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Functions with α * Closed Sets in Topological Spaces

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Abstract: *The aim of this paper is to define a new class of functions namely α * homeomorphism and strongly α * homeomorphism and study their properties. Additionally, we relate and compare these functions with some other functions in topological spaces.*

Keywords: α * *homeomorphism, strongly* α * *homeomorphism.*

1. Introduction

The notion of Homeomorphism plays a very important role in Topology. In the course of generalization of the notion of Homeomorphism, Maki et al [3] introduced g-homeomorphisms in topological spaces. Devi et al [1] introduced the concept of α-homeomorphisms and then we introduce α *-homeomorphisms and strongly α *-homeomorphisms and discuss some of their basic properties.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The power set of X is denoted by P(X).

Definition 2.1: A subset A of a topological space X is said to be a α ***open*** [5] if A $\subseteq$ int* (cl (int* (A))).

Definition 2.2: A function f: (X, τ) ⟶ (Y, σ) is called a α *** continuous*** [6]if f^{-1}(O) is a α *open set of (X, τ) for every open set O of (Y, σ).

Definition 2.3: A map f*:* (X, τ) ⟶ (Y, σ) is called a α *** open map* [4]**if image of each open set in X is α * open in Y.

Definition 2.4: A map f*:* (X, τ) ⟶ (Y, σ) is called a ***Homeomorphism*** if f is both open and continuous.

Definition 2.5: A map f*:* (X, τ) ⟶ (Y, σ) is called a ***semi- homeomorphism*** [2]if f is both irresolute and pre semi-open .

Definition 2.6: A function f*:* (X, τ) ⟶(Y, σ) is said to be ***Irresolute*** [7]if f^{-1}(O) is semi-open in (X, τ) whenever O is semi-open in (Y, σ).

Definition 2.7: A function f*:* (X, τ) ⟶(Y, σ) is called ***pre semi open* [7]**f(O) is semi-open in (Y, σ) for all O semi-open in (X, τ) .

Definition 2.8: A map f*:* (X, τ) ⟶ (Y, σ) is called a ***g- homeomorphism*** [3]if f is both g-open and g-continuous.

Definition 2.9: A map f*:* (X, τ) ⟶ (Y, σ) is called a α ***g- homeomorphism*** [1]if f is both α ***g*** - open and α *g* - continuous.

Definition 2.10: A map f: (X, τ) ⟶ (Y, σ) is called a ***gα - homeomorphism*** [1]if f is both gα - open and gα -continuous.

Definition 2.11: A function f: (X, τ) ⟶(Y, σ) is said to be α ****Irresolute*** [6] if $f^{-1}(O)$ is a α *open in (X, τ) for every α *open set O in (Y, σ).

Theorem 2.12 [5]**:** Every open set is α * open.

Theorem 2.13 [6]: Every g-continuous map is α * continuous.

Theorem 2.14 [4]: Every g-open map is α * open.

Theorem 2.15 [4]**:** Let f: (X, τ) ⟶ (Y, σ) be a bijective map. Then the following are equivalent:

(1) f is a α * open map.

(2) f is a α * closed map.

(3) f^{-1} is a α * continuous map.

Theorem 2.16[6]: Every α *irresolute map is α *continuous.

3. α * HOMEOMORPHISMS

Definition 3.1: A bijection f: (X, τ) ⟶ (Y, σ) is called α * **Homeomorphisms** if f is both α *continuous and α *open.

Example 3.2: Let X = Y= {a, b, c, d}, τ = {ϕ, {ab}, X} and σ = {ϕ, {a},{abc},Y}, α *O(X, τ) = P(X) and α *O(Y, σ) = { ϕ, {a} , {b} , {c} , {ab} , {ac} , {ad} , {bc} , {abc} , {abd} , {acd} , Y} .Let f: (X, τ) ⟶(Y, σ) be defined by f(c) = f(d) = a, f(b) = c , f(a) = b. Clearly , f is α * Homeomorphisms.

Theorem 3.3: Let f: (X, τ) ⟶ (Y, σ) be a bijective, α *continuous map. Then the following statements are equivalent

(i) f is a α *open.

(ii) f is a α * Homeomorphisms.

(iii)f is a α *closed.

Proof:

(i)⇔(ii) Obvious from the definition.

(ii)⇔(iii) Let V be a closed set in (X, τ). Then V^c is open in (X, τ). By hypothesis, $f(V^c) = (f(V))^c$ is α *open in (Y, σ). That is, f(V) is α *closed in (Y, σ). Therefore, f is a α *closed.

(iii)⇔(i) Let V be a open set in (X, τ). Then V^c is closed in (X, τ). By hypothesis, $f(V^c) = (f(V))^c$ is α *closed in (Y, σ). That is, f(V) is α *open in (Y, σ). Therefore, f is a α *open map.

Theorem 3.4: Every homeomorphism is α * homeomorphism.

Proof: Let f: (X, τ) ⟶ (Y, σ) be an homeomorphism, then f is bijective, continuous and open.. Let V be an open set in Y. Since, f is continuous, $f^{-1}(V)$ is open in X. Since, every open set is α *open, $f^{-1}(V)$ is α *open in X which implies f is α *continuous. Let W be an open set in X. Since, f is open, f (W) is open in Y. Since, every open set is α *open, f (W) is α *open in Y which implies f is α *open. Thus, f is α * homeomorphism.

Remark 3.5: The converse of the above theorem need not be true.

Example 3.6: Let X = Y= {a, b, c, d}, τ = { ϕ, {ab}, X} and σ = {ϕ, {a},{abc},Y}, α *O(X, τ) = P(X) and α *O(Y, σ) = {ϕ, {a},{b},{c},{ab},{ac},{ad},{bc},{abc},{abd},{acd},Y} .

Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = b, f(b) = c, f(c) = f(d) = a. Clearly, f is α * Homeomorphisms. Here, {ab} is open in X, but f{ab}= {bc} is not open in Y. Hence, f is not an open map. Therefore, f is not homeomorphism.

Theorem 3.7: Every α homeomorphism is α * homeomorphism.

Proof: Let f: (X, τ) ⟶ (Y, σ) be an α homeomorphism, then f is bijective, α continuous and α open. Let V be an open set in Y. Since, f is α continuous. $f^{-1}(V)$ is α open in X. Since, every α open set is α *open, $f^{-1}(V)$ is α *open in X which implies f is α *continuous. Let W be an open set in X. Since, f is α open, f (W) is α open in Y. Since, every α open set is α *open, f (W) is α *open in Y which implies f is α *open. Thus, f is α * homeomorphism.

Remark 3.8: The converse of above theorem need not be true.

Example 3.9: Let X = Y= {a, b, c,d}, τ = {ϕ, {a},{ab},{abc}, X} and σ = {ϕ, {ab},{abc},Y}, α *O(X, τ) = {ϕ, {a} ,{b},{c},{ab},{ac},{ad},{bc},{abc},{abd},{acd}, X} , α O(X, τ) = {ϕ, {a}, {ab} , {ac} , {ad} , {abc} , {abd} , {acd} , X } and α *O(Y, σ)={ϕ, {a} , {b} , {c} , {ab} , {ac} , {ad} , {bc}, {bd} , {abc} , {abd} , {acd} , Y } , α O(Y, σ) = {ϕ, {ab},{abc},{abd},Y}. Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = a, f(b) = d, f(c) = b , f(d) = c. Clearly, f is α * Homeomorphisms. Here, {a} is open in X, but f{a}= a is not α open in Y. Hence, f is not a α open map. Therefore, f is not α homeomorphism.

Theorem 3.10: Every g -homeomorphism is α * homeomorphism.

Proof: Let f: (X, τ) ⟶ (Y, σ) be an g-homeomorphism, then f is bijective, g-continuous and g-open. Since, every g-continuous map is α *continuous and g-open map is α *open which implies f is both α *continuous and α *open. Therefore, f is α * homeomorphism.

Remark 3.11: The converse of above theorem need not be true.

Example 3.12: Let X = Y= {a, b, c}, τ = {ϕ, {ab}, X} and σ = {ϕ, {a},{ab},Y}, α *O(X, τ) = {ϕ, {a},{b},{ab},{ac},{bc}, X} , GO(X, τ) = {ϕ, {a},{b},{ab}, X} and α *O(Y, σ) = {ϕ, {a},{b},{ab},{ac},Y} , GO(Y, σ) ={ϕ, {a},{b},{ab},Y} .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = c, f(b) = a, f(c) = b. Clearly, f is α * Homeomorphisms. But, f is not g-homeomorphism for the open set V= {ab} in X, f (V) = {ac} is not g-open in Y. Hence, f is not g-open map. Therefore, f is not g -homeomorphism.

Remark 3.13: The concept of α * homeomorphism and semi- homeomorphism are independent as can be seen from the following examples.

Example 3.14: Let X = Y= {a, b, c}, τ = {ϕ, {ab}, X} and σ = {ϕ, {a},{ab},Y}, α *O(X, τ) = {ϕ, {a},{b},{ab},{ac},{bc}, X} , SO(X, τ) = {ϕ, {ab}, X} and α *O(Y, σ) = {ϕ, {a},{b},{ab},{ac},Y} , SO(Y, σ) ={ϕ, {a},{ab},{ac},Y} .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = c, f(b) = a, f(c) = b. Clearly, f is α * Homeomorphisms. But, f is not semi-homeomorphism for the semi open set V= {ab} in Y, $f^{-1`}$ (V) = {bc} is not semi-open in X. Hence, f is not irresolute map. Therefore, f is not semi -homeomorphism.

Example 3.15: Let X = Y= {a, b, c, d}, τ = {ϕ, {a},{ab},{abc}, X} and σ = {ϕ, {a} , {b} , {c} , {ab} , {ac} , {bc} , {abc} , Y } , α *O(X, τ) = {ϕ , {a} , {b} , {c} , {ab} , {ac} , {ad} , {bc} , {abc} , {abd} , {acd} , X }, SO(X, τ) = {ϕ, {a} , {ab} , {ad} , {abc} , {abd} , {acd} , X } and α *O(Y, σ) = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},Y}, SO(Y, σ) = P(X) / {d} . Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = f(b) = f(d) = a , f(c) = d. Clearly, f is semi homeomorphisms. But for the open set V = {abc} in X, but f{V}= f{abc}= {ad} is not α *open in Y. Hence, f is not α * open map. Therefore, f is not α *homeomorphism.

Remark 3.16: The concept of α * homeomorphism and α g- homeomorphism are independent as can be seen from the following examples.

Example 3.17: Let X = Y= {a, b, c}, τ = {ϕ, {ab}, X} and σ = {ϕ, {a},{ab},Y}, α *O(X, τ) = {ϕ, {a},{b},{ab},{ac},{bc}, X} , α g(X, τ) = {ϕ, {a},{b},{ab}, X} and α *O(Y, σ) = {ϕ, {a},{b},{ab},{ac},Y} , α g(Y, σ) ={ϕ,{b}, {a},{ab},{ac},Y} .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = c, f(b) = a, f(c) = b. Clearly, f is α * Homeomorphisms. But, f is not α g - homeomorphism for the open set V= {ab} in Y, f^{-1} (V) = {bc} is not α g -open in X. Hence, f is not α g -continuous map. Therefore, f is not α g -homeomorphism.

Example 3.18: Let X = Y= {a, b, c, d}, τ = {ϕ, {a},{b} , {ab} , {abc} , X } and σ = {ϕ, {a} , {b} , {ab} , {bc} , {abc} , Y }, α *O(X, τ) = {ϕ , {a} , {b} , {c} , {ab} , {ac} , {bc} , {abc} , {abd} , X }, α g(X, τ)= {ϕ, {a},{b},{c},{ab},{ac},{ad},{bc},{bd},{abc},{abd},X} and α *O(Y, σ) = {ϕ, {a} , {b} , {c} , {ab} , {ac} , {bc} , {abc} , {abd} , Y } , α g (Y, σ) = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},{abd},Y}.Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = a , f(b) = f(d) = b , f(c) = d. Clearly, f is α g homeomorphisms. But for the open set V = {b} in Y, f^{-1}{V}= {bd} is not α *open in X . Hence, f is not α * continuous map. Therefore, f is not α *homeomorphism.

Remark 3.19: The concept of α * homeomorphism and g α - homeomorphism are independent as can be seen from the following examples.

Example 3.20: Let X = Y= {a, b, c}, τ = {ϕ, {ab}, X} and σ = {ϕ, {a},{ab},Y}, α *O(X, τ) = {ϕ, {a},{b},{ab},{ac},{bc}, X} , g α (X, τ) = {ϕ, {a},{b},{ab}, X} and α *O(Y, σ) = {ϕ, {a},{b},{ab},{ac},Y}, g α (Y, σ) ={ϕ,{b}, {a},{ab},{ac},Y} .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = c, f(b) = a, f(c) = b. Clearly, f is α * Homeomorphisms. But, for the open set V= {ab} in Y, f^{-1} (V) = {bc} is not g α -open in X. Hence, f is not g α -continuous map. Therefore, f is not g α -homeomorphism.

Example 3.21: Let X = Y= {a, b, c, d}, τ = {ϕ, {a} , {b} , {ab} , {abc} , X } and σ = {ϕ, {a} , {abc} , Y }, α *O(X, τ) = { ϕ , {a} , {b} , {c} , {ab} , {ac} , {bc} , {abc} , {abd} , X }, g α (X, τ)= {ϕ, {a} , {b} , {ab} , {ac} , {ad} , {bc} , {bd} , {abc} , {abd} , X } and α *O(Y, σ) = {ϕ, {a} , {b} , {c} , {ab} , {ac} , {ad},{bc} , {abc} , {abd} , {acd}, Y } , g α (Y, σ) = {ϕ, {a},{b},{ab},{ac},{ad},{abc},{abd},{acd}, Y}.Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = a = f(d) , f(b) = b , f(c) = c. Clearly, f is g α homeomorphisms. But for the open set V = {a} in Y, f^{-1}{V}= {ad} is not α *open in X . Hence, f is not α * continuous map. Therefore, f is not α *homeomorphism.

Remark 3.22: The composition of two α *homeomorphism need not be a α *homeomorphism.

Example 3.23: Let X = Y= Z = {a, b, c, d}, τ = {ϕ, {a}, {abc} , X } and σ = {ϕ, {ab}, {abc} , Y } , η = {ϕ, {a}, {b} , {c} , {ab} , {ac} , {bc} , {abc} , Z } , α *O(X, τ) = {ϕ , {a}, {b} , {c} , {ab} , {ac} , {ad} ,{bc} , {abc} , {abd} , {acd} , X }, α *O(Y, σ) = {ϕ, {a} , {b} , {c} , {ab} , {ac} ,{bc} , {bd} , {abc} , {abd} , {acd}, Y } , α *O(Z, η) = {ϕ, {a} , {b} , {c} , {ab} , {ac} , {bc} , {abc} , Z } .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = a , f(b) = d , f(c) = b , f(d) = c. Clearly, f is α *homeomorphism. Let g: (Y, σ) ⟶(Z, η) be an identity map. Clearly, g is α *homeomorphism. Here, f and g are α *homeomorphism. But $(g\circ f)^{-1}$ {bc} = f^{-1}(g^{-1} {bc})) = f^{-1} {bc} = cd, {cd} is not α *open in X. Therefore, $(g\circ f)$ is not α *homeomorphism.

4. STRONGLY α *HOMEOMORPHISM

Definition 4.1: A bijection f: (X, τ) ⟶ (Y, σ) is said to be ***strongly*** α **** -homeomorphism*** if both f and f^{-1} are α *Irresolute .

Example 4.2: Let X = Y= {a, b, c}, τ = {ϕ, {a},{ab}, X} and σ = {ϕ, {a}, {b},{ab},Y}, α *O(X, τ) = {ϕ, {a},{b},{ab},{ac}, X} and α *O(Y, σ) = {ϕ, {a},{b},{ab},{ac},Y}.Let f: (X, τ) ⟶(Y, σ) is an identity map. Clearly, f is strongly α * -Homeomorphisms.

We denote the family of all strongly α * -homeomorphism of a topological space X into itself by **S** α * -*h(X)*.

Theorem 4.3: Every strongly α * -homeomorphism is a α * -homeomorphism. In other words, for any space strongly α * -homeomorphism(X) $\subset$ α * -homeomorphism(X).

Proof: Let f: (X, τ) ⟶ (Y, σ) be a bijective map which is strongly α * -homeomorphism. Then f and f^{-1} are α *irresolute. Since, every α *irresolute are α * continuous, f and f^{-1} are α * continuous. Since, f^{-1} is α * continuous, by thm []f is α * open map. Thus, f is both α * continuous and α * open. Therefore, f is α * -homeomorphism.

Remark 4.4: The converse of the above theorem need not be true.

Example 4.5: Let X = Y= {a, b, c, d}, τ = {ϕ, {a},{abc}, X} and σ = {ϕ, {ab},{abc},Y}, α *O(X, τ) = {ϕ, {a},{b},{c},{ab},{ac},{ad},{bc},{abc},{abd},{acd},X} and α *O(Y, σ) = {ϕ, {a},{b},{c},{ab},{ac},{bc},{bd},{abc},{abd},{acd},Y} .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = a, f(b) = d, f(c) = b , f(d) =c. Clearly, f is α * Homeomorphisms. But for the α * open set V = {c} in (Y, σ), f^{-1} ({c}) = d is not α * open in(X, τ). Therefore, f is not strongly α * -homeomorphism.

Theorem 4.6: If f: (X, τ) ⟶ (Y, σ) and g: (Y, σ) ⟶(Z, η) are strongly α * -homeomorphism then their (g∘f) : (X, τ) ⟶ (Z, η) is also strongly α * -homeomorphism.

Proof:

(i) **(g∘f) is α *irresolute**.

Let U be a α * open in Z. Now, $(g\circ f)^{-1}$ (U) = $f^{-1}(g^{-1}(U))$ = $f^{-1}(V)$ where V = $g^{-1}(U)$. By hypothesis, V = $g^{-1}(U)$ is α * open in Y and so again, by hypothesis $f^{-1}(V)$ is α * open in X.

(ii) **$(g\circ f)^{-1}$ is α *irresolute.**

Let G be a α * open in X. By hypothesis, f(G) is α * open in Y. Again, by hypothesis (g∘f) (G) = g(f(G)) is α * open in Z. Thus, $(g\circ f)^{-1}$ is α *irresolute.

From (i) and (ii) , (g∘f) : (X, τ) ⟶ (Z, η) is also strongly α * -homeomorphism.

Theorem 4.7: Every strongly α * -homeomorphism is α *irresolute.

Proof: It is the consequence of the definition.

Remark 4.8: The converse of the above theorem need not be true.

Example 4.9: Let X = Y= {a, b, c, d}, τ = {ϕ, {ab},X} and σ = {ϕ, {a},{abc},Y}, α *O(X, τ) = P(X) and α *O(Y, σ) = {ϕ, {a},{b},{c},{ab},{ac},{ad},{bc},{abc},{abd},{acd},Y} Let f: (X, τ) ⟶(Y, σ) be a identity map . Clearly, f is α * irresolute. But for the α * open set V = {d} in (X, τ), f^{-1} ({d}) = d is not α * open in (Y, σ) . Therefore, f is not strongly α * -homeomorphism.

Theorem 4.10: The set S α * -*h(X)* is a group under the composition of maps.

Proof: Define a binary operation ‘ * ‘ by S α * -*h(X)* × S α * -*h(X)* ⟶ S α * -*h(X)* , by f * g = f ∘ g for all f and g in S α * -*h(X)* and ∘ is the usual operation of composition of maps. Then by theorem 4.6, f ∘ g ∈ S α * -*h(X)* . We know that the composition of maps are associative and the identity map i: X ⟶ X belonging to S α * -*h(X)* serves as the identity element. If f ∈ S α * -*h(X)* then f^{-1} ∈ S α * -*h(X)* such that f ∘ f^{-1} = f^{-1} ∘ f = i and so inverse exists for each element of S α * -*h(X)*. Therefore, S α * -*h(X)* is a group under the composition of maps.

Theorem 4.11: Let f: (X, τ) ⟶(Y, σ) be a strongly α * -homeomorphism. Then f induces an isomorphism from the group S α * -*h(X)* onto the group S α * -*h(Y)*.

Proof: Using the map f, we define a map ψ_f : S α * -*h(X)* ⟶ S α * -*h(Y)* by ψ_f (h) = f ∘h∘f^{-1} for each h ∈ S α * -*h(X)*. By theorem 4.6, ψ_f is well defined in general, because f ∘h∘ f^{-1} is a strongly α * -homeomorphism for every strongly α * -homeomorphism h: X $\longrightarrow$ Y. To show that ψ_f is a bijective homeomorphism. Bijective of ψ_f is clear. Further for all h_1 , $h_2 \in S\alpha * \text{-}h(X)$, $\psi_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \psi_f(h_1) \circ \psi_f(h_2)$. Therefore , ψ_f is a homeomorphism and hence it induces an isomorphism induced by f.

Theorem 4.12: strongly α * -homeomorphism is an equivalence relation on the collection of all topological spaces.

Proof: Reflexivity and symmetry are immediate and transitivity follows from Theorem 4.6

REFERENCES

[1]. Balachandran K , Devi R: Some generalizations of α -homeomorphisms in topological spaces, Indian J. Pure. Appl.Math. 32(4), (2001), 551-563.

[2]. Chandra Sekhara Rao .K : On Semi- Homeomorphism, International Journal of Pure Algebra, -3(6), 2013, 222-223

[3]. Maki H ; Sundaram P. ; Balachandran K : Generalized homeomorphisms in topological spaces, Bulletin of Fukuoka University of Education, Vol 40,Part-III (1991)13-21.

[4]. S.Pious Missier and P. Anbarasi Rodrigo, Strongly α * continuous functions in Topological spaces,IOSR – JM , Volume 10, Issue 4 Ver. I (Jul-Aug. 2014), PP 55-60

[5]. S.Pious Missier and P. Anbarasi Rodrigo, Some notions of nearly open sets in Topological spaces, IJMA 4(12), 2013, 1-7

[6]. S.Pious Missier and P. Anbarasi Rodrigo, On α * continuous functions in Topological spaces, OUTREACH Volume VII 2014 148 – 154

[7]. D. Sivaraj, Semi-homeomorphisms, Ann Math. Hung. 48(1986), 139-145

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