International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 3, Issue 3, March 2015, PP 74-80
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

On θ-Semigeneralized Pre Closed Sets in Topological Spaces

Md. Hanif PAGE
Asst. Professor,
Department of Mathematics,
B.V.B. College of Engg. and Technology,
Hubli-580031, Karnataka, India.
hanif01@yahoo.com

V.T. Hosamath
Asst. Professor,
Department of Mathematics,
K.L.E. Institute of Technology,
Hubli-580030, Karnataka, India.
vthosamath@gmail.com

Abstract: *This paper introduces new class of sets called θ-semigeneralized pre closed set in topological spaces.Basic properties of this new generalized closed sets are analysed.*

2010 Mathematics Subject Classification: *57N505*

Keywords: *θ-sgp-closed, θ-sg-closed, θ-g-closed set, sg-closed set, sgp-closed set, semi-open set, pre-θ-closed set.*

1. INTRODUCTION

General topology is important in many fields of applied sciences as well as in all branches of mathematics. The concept of generalized closed sets introduced by Levine[13] plays important role in general topology. This notion has been extensively studied in recent years by many topologists. Bhattacharyya and Lahiri [2] continued the work of Levine and offered another notion analogous to Levine's g-closed sets called semi-generalized closed set (briefly sg-closed) by replacing the closure operator in Levine's g-closed set by semi-closure operator and by replacing its open super set by semi-open super set. Recently, Dontchev and Maki [9] gave another new generalization of Levin's g-closed set by utilizing θ-closure operator called θ-g-closed set. The concept of θ-g-closed set was applied to the digital line. In 2003, Caldas and Jafari defined θ-semigeneralized closed set using semi-θ-closure operator.

In section three, we introduce a new form of generalized closed set called θ-semigeneralized pre closed set (briefly, θ-sgp-closed set) by utilizing pre-θ-closure operator. We investigate its relation to θ-g-closed sets, θ-sg-closed sets and other generalized closed sets. We have proved that the class of θ-sg-closed sets and the class of θ-sgp-closed sets are independent.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of space X, then Cl(A) and Int(A) denote the closure of A and the interior of A in X respectively.

The following definitions are useful in the sequel.

Definition 2.1: A subset A of space X is called
(i) a semi-open set [12] if $A \subseteq Cl(Int(A))$.

(ii) a semi-closed set [5] if $Int(Cl(A)) \subseteq A$.

(iii) a pre-open set[15] if $A \subseteq Int(Cl(A))$.

(iv) a pre-closed set[15] if $Cl(Int(A)) \subseteq A$.

(v) an α-closed set[16] if $Cl(Int(Cl(A))) \subseteq A$.

(vi) a regular open set[21](resp. a regular closed set[21]) if $A = Int(Cl(A))$(resp. $A = Cl(Int(A))$).

Definition 2.2: A subset A of a topological space X is called

(i) a generalized-closed (briefly g-closed) set[13] if $Cl(A) \subseteq U$ and U is open in X.

(ii) a semi-generalized closed set (briefly sg-closed)[2] if $sCl(A) \subseteq U$ and U is semi-open in X. The complement of a sg-closed set is called a sg-open set.

(iii) a semi-generalized pre closed set (briefly sgp-closed)[17] if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

(iv) a generalized preregular closed set(briefly gpr-closed)[11] if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(v) an α-generalized semi-closed set(briefly αgs-closed)[20] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is semi-open in X.

(vi) a generalized preclosed set(briefly gp-closed)[14] if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(vii) a generalized semi-preclosed set(briefly gsp-closed)[8] if $spCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(viii) θ-generalized closed set(briefly θ-g-closed)[9] if $Cl_\theta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(ix) θ-generalized semi-closed set(briefly θ-gs-closed)[18] if $sCl_\theta(A) \subset U$ whenever $A \subset U$ and U is open in X.

(x) θ-semigeneralized closed set(briefly θ-sg-closed)[4] if $sCl_\theta(A) \subset U$ whenever $A \subset U$ and U is semi-open in X.

Definition 2.3: The semi-closure [5] of a subset A of X is the intersection of all semi-closed sets that contain A and is denoted by sCl(A).

Definition 2.4: The pre-closure [6] of a subset A of X is the intersection of all pre-closed sets that contain A and is denoted by pCl(A).

Definition 2.5: The θ-closure [22] of a set A is denoted by $Cl_\theta(A)$ and is defined by $Cl_\theta(A) = \{x \in X : Cl(U) \cap A \neq \emptyset, U \varepsilon \tau, x \varepsilon U\}$ and a set A is θ-closed if and only if $A = Cl_\theta(A)$.

Definition 2.6: A point $x \in X$ is called a semi-θ-cluster point of A [7] if $sCl(U) \cap A \neq \emptyset$, for each semi-open set U containing x.

Definition 2.7: A point $x \in X$ is called a pre-θ-cluster point of A[19] if $pCl(U) \cap A \neq \emptyset$, for each pre-open set U containing x.

Definition 2.8: The semi-θ-closure [7] denoted by $sCl_\theta(A)$, is the set of all semi-θ-cluster points of A. A subset A is called semi-θ-closed set [7] if $A = sCl_\theta(A)$. The complement of semi-θ-closed set is semi-θ-open set.

Definition 2.9: The pre-θ-closure denoted by $pCl_\theta(A)$, is the set of all pre-θ-cluster points of A. A subset A is called pre-θ-closed set [19] if $A = pCl_\theta(A)$. The complement of pre-θ-closed set is pre-θ-open set.

Definition 2.10: The set $\{x \in X \mid sCl(U) \subset A$ for some $U \varepsilon SO(X, x)\}$ is called the semi-θ-interior of A and is denoted by $sInt_\theta(A)$. A subset A is called semi-θ-open[10] if $A = sInt_\theta(A)$.

Definition 2.11: A topological space X is a pre-θ-R_0 space[1] if every pre-θ-open set contains pre-θ-closure of each of its singletons.

Definition 2.12: Let A be subset of a topological space X. The pre-θ-kernal[1] of $A \subset X$, denoted by $pKer_\theta(A)$, is defined to be the set $\cap\{O : O \in P_\theta O(X, \tau)$ and $A \subset O\}$.

Lemma 2.13[3]: For any subset A of a topological space X, $pCl(A) \subset pCl_\theta(A)$.

3. θ- SEMIGENARALIZED PRE CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of a topological space X is called θ-Semigeneralized pre closed set (briefly, θ-sgp-closed set) if $pCl_\theta(A) \subset U$ whenever $A \subset U$ and U is semi-open in X.

The complement of θ-Semigeneralized pre closed set is called θ-Semigeneralized pre open set (briefly, θ-sgp-open).

Remark 3.2: The concept of θ-sgp-closed sets and closed sets are independent of each other as seen from the following examples.

Example 3.3: Let X = {a, b, c} and τ = {X, Ø, {a, b}}. Then the subset A = {a, c} is θ-sgp-closed set but it is not closed set in X.

Example 3.4: Let X = {a, b, c} and τ = {X, Ø, {a}, {a, b}, {a, c}}. Then the subset A = {b, c} is closed set but it is not θ-sgp-closed set in X.

Theorem 3.5: Every pre-θ-closed set is θ-sgp-closed set but not conversely.

Proof: Let $A \subset U$ be pre-θ-closed. Then $A = pCl_\theta(A)$. Let $A \subset U$ and U is semi-open in X. It follows that $pCl_\theta(A) \subset U$. This means that A is θ-sgp-closed set.

Example 3.6: Let X = {a, b, c} and τ = {X, Ø, {a}, {b}, {a, b}, {a, c}}. Then the subset A = {b, c} is θ-sgp-closed set but it is not pre-θ-closed set in X.

Theorem 3.7: Every θ-sgp-closed set is sgp-closed set but not converse.

Proof: It is true that $pCl(A) \subset pCl_\theta(A)$ for every subset A of X.

Example 3.8: Let X = {a, b, c} and τ = {X, Ø, {a}, {a, b}}. Set A = {b} and U = {a, b}. But $pCl_\theta(A) = X$ which is not a subset of U, where U is semi-open in X. Hence A = {b} is not θ-sgp-closed set. But it is sgp-closed set.

Theorem 3.9: Every θ-sgp-closed set is gp-closed set.

Proof: Let A be an θ-sgp-closed set in a topological space X. Let U be an open set and so it is semi-open such that $A \subseteq U$. Then $pCl(A) \subseteq U$. Hence A is gp-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.10: Let X = {a, b, c} and τ = {X, Ø, {a}, {c}, {a, c}}. Then a subset A = {a, b} is gp-closed set but it is not θ-sgp-closed set.

Theorem 3.11: Every θ-sgp-closed set is gsp-closed set.

Proof: Let A be a θ-sgp-closed set in X. Let $A \subseteq U$, where U is open and so it is semi-open set in X. Then $pCl_\theta(A) \subseteq U$. But $spCl(A) \subseteq pCl(A) \subseteq pCl_\theta(A)$. Therefore $spCl(A) \subseteq U$. Hence A is gsp-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.12: Let X = {a, b, c} and τ = {X, Ø, {a}}. Then a subset A = {a, c} is gsp-closed set but it is not θ-sgp-closed set.

Remark 3.13: The concept of θ-sgp-closed sets and θ-gs-closed sets are independent of each other as seen from the following examples.

Example 3.14: Let X = {a, b, c} and τ = {X, Ø, {a}, {c}, {a, c}}. Then the subset A = {a, b} is θ-gs-closed set but it is not θ-sgp-closed set in X.

Example 3.15: Let X = {a, b, c} and τ = {X, Ø, {b}, {b, c}, {a, c}}. Then the subset A = {c} is θ-sgp-closed set but it is not θ-gs-closed set in X.

Remark 3.16: The notion of θ-sgp-closed sets and α-closed sets are independent of each other as seen from the following examples.

Example 3.17: Let X = {a, b, c} and τ = {X, Ø, {a, b}}. Then the subset A = {a, c} is θ-sgp-closed set but it is not α-closed set in X.

Example 3.18: Let X = {a, b, c} and τ = {X, Ø, {a}, {a, b}, {a, c}}. Then the subset A = {b, c} is α-closed set but it is not θ-sgp-closed set in X.

Remark 3.19: The concept of θ-sgp-closed sets and αgs-closed sets are independent of each other as seen from the following examples.

Example 3.20: Let X = {a, b, c} and τ = {X, Ø, {b}, {b, c}, {a, c}}. Then a subset A = {c} is a θ-sgp-closed set but it is not αgs-closed set.

Example 3.21: Let X = {a, b, c} and τ = {X, Ø, {a}, {c}, {a, c}}. Then a subset A = {a, b} is αgs-closed set but it is not θ-sgp-closed set.

Theorem 3.22: Every θ-g-closed set is θ-sgp-closed set.

Proof: Let A be a θ-g-closed set in X. Let A ⊆ U, where U is open set in X. Then $Cl_\theta(A) \subseteq U$. But $pCl_\theta(A) \subseteq Cl_\theta(A)$. Therefore $pCl_\theta(A) \subseteq U$. Hence A is θ-sgp-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.23: Let X = {a, b, c} and τ = {X, Ø, {b}, {b, c}, {a, c}}. Then a subset A = {c} is θ-sgp-closed set but it is not θ-g-closed set.

Remark 3.24: The notion of θ-sgp-closed sets and θ-sg-closed sets are independent of each other as seen from the following examples.

Example 3.25: Let X = {a, b, c} and τ = {X, Ø, {a}, {c}, {a, c}}. Then a subset A = {a, b} is θ-sg-closed set but it is not θ-sgp-closed set.

Example 3.26: Let X = {a, b, c} and τ = {X, Ø, {b}, {b, c}, {a, c}}. Then a subset A = {c} is θ-sgp-closed set but it is not θ-sg-closed set.

Theorem 3.27: Every θ-sgp-closed set is gpr-closed set.

Proof: Let A be a θ-sgp-closed set in X. Let A ⊆ U, where U is regular-open and so it is semi-open set in X. Then $pCl_\theta \subseteq U$. Hence A is gpr-closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.28: Let X = {a, b, c} and τ = {X, Ø, {a}, {c}, {a, c}}. Then a subset A = {a, b} is a gpr-closed set but it is not θ-sgp-closed set.

Remark 3.29: Union of θ-sgp-closed sets need not be a θ-sgp-closed set as seen from the following example.

Example 3.30: Let X = {a, b, c} and τ = {X, Ø, {a}, {b}, {a, b}, {a, c}, {b, c}}. Then the subsets {a} and {b} are θ-sgp-closed sets but their union {a} ∪ {b} = {a, b} is not a θ-sgp-closed set in X.

Remark 3.31: Intersection of θ-sgp-closed sets need not be a θ-sgp-closed set as seen from the following example.

Example 3.32: Let X = {a, b, c} and τ = {X, Ø, {a}, {b}, {a, b} {a,c}}. Then the subsets {a, b} and {a, c} are are θ-sgp-closed sets but their intersection {a, b} ∩ {a, c} = {a} is not a θ-sgp-closed set in X.

Theorem 3.33: A set $A \subset X$ is θ-sgp-open set if and only if $F \subset pInt_\theta(A)$ whenever F is semi-closed set in X and $F \subset A$.

Proof: Necessity. Let A be θ-sgp-open set and $F \subset A$, where F is semi-closed set. It is obvious that A^c (complement of A) is contained in F^c. This implies that $pCl_\theta(A^c) \subset F^c$. Hence $pCl_\theta(A^c) = (pInt_\theta(A))^c \subset F^c$, i.e. $F \subset pInt_\theta(A)$.

Sufficiency. If F is a semi-closed set with $F \subset pInt_\theta(A)$ whenever $F \subset A$, then it follows that $A^c \subset F^c$ and $(pInt_\theta(A))^c \subset F^c$ i.e. $pCl_\theta(A^c) \subset F^c$. Therefore A^c is θ-sgp-closed set and therefore A is θ-sgp-open set.

Lemma 3.34: Let A be a θ-sgp-closed subset of X. Then,

(i) $pCl_\theta(A)\backslash A$ does not contain a nonempty semi-closed set.

(ii) $pCl_\theta(A)\backslash A$ is θ-sgp-open set.

Proof: (i). Let F be semi-closed set such that $F \subset pCl_\theta(A)\backslash A$. Since F^c is semi-open set and $A \subset F^c$, it follows that $pCl_\theta(A) \subset F^c$, i.e. $F \subset (pCl_\theta(A))^c$. This implies that $F\subset(pCl_\theta(A))^c\cap pCl_\theta(A) = Ø$.

(ii) If A is θ-sgp-closed set and F is a semi-closed set such that $F \subset pCl_\theta(A)\backslash A$, then by (i), F is empty and therefore $F \subset pInt_\theta(pCl_\theta(A)\backslash A)$. By theorem 3.33, $pCl_\theta(A)\backslash A$ is θ-sgp-open set.

Lemma 3.35: For any subset A of a topological space X, $pCl_\theta(A)$ is pre θ-closed set.

Lemma 3.36: If A is a θ-sgp-closed set of a topological space X such that $A \subset B \subset pCl_\theta(A)$ then B is also a θ-sgp-closed set of X.

Proof: Let O be a semi-open set of X such that $B \subset O$. Then $A \subset O$. Since A is θ-sgp-closed set, $pCl_\theta(A) \subset O$. By using Lemma 3.35, $pCl_\theta(B) \subset pCl_\theta(pCl_\theta(A)) = pCl_\theta(A) \subset O$. Therefore B is also a θ-sgp-closed set of X.

Lemma 3.37: Let X be a topological space and $x \in X$. The following two statements are equivalent:

(i) $y \in pKer_\theta(\{x\})$;

(ii) $x \in pCl_\theta(\{y\})$.

Proof: Let $y \notin pKer_\theta(\{x\})$. It follows that there exists a semi θ-open set U containing x such that $y \notin U$. This means that $x \notin pCl_\theta(\{y\})$. The converse can be proved by the same taken.

Lemma 3.38: The following statements are equivalent for any points x and y in a topological space X: (i) $pKer_\theta(\{x\}) \neq pKer_\theta(\{y\})$;

(ii) $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$.

Proof: (i)→(ii): Let $pKer_\theta(\{x\}) \neq pKer_\theta(\{y\})$. Then there exists a point z in X such that $z \in pKer_\theta(\{x\})$ and $z \notin pKer_\theta(\{y\})$. By $z \in pKer_\theta(\{x\})$, it follows that $\{x\} \cap pCl_\theta(\{z\}) \neq Ø$. This implies $x \in pCl_\theta(\{z\})$. By $z \notin pKer_\theta(\{y\})$, we obtain $\{y\} \cap pCl_\theta(\{z\}) = Ø$. Since $x \in pCl_\theta(\{z\})$, $pCl_\theta(\{x\}) \subset pCl_\theta(\{z\})$ and $\{y\} \cap pCl_\theta(\{x\}) = Ø$. Hence it follows that $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$. Now $pKer_\theta(\{x\}) \neq pKer_\theta(\{y\})$ implies that $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$.

(ii)→(i): Let $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$. Then there exists a point z in X such that $z \in pCl_\theta(\{x\})$ and $z \notin pCl_\theta(\{y\})$. This means that there exists a pre-θ-open set containing z and therefore x but not y, i.e., $y \notin pKer_\theta(\{x\})$. Hence $pKer_\theta(\{x\}) \neq pKer_\theta(\{y\})$.

Theorem 3.39: A topological space X is a pre-θ-R_0 space if and only if for x and y in X, $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$ implies $pCl_\theta(\{x\}) \cap pCl_\theta(\{y\}) = Ø$.

Proof: Suppose that X is pre-θ-R_0 and $x, y \in X$ such that $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$. Then, there exist $z \in pCl_\theta(\{x\})$ such that $z \notin pCl_\theta(\{y\})$ (or $z \in pCl_\theta(\{y\})$ such that $z \notin pCl_\theta(\{x\})$). There exists $V \in SO(X, \tau)$ such that $y \notin V$ and $z \in V$; hence $x \in V$. Therefore, we have $x \notin pCl_\theta(\{y\})$. Thus $x \in X \setminus pCl_\theta(\{y\})$, which implies $pCl_\theta(\{x\}) \subset X \setminus pCl_\theta(\{y\})$ and $pCl_\theta(\{x\}) \cap pCl_\theta(\{y\}) = Ø$. The proof for otherwise is similar.

Sufficiency. Let V be pre-θ-open set and let $x \in V$. We will show that $pCl_\theta(\{x\}) \subset V$. Let $y \notin V$, i.e., $y \in X \setminus V$. Then $x \neq y$ and $x \notin pCl_\theta(\{y\})$. This shows that $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$. By assumption, $pCl_\theta(\{x\}) \cap pCl_\theta(\{y\}) = Ø$. Hence $y \notin pCl_\theta(\{x\})$. Therefore $pCl_\theta(\{x\}) \subset V$.

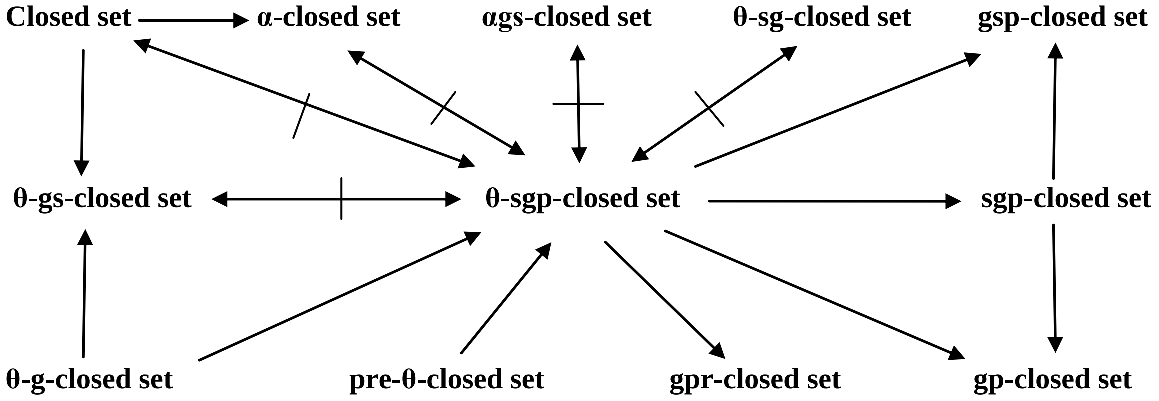
Theorem 3.40: A topological space X is a pre-θ-R_0 space if and only if for any points x and y in X, $pKer_\theta(\{x\}) \neq pKer_\theta(\{y\})$ implies $pKer_\theta(\{x\}) \cap pKer_\theta(\{y\}) = Ø$.

Proof: Suppose that X is pre-θ-R_0 space. Thus by Lemma 3.38, for any points x and y in X if $pKer_\theta(\{x\}) \neq pKer_\theta(\{y\})$ then $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$. Now we prove that $pKer_\theta(\{x\}) \cap pKer_\theta(\{y\}) = Ø$. Asuume that $z \in pKer_\theta(\{x\}) \cap pKer_\theta(\{y\})$. By $z \in pKer_\theta(\{x\})$ and Lemma 3.37, it follows that $x \in pCl_\theta(\{z\})$. Since $x \in pCl_\theta(\{x\})$, by Theorem 3.39, $pCl_\theta(\{x\}) = pCl_\theta(\{z\})$.

Similarly, we have $pCl_\theta(\{y\}) = pCl_\theta(\{z\}) = pCl_\theta(\{x\})$. This is a contradiction. Therefore, we have $pKer_\theta(\{x\}) \cap pKer_\theta(\{y\}) = \emptyset$.

Conversely, let X be a topological space such that for any points x and y in X, $pKer_\theta(\{x\}) \neq pKer_\theta(\{y\})$ implies $pKer_\theta(\{x\}) \cap pKer_\theta(\{y\}) = \emptyset$. If $pCl_\theta(\{x\}) \neq pCl_\theta(\{y\})$, then by Lemma 3.38, $pKer_\theta(\{x\}) \neq pKer_\theta(\{y\})$. Because $z \in pCl_\theta(\{x\})$ implies that $x \in pKer_\theta(\{z\})$ and therefore $pKer_\theta(\{x\}) \cap pKer_\theta(\{z\}) \neq \emptyset$. By hypothesis, we therefore have $pKer_\theta(\{x\}) = pKer_\theta(\{z\})$. Then $z \in pCl_\theta(\{x\}) \cap pCl_\theta(\{y\})$ implies that $pCl_\theta(\{x\}) = pCl_\theta(\{z\}) = pCl_\theta(\{y\})$. This is a contradiction. Hence, $pCl_\theta(\{x\}) \cap pCl_\theta(\{y\}) = \emptyset$ and by Theorem 3.39, X is a pre-θ-R_0 space.

3.41 Remark: The **“Implication Diagram”** about θ-sgp-closed set.



where $A \longrightarrow B$ (resp. $A \nleftrightarrow B$) represents A implies B but not conversely (resp.A and B are independent).

4. CONCLUSION

In the class of θ-sgp-closed sets defined using semi-open sets lies between the class of θ-g-closed sets and the class of sgp-closed set. The θ-sgp-closed set can be used to derive a new decomposition of continuity and new separation axioms. This concept can be extended to bitopological and fuzzy topological spaces.

REFERENCES

[1]. A. A. El-Atik, Some more results on pre-θ-open sets, Antarctica J. Math. 2(1)(2005), 111-121.

[2]. P.Bhattacharyya and B.K.Lahiri, Semi-generalized closed sets in topology, Indian J. Math. 29(3)(1987), 375-382.

[3]. Miguel Caldas, Saeid jafari and Takashi Noiri, Characterizations of Pre-R_0 and Pre-R_1 topological spaces, Topological proceedings, 25(2000), 17-30.

[4]. Miguel Caldas and Saeid Jafari, On θ-semigeneralized closed sets in topology, Kyungpook Math J. 43(2003), 135-148.

[5]. S.G.Crossely and S.K.Hildbrand, On semi-closure, Texas J. Sci, 22(1971), 99-112.

[6]. N.El-Deeb, I.A.Hasanein, A.S.Mashhour and T.Noiri, On p-regular spaces, Bull. Math. Soc. Sci. Math. R.S.Roumanie, 22(75)(1983), 311-315.

[7]. G. Di Maio, T.Noiri, On s-closed spaces, Indian J. Pure Appl. Math. 18(1987), 226-233.

[8]. J. Dontchev, On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 16(1995), 35-48.

[9]. J. Dontchev and H. Maki, On θ-generalized closed sets, Internet. J. Math. & Math. Sci., 22(1999), 239-249S.

[10]. Ganguly and C.K.Basu, Further characterizations of s-closed spaces, Indian J. Pure Appl. Math., 23(9)(1992), 635-641.Y.

[11]. Gnanambal, On generalized preregular closed sets in topological spaces, Indian J. Pure. Appl. Math., 28(3)(1997), 351-360.

[12]. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70(1963), 36-41.

[13]. N.Levine, Generalized closed sets in topology, Rend. Circ. Math. Paleomo 19(1970), 89-96.

[14]. H.Maki, J.Umehara and T.Noiri, Every topological space is pre-$T_{1/2}$, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 17(1996), 33-42.

[15]. A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, On pre-continuous and weak pre-continuous mapping, Proc. Math. Phys. Soc. Egypt 53(1982), 47-53.

[16]. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb, α-continuous and α-open mappings, Acta Math. Hung., 41(3-4)(1983), 213-218.

[17]. Govindappa Navalagi and Mahesh Bhat, sgp-closed sets in topological spaces, Jour. Appl. Math. Ana. Appl.. Vol.3 Nr.1(2007), 45-58.

[18]. Govindappa Navalagi and Md.Hanif Page, On θgs-Neighbourhoods, Indian Journal of Mathematics and Mathematical Sciences, Vol.4, No.1, (June2008), 21-31.

[19]. M.C.Pal and P.Bhattacharyya, Feeble and strong forms of preirresolute functions, Bull. Malaysian Math. Soc., 19 (1996), 63-75.

[20]. Rajamani M, K.Viswanathan, On αgs-closed sets in topological spaces, Acta Ciencia Indica, XXXM(3)(2004), 21-25.

[21]. M.H.Stone, Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41(1937), 375-381.

[22]. N.V.Velicko, On H-closed topological spaces, Amer. Math. Soc. Transl., 78(1968), 103-118.