing its parameter values. In this section, we study the sensitivity of total cost TC per time unit with respect to the changes in the values of the parameters A, p, h, r, C_p, α, C_d, β, λ, k, C_s, θ and C_a

This analysis is performed by considering 10% and 20% increase and decrease in each one of the above parameters keeping all other remaining parameter as fixed. The results are presented in the Table below. The last column of the table shows the % change in TC as compared to the original solution corresponding to the change in parameters values, taken one by one.

Sensitivity Analysis

Table: *Sensitivity analysis of parameters considered in defining inventory model*

Parameter	% change	μ	t_1	T	TC	% change in TC
A	– 20	0.83175589	1.266598147	3.629045911	562.6778426	– 3.154367106
	– 10	0.89102353	1.360275867	3.824622008	572.0686178	– 1.538068228
	+ 10	0.998302145	1.530564752	4.181725486	589.5515557	1.471018078
	+ 20	1.047335814	1.60871594	4.346500791	597.759319	2.883702162
p	– 20	0.859701786	1.31110523	3.769610753	589.1869838	1.408269563
	– 10	0.903711768	1.380337631	3.888256076	585.1925571	0.720766421
	+ 10	0.987897045	1.514481235	4.130136151	576.6313298	– 0.75275432
	+ 20	1.028580335	1.580204783	4.254440932	572.0774381	– 1.53655011
h	– 20	1.00621809	1.54390374	4.107218692	578.3496366	– 0.457006928
	– 10	0.97520929	1.494188491	4.055958443	579.7142188	– 0.222141037

	+ 10	0.919440181	1.404948498	3.963745791	582.2279515	0.210511559
	+ 20	0.894291303	1.364778613	3.922158	583.3891243	0.410367515
r	– 20	0.97836117	1.499141683	4.065402182	580.1096796	– 0.154076074
	– 10	0.96175812	1.472597431	4.03572866	580.5675553	– 0.075268529
	+ 10	0.93199116	1.425056323	3.982674506	581.4235578	0.072062863
	+ 20	0.918550719	1.403611046	3.95878191	581.825281	0.141205688
C_p	– 20	1.037274297	1.540212251	3.939419913	512.7097178	– 11.75466074
	– 10	0.990215601	1.49328101	3.975277894	547.1746475	– 5.822708784
	+ 10	0.905757256	1.404892347	4.039542864	614.265725	5.724712076
	+ 20	0.868429347	1.364413278	4.07016762	647.014448	11.3612781
α	– 20	0.946241944	1.447780841	4.007998621	581.0059841	0.000191915
	– 10	0.946296945	1.447882528	4.008122461	581.0054268	0.000095995
	+ 10	0.946407202	1.448086356	4.008370699	581.0043233	– 0.00009393
	+ 20	0.946462458	1.448188499	4.0084951	581.0037523	– 0.00019222
C_d	– 20	0.958475485	1.51802264	4.04577743	576.0678279	– 0.849741786
	– 10	0.9529651	1.482227301	4.027633266	578.4065998	– 0.447202661
	+ 10	0.939019323	1.415379682	3.988252259	583.6186477	0.449872076
	+ 20	0.93123837	1.384425497	3.968080257	585.9711122	0.854767899
β	– 20	0.945924244	1.447251212	4.007352972	581.0088104	0.000678365
	– 10	0.946095172	1.447546686	4.007712133	581.0071054	0.000384908
	+ 10	0.946727253	1.448617365	4.009021841	581.0019521	– 0.000502048
	+ 20	0.947266002	1.449519586	4.01012983	580.998157	– 0.00115526
λ	– 20	0.711698697	1.126870535	3.942689058	526.8454182	– 9.321686228
	– 10	0.831906411	1.294211397	3.975372781	555.6323446	– 4.367007193
	+ 10	1.056702954	1.591573486	4.042661113	603.4029699	3.855062498
	+ 20	1.1646265	1.728059591	4.079945996	623.1660866	7.256603129
k	– 20	1.342361729	1.984224462	4.476893386	522.035381	– 10.14956865
	– 10	1.12600564	1.696279152	4.228863153	553.6095199	– 4.71516688
	+ 10	0.792027081	1.227334262	3.802834813	604.8840884	4.109986095
	+ 20	0.6571643	1.028832758	3.607387127	625.7092291	7.694317618
C_s	– 20	0.860126072	1.311404129	4.296908106	555.2920885	– 4.42557059
	– 10	0.90565684	1.383448927	4.137856415	568.9304429	– 2.078197073
	+ 10	0.982935162	1.50611407	3.900384517	591.7867648	1.855732415
	+ 20	1.015999855	1.558748329	3.809082029	602.2973344	3.664765379
θ	– 20	0.980314302	1.636660992	4.139036945	568.0747475	– 2.225475592
	– 10	0.964425575	1.53543882	4.070841421	575.1847286	– 1.001736951
	+ 10	0.927502049	1.372108242	3.95131385	585.8344683	0.831249364
	+ 20	0.908736406	1.30600132	3.899810838	589.8909184	1.529427684
C_a	– 20	0.946575248	1.44833933	4.008676399	581.0026323	– 0.000384979
	– 10	0.946463543	1.448161694	4.008461255	581.0037512	– 0.000192402
	+ 10	0.946240711	1.447807344	4.008032093	581.0059859	0.000192226
	+ 20	0.946129582	1.447630627	4.007818073	581.0071017	0.000384278

9. Graphical Presentation

Graphical presentation of the above sensitivity analysis is shown in Fig. 2.

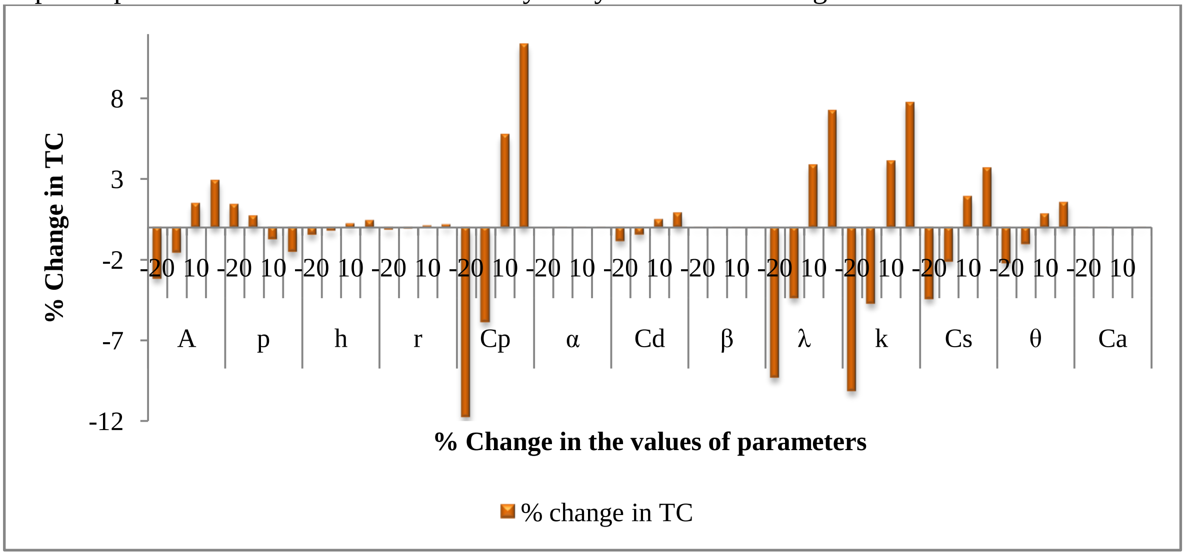

Fig. 2. *Graphical presentation of the above sensitivity analysis*

10. Conclusion

From the above sensitivity analysis we may conclude that the total cost TC per time unit is highly sensitive to the changes in the values of the parameters C_p, k and λ; moderately sensitive to the changes in the values of the parameters C_s, A and θ and less sensitive to the changes in the values of the parameters p, C_d, h, r, α, β and C_a.

Moreover, It can also be observed from Fig. 2. that there is an opposite change in total cost TC per time unit for parameters p, α and β where as simultaneous change can be found in total cost TC per time unit for the remaining parameters C_p, k, λ, C_s, A, θ, C_d, h, r and C_a.

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