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On Almost Supra N-continuous Function

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Abstract: *In this paper, we introduce the concept of almost supra N-continuous function and investigated the relationship of this functions with other functions. Also we have defined mildly supra N-normal space.*

Mathematics subject classification: *54C10, 54C05*

Keywords: *almost supra N-continuous function.*

1. INTRODUCTION

Supra topological spaces was introduced by A.S.Mashhour et al [3] in 1983. The concept of almost continuity was introduced by M.K.Singal and A.R.Singal[7] using regular closed sets. Takashi Noiri[6] obtained some characterizations of almost continuity.

In this paper, we bring out the concept of almost supra N-continuous function and investigated the relationship with other functions in supra topological spaces. Also a new type of normal space called mildly supra N-normal space is also defined and its properties are investigated.

2. PRELIMINARIES

Definition 2.1[3]

A subfamily μ of X is said to be supra topology on X if

i) $X, \varphi \in \mu$

ii)If $A_i \in \mu, \forall i \in j$ then $\cup A_i \in \mu$

(X, μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c.

Definition 2.2[3]

The supra closure of a set A is denoted by $cl^{\mu}(A)$, and is defined as supra $cl(A) = \cap\{B : B$ is supra closed and $A \subseteq B\}$.

The supra interier of a set A is denoted by $int^{\mu}(A)$, and is defined as supra $int(A) = \cup\{B : B$ is supra open and $A \supseteq B\}$.

Definition 2.3[3]

Let (X, τ) be a topological space and μ be a supra topology on X. We call μ be a supra topology associated with τ, if $\tau \subseteq \mu$.

Definition 2.4

A subset A of a space X is called

(i) supra semi-open set[2], if $A \subseteq cl^{\mu}(int^{\mu}(A))$.

(ii) supra α -open set[1], if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$.

(iii) supra Ω- closed set[5], if $scl^{\mu}(A) \subseteq int^{\mu}(U)$,whenever $A \subseteq U$, U is supra open set.

(iv) supra N-closed set[7], if $\Omega cl^{\mu}(A) \subseteq U$, whenever $A \subseteq U$, U is supra α-open set.

(v) supra regular open[10], if $A=int^{\mu}cl^{\mu}(A)$

The complement of the above mentioned sets are their respective open and closed sets and vice-versa.

Definition 2.5 A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is said to be

(i) supra N-continuous [8] if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).

(ii) supra N-irresolute[8] if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra N-closed set V of (Y, σ).

(iii) perfectly supra N-continuous[10] if $f^{-1}(V)$ is supra clopen in (X, τ) for every supra N-closed set V of (Y, σ).

(iv) Strongly supra N-continuous[10] if $f^{-1}(V)$ is supra closed in (X, τ) for every supra N-closed set V of (Y, σ).

(v) perfectly contra supra N-irresolute[9] if $f^{-1}(V)$ is supra N-closed and supra N-open in (X, τ) for every supra N-open set V of (Y, σ).

(vi)Contra supra N-irresolute[9], if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra N-open set V of (Y, σ).

(vii) Almost contra supra N-continuous[9], if $f^{-1}(V)$ is supra N-closed in (X, τ) for every supra regular open set V of (Y, σ).

Definition 2.6[11] A Space (X, τ) is said to be

(i) supra N-normal if for any pair of disjoint supra closed sets A and B, there exist disjoint supra N-open sets U and V such that $A \subset U$ and $B \subset V$.

(ii)weakly supra N-normal if for any pair of disjoint supra N-closed sets A and B, there exist disjoint supra open sets U and V such that $A \subset U$ and $B \subset V$.

3. ALMOST SUPRA N-CONTINUOUS FUNCTION

Definition 3.1 A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is called Almost supra continuous function if $f^{-1}(V)$ is supra open set in (X, τ) for every supra regular open set V of (Y, σ).

Definition 3.2 A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is called Almost supra N-continuous function if $f^{-1}(V)$ is supra N-open in (X, τ) for every supra regular open set V of (Y, σ).

Theorem 3.3 For a function $f:(X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent:

i) f is almost supra N-continuous.

ii) $f^{-1}(V)$ is supra N-closed in X for every supra regular closed set V of Y.

iii) $f^{-1}(cl^{\mu}int^{\mu}(V))$ is supra N-closed in X, for every supra closed set V of Y.

iv) $f^{-1}(int^{\mu}cl^{\mu}(V))$ is supra N-open in X, for every supra open set V of Y.

Proof

(i)⇒(ii) Let V be supra regular closed set in Y. Then Y-V is supra regular open set in Y. Since f is almost supra N-continuous, f^{-1} (Y-V)=X-f^{-1}(V) is supra N-open in X. Hence f^{-1}(V) is supra N-closed in X.

(ii)⇒(iii) Let V be supra closed set in Y. Then V=$cl^{\mu}int^{\mu}(V)$ is supra regular closed set in Y, then by hypothesis, $f^{-1}(cl^{\mu}int^{\mu}(V))$ is supra N-closed in X.

(iii)⇒(iv) Let V be supra open set in Y. Then V=$int^{\mu}cl^{\mu}(V)$ is supra regular open set in Y.Then Y- $int^{\mu}cl^{\mu}(V)$ is supra regular closed set in Y. Then by hy- pothesis, f^{-1}(Y-$int^{\mu}cl^{\mu}$(V))=X-$f^{-1}(int^{\mu}cl^{\mu}(V))$ is supra N-closed in X. Hence $f^{-1}(int^{\mu}cl^{\mu}(V))$ is supra N-open in X.

(iv)⇒(i) Let V be supra open set in Y. Then V=$int^{\mu}cl^{\mu}$ (V) is supra regular open set and every regular open set is open set in Y. Then by hypothesis, $f^{-1}(int^{\mu}cl^{\mu}(V))=f^{-1}(V)$ is supra N-open in X. Hence f is almost supra N-continuous.

Theorem 3.4 Every supra N-continuous function is almost supra N-continuous function.

Proof Let f:(X, τ) → (Y, σ) be a supra N-continuous function. Let V be supra regular open set in (Y,σ). Then V is supra open set in (Y,σ), since every supra regular open set is supra open set. Since f is supra N-continuous function f^{-1}(V) is both supra N-open in (X, τ). Therefore f is almost supra N-continuous function. The converse of the above theorem need not be true. It is shown by the following example.

Example 3.5 Let X=Y={a, b, c} and τ= {X, φ, {a}, {a, b}}, σ = {Y, φ, {a}, {b}, {a, b}, {b, c}}. N-open set in (X,τ) are {X, φ, {a}, {b}, {a, b}, {a, c}}. N-open set in (Y,σ) are {Y, φ, {a}, {b}, {a, b}, {b, c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is almost supra N-continuous but not supra N-continuous, since V={a, b} is supra open in (Y, σ) but f^{-1}({a, b})= {b, c} is not supra N-open set in (X,τ).

Theorem 3.6 Every strongly supra N-continuous function is almost supra N- continuous function.

Proof Let f:(X, τ) → (Y, σ) be a strongly supra N-continuous function. Let V be supra regular open set in (Y,σ), then V is supra N-open set in (Y,σ), since every supra regular open set is supra open set and every supra open set is supra N-open set. Since f is strongly supra N-continuous function, then f^{-1}(V) is supra N-open in (X,τ). Therefore f is Almost supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.7 Let X=Y={a, b, c} and τ= {X, φ, {a}, {a, b}}, σ = {Y, φ, {a}, {b}, {a, b}, {b, c}}. N-open set in (X,τ) are {X, φ, {a}, {b}, {a, b}, {a, c}}. N-open set in (Y,σ) are {Y, φ, {a}, {b}, {a, b}, {b, c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is almost supra N-continuous but not strongly supra N-continuous, since V={a, b} is supra N-open in (Y, σ) but f^{-1}({a, b}) = {b, c} is not supra open set in (X,τ).

Theorem 3.8 Every perfectly supra N-continuous function is almost supra N- continuous function.

Proof Let f:(X, τ) → (Y, σ) be a perfectly supra N-continuous function. Let V be supra regular open set in (Y,σ), then V is supra N-open set in (Y,σ), since every supra regular open set is supra open set and every supra open set is supra N-open set. Since f is perfectly supra N-continuous function, then $f^{-1}(V)$ is supra clopen in (X,τ), then $f^{-1}(V)$ is supra N-clopen in (X,τ), implies $f^{-1}(V)$ is supra N-open in (X,τ). Therefore f is Almost supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.9 Let X=Y={a, b, c} and τ= {X, φ, {a}, {a, b}}, σ = {Y, φ, {a}, {b}, {a, b}, {b, c}}. N-open set in (X,τ) are {X, φ, {a}, {b}, {a, b}, {a, c}}. N-open set in (Y,σ) are {Y, φ, {a}, {b}, {a, b}, {b, c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is almost supra N-continuous but not perfectly supra N-continuous, since V={a, b} is supra N-open in (Y, σ) but $f^{-1}(\{a, b\}) = \{b, c\}$ is not supra clopen set in (X,τ).

Theorem 3.10 Every almost supra continuous function is almost supra N-continuous function.

Proof Let f:(X, τ) → (Y, σ) be a almost supra continuous function. Let V be supra regular open set in (Y, τ). Since f is almost supra continuous function, then $f^{-1}(V)$ is supra open in (X,τ), implies $f^{-1}(V)$ is supra N-open in (X,τ). Therefore f is almost supra N-continuous function.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.11 Let X=Y={a, b, c} and τ= {X, φ, {a}, {a, b}}, σ = {Y, φ, {a}, {b}, {a, b}, {b, c}}. N-open set in (X,τ) are {X, φ, {a}, {b}, {a, b}, {a, c}}. N-open set in (Y,σ) are {Y, φ, {a}, {b}, {a, b}, {b, c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is almost supra N-continuous but not almost supra continuous, since V={a} is supra regular open in (Y, σ) but $f^{-1}(\{a\}) = \{c\}$ is not supra open set in (X,τ).

Theorem 3.12 If f:(X, τ) → (Y, σ) is supra N-irresolute and g:(Y, σ) → (Z, η) is almost supra N-continuous then g∘f:(X, τ) → (Z, η) is almost supra N-continuous. **Proof** Let V be supra regular open set in Z. Since g is almost supra N-continuous, then $g^{-1}(V)$ is supra N-open set in Y. Since f is supra N-irresolute, then $f^{-1}(g^{-1}(V))$ is supra N-open in X. Hence g∘f is almost supra N-continuous.

Theorem 3.13 If f:(X, τ) → (Y, σ) is strongly supra N-continuous and g:(Y, σ) → (Z, η) is almost supra N-continuous then g∘f:(X, τ) → (Z, η) is almost supra N- continuous.

Proof Let V be supra regular open set in Z. Since g is almost supra N-continuous, then $g^{-1}(V)$ is supra N-open set in Y. Since f is strongly supra N-continuous, then $f^{-1}(g^{-1}(V))$ is supra open in X. Implies $f^{-1}(g^{-1}(V))$ is supra N-open in X. Hence g∘f is almost supra N-continuous.

Theorem 3.14 If f:(X, τ) → (Y, σ) is contra supra N-irresolute and g:(Y, σ) → (Z, η) is almost contra supra N-continuous then g∘f:(X, τ) → (Z, η) is almost supra N-continuous.

Proof Let V be supra regular open set in Z. Since g is almost contra supra N-continuous, then $g^{-1}(V)$ is supra N-closed set in Y. Since f is contra supra N-irresolute, then $f^{-1}(g^{-1}(V))$ is supra N-open in X. Hence g∘f is almost supra N-continuous.

Definition 3.15 A space X is said to be mildly supra N-normal if for every pair of disjoint supra regular closed sets A and B of X, there exist disjoint supra N-open sets U and V such that A⊂U and B⊂V.

Theorem 3.16 Every supra normal space is mildly supra N-normal.

Proof Let A and B be disjoint supra regular closed sets of X, then A and B are disjoint supra closed sets of X, since every supra regular closed set is supra closed set. Since X is supra normal, there exist disjoint supra open sets U and V such that A⊂U and B⊂V. Since every supra open set is supra N-open set, then U and V are disjoint supra N-open sets. Hence X is mildly supra N-normal.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.17 Let X={a, b, c, d} and τ = {X, φ, {a}, {d}, {a, b}, {c, d}, {a, b, c}, {b, c, d}} supra N-open sets in (X,τ) are {X, φ, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d},{b, c}, {a, c}, {a, b, c}, {a, b, d}, {b, c, d}, {a, c, d}}.Here (X,τ) is mildly supra N- normal but not supra normal, since A={a, b} and B={d} is supra closed in (X,τ) but A and B is not contained in disjoint supra open sets.

Theorem 3.18 Every supra N-normal space is mildly supra N-normal.

Proof Let A and B be disjoint supra regular closed sets of X, then A and B are disjoint supra closed sets of X, since every supra regular closed set is supra closed set. Since X is supra N-normal, there exist disjoint supra N-open sets U and V such that A⊂U and B⊂V. Hence X is mildly supra N-normal.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.19 Let X={a, b, c, d} and τ = {X, φ, {a}, {d}, {a, b}, {c, d}, {a, b, c}, {b, c, d}}

supra N-open sets in (X,τ) are {X, φ, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d},{b, c}, {a, c}, {a, b, c}, {a, b, d}, {b, c, d}, {a, c, d}}.Here (X,τ) is mildly supra N- normal but not supra N-normal, since A={a, b} and B={d} is supra closed in (X,τ) but A and B is not contained in disjoint supra N-open sets.

Theorem 3.20 Every weakly supra N-normal space is mildly supra N-normal.

Proof Let A and B be disjoint supra regular closed sets of X, then A and B are disjoint supra closed sets and hence supra N-closed sets of X, since every supra regular closed set is supra closed set. Since X is weakly supra N-normal, there exist disjoint supra open sets U and V such that A⊂U and B⊂V. Since every supra open set is supra N-open set, U and V are supra N-open sets. Hence X is mildly supra N-normal.

The converse of the above theorem need not be true. It is shown by the following example.

Example 3.21 Let X={a, b, c, d} and τ = {X, φ, {a}, {d}, {a, b}, {c, d}, {a, b, c}, {b, c, d}} supra N-open sets in (X,τ) are{X, φ, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {b, c}, {a, c}, {a, b, c}, {a, b, d}, {b, c, d}, {a, c, d}}.Here (X,τ) is mildly supra N-normal but not weakly supra N-normal, since A={a, b} and B={d} is supra N- closed in (X,τ) but A and B is not contained in disjoint supra open sets.

Theorem 3.20 If f:(X, τ) → (Y, σ) be supra N-open map, almost supra N- continuous surjective, and if X is weakly supra N-normal, then Y is mildly supra N-normal.

Proof Let A and B be disjoint regular closed set in Y. Since f is almost supra N -continuous, then f^{-1}(A) and f^{-1}(B) are supra N-closed set in X. Since X is weakly supra N-normal, there exist disjoint supra open set U and V in X such that f^{-1}(A) ⊂ U and f^{-1} (B) ⊂ V. Since f is supra N-closed map, f(U) and f(V) are disjoint supra N-open set in Y. Hence Y is mildly supra N-normal.

4. CONCLUSION

We introduced the concept of almost supra N-continuous function on supra topological space and investigated its relationship with other functions. Also a new type of normal space called mildly supra N-normal space was introduced and studied some of its properties.

REFERENCES

[1]. R.Devi, S.Sampathkumar and M.Caldas, On supra α-open sets and sα-continuous maps, General Mathematics, 16(2)(2008), pp 77-84.

[2]. N.Levine, Semi-open sets and Semi-continuity in topological spaces, Amer.Math., 12(1991), pp 5-13.

[3]. A.S.Mashhour, A.A.Allam, F.S.Mahmoud and F.H.Khedr, On supra topolog- ical spaces, Indian J.Pure and Appl.Math.,14(A)(1983), pp 502-510.

[4]. T.Noiri, J.Bhuvaneswari and A.Rajesh, On totally πg-continuous functions, Fasciculi Mathematici, 2013, Nr 50, pp 101-108.

[5]. T.Noiri and O.R.Sayed, On Ωclosed sets and Ωs closed sets in topological spaces, Acta Math,4(2005), pp 307-318.

[6]. T.Noiri, Indian J. Pure appl. Math. 20(6):571-576, June 1989.

[7]. [7] M.K.Singal and A.R.Singal, Almost continuous mappings, yokohama Math J.16(1968), 63-73.

[8]. L.Vidyarani and M.Vigneshwaran, On supra N-closed and sN-closed sets in supra Topological spaces, International Journal of Mathematical Archieve, Vol-4, Issue-2, 2013, pp 255-259.

[9]. L.Vidyarani and M.Vigneshwaran, contra supra N-continuous functions in supra Topological spaces, Journal of Global Research in Mathematical Archieves, Vol 1, No.9 2013, pp 27-33.

[10]. L.Vidyarani and M.Vigneshwaran, N-Homeomorphism and N*-Homeomorphism in supra topological spaces, International Journal of Mathematics and Statistics Invention, Vol-1, Issue-2, 2013, pp 79-83.

[11]. L.Vidyarani and M.Vigneshwaran, Quasi supra N-closed map and supra N- normal spaces(submitted).