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Ecological Harvested Ammensal Model- A Homotopy Analysis

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Abstract: *The paper explicates to derive a series solution in a special case of ecological Ammensalism. In this model the harvested Enemy species is considered. Unlimited resources are given to Ammensal species. This model is formed by a couple of first order nonlinear differential equations. The series solution is obtained by utilizing homotopy analysis. In addition to this numerical solutions analyze various possibilities of interactions between Ammensal and Enemy species*

Keywords: *Ammensalism, Homotopy Analysis, Stability, Dominance Reversal time*.

1. Introduction

Homotopy Analysis method mainly aims to supply a convergent series solution for nonlinear differential equations.The special features of HAM are flexibility on choice of base function and initial guess of solutions.It has been employed in many fields of science and technology. Homotopy analysis method involves Taylor's series which helps to represent the functions as a power series. In the concept of homotopy,it helps to provide a connection between different systems. HAM was developed by Liao[4-7].HAM consists of Euler transforms which a can provide convergent series solution.Later few mathematicians like Liao. Abhashandy [1], Hilton,P.J [2], Liao Shijun [3] discussed the applications of HAM for enhancing computational efficiency to derive multiple solutions of nonlinear problems. HAM is defined to be independent of small and large physical parameters.

HAM method is atmost valid procedure for getting a guaranteed convergence of solution series. It has a large freedom to consider any type of equations which divides into many possible linear sub problems.

2. Basic Idea of Homotopy Analysis Method

Step (1): Let us consider nonlinear differential equation:

$$A\ u - f\ r = 0,\ r \in \Omega \qquad \text{(I)}$$

With the the boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad r \in \Gamma$$

where A is a general differential operator, B a boundary operator, $f\ r$ is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω.

Step (2): In general the operator A, is divided into two parts: a linear part L and a nonlinear part N. Therefore above differential equation(I) is expressed in the form of

$$L(u) - N(u) - f(r) = 0 \qquad \text{(II)}$$

Step (3):

With the help of homotopy Analysis, one can constitute a homotopy $v(r,p) : \Omega \times [0,1] \to \mathbb{R}$ which satisfies

$$H(v,p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[A(v) - f(r)\right] = 0,\ p \in [0,1], r \in \Omega \qquad \text{(III)}$$

It is nothing but

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p\left[N(v) - f(r)\right] = 0 \qquad \text{(IV)}$$

where $p \in [0,1]$ is named as an embedding parameter ,and u_0 is an initial approximation of equation(1),which satisfies the boundary conditions.

Step (4): Then equations (III), (IV) follow that

$$H(v,0) = L(v) - L(u_0) = 0$$

and $H(v,1) = A(v) - f(r) = 0$

Thus the changing process of p from zero to unity is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$.

Step (5): According to the HPM, we can first use the imbedding parameter p as a 'small parameter' and assume that the solutions of the equations (III) and (IV) can be written as a power series in p :

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + p^4v_4 + \text{----------}$$

The approximate solution of equation (I) can be obtained as

$$u = \underset{p\to 1}{Lt}\, v = v_0 + v_1 + v_2 + v_3 + v_4 + \text{-----------}$$

3. NOTATIONS ADOPTED

N_1 (t) : The population rate of the species S_1 at time t

N_2 (t) :The population rate of the species S_2 at time t

a_i : The natural growth rate of S_i, i = 1, 2.

a_{ii} : The rate of decrease of S_i; due to its own insufficient resources ,i=1,2.

a_{12} :The inhibition coefficient of S_1 due to S_2 i.e The Ammensal coefficient.

$H_2(t)$:The replenishment or renewal of S_2 per unit time

h_2 :a_{22} H_2 is rate of harvest of the enemy.

The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α, h_1, h_2 are assumed to be non-negative constants.

4. THE BASIC MODEL EQUATIONS

$$\frac{dN1}{dt} = a_1N_1 - a_{12}N_1N_2 \qquad (1)$$

$$\frac{dN2}{dt} = a_2N_2 - a_{22}N_2^2 - a_{22}H_2 \quad \text{With } N_1(0) = c_1,\ N_2(0) = c_2 \qquad (2)$$

with the conditions $N_i(0) = N_{i0} \geq 0, i = 1,2$;

A homotopy system of (1) & (2) can be constructed as

$$v_1^1 - N_{1,0}^1 + p\,(N_{1,0}^1 - a_1 v_1 + a_{12}v_1v_2) = 0 \qquad (3)$$

$$v_2^1 - N_{2,0}^1 + P\,(N_{2,0}^1 - a_2 N_2 + a_{22} N_2^2 + a_{22}H_2) = 0 \qquad (4)$$

the initial approximations are assumed as

$v_{1,0}(t) = N_{1,0}(t) = v_1(0) = c_1$ (5)

$v_{2,0}(t) = N_{2,0}(t) = v_2(0) = c_2$ (6)

and $v_1(t) = v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + \ldots$ (7)

$v_2(t) = v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + \ldots$ (8)

here $v_{i,j}$ (i=1,2, j= 1,2,3,…) which are to be determined by substituting (5),(6),(7),(8) in (3) & (4),

After doing this, we obtain

$v_{1,0}^1(t) + pv_{1,1}^1(t) + p^2v_{1,2}^1(t) + p^3v_{1,3}^1(t) + p^4v_{1,4}^1(t) + \ldots - N_{1,0}^1(t)$

$+ p[N_{1,0}^1 - a_1(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + \ldots) + a_{12}(v_{1,0}(t) + pv_{1,1}(t) + p^2v_{1,2}(t) + p^3v_{1,3}(t) + p^4v_{1,4}(t) + \ldots)(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + \ldots)] = 0$ (9)

$v_{2,0}^1(t) + pv_{2,1}^1(t) + p^2v_{2,2}^1(t) + p^3v_{2,3}^1(t) + p^4v_{2,4}^1(t) + \ldots - N_{2,0}^1(t)$

$+ p[N_{2,0}^1(t) - a_2(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + \ldots) + a_{22}(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + \ldots)(v_{2,0}(t) + pv_{2,1}(t) + p^2v_{2,2}(t) + p^3v_{2,3}(t) + p^4v_{2,4}(t) + \ldots) + a_{22}H_2] = 0$ (10)

From equation (9)

$0 + pv_{1,1}^1(t) + p^2v_{1,2}^1(t) + p^3v_{1,3}^1(t) + p^4v_{1,4}^1(t) + \ldots - 0$

$+ p[0 - a_1v_{1,0}(t) - a_1pv_{1,1}(t) - a_1p^2v_{1,2}(t) - a_1 p^3v_{1,3}(t) - a_1 p^4v_{1,4}(t) + \ldots$

$+ a_{12} v_{1,0}(t)v_{2,0}(t) + p\, a_{12} v_{1,0}(t)v_{2,1}(t) + p^2 a_{12} v_{1,0}(t)v_{2,2}(t) + p^3 a_{12} v_{1,0}(t)v_{2,3}(t) + p^4 a_{12} v_{1,0}(t)v_{2,4}(t) + \ldots + pa_{12}v_{1,1}(t) v_{2,0}(t) + p^2 a_{12}v_{1,1}(t)v_{2,1}(t) + p^3 a_{12}v_{1,1}(t)v_{2,2}(t) + p^4 a_{12}v_{1,1}(t)v_{2,3}(t) + p^5 a_{12}v_{1,1}(t)v_{2,4}(t) + \ldots$

$+ p^2 a_{12}v_{1,2}(t) v_{2,0}(t) + p^3 a_{12}v_{1,2}(t)v_{2,1}(t) + p^4 a_{12}v_{1,2}(t)v_{2,2}(t) + p^5 a_{12}v_{1,2}(t)v_{2,3}(t) + p^6 a_{12}v_{1,2}(t)v_{2,4}(t) + \ldots + p^3 a_{12}v_{1,3}(t) v_{2,0}(t) + p^4 a_{12}v_{1,3}(t)v_{2,1}(t) + p^5 a_{12}v_{1,3}(t)v_{2,2}(t) + p^6 a_{12}v_{1,3}(t)v_{2,3}(t) + p^7 a_{12}v_{1,3}(t)v_{2,4}(t) + \ldots + p^4 a_{12}v_{1,4}(t) v_{2,0}(t) + p^5 a_{12}v_{1,4}(t)v_{2,1}(t) + p^6 a_{12}v_{1,4}(t)v_{2,2}(t) + p^7 a_{12}v_{1,4}(t)v_{2,3}(t) + p^8 a_{12}v_{1,4}(t)v_{2,4}(t) + \ldots] = 0$ (11)

From equation (10)

$0 + pv_{2,1}^1(t) + p^2v_{2,2}^1(t) + p^3v_{2,3}^1(t) + p^4v_{2,4}^1(t) + \ldots - 0$

$P\,[0 - a_2 v_{2,0}(t) - p\, a_2v_{2,1}(t) - p^2 a_2v_{2,2}(t) - p^3 a_2v_{2,3}(t) - p^4 a_2v_{2,4}(t) - \ldots$

$+ a_{22} v_{2,0}^2(t) + pa_{22} v_{2,0}(t) v_{2,1}(t) + p^2a_{22}v_{2,0}(t) v_{2,2}(t) + p^3 a_{22}v_{2,0}(t) v_{2,3}(t) + p^4 a_{22}v_{2,0}(t) v_{2,4}(t) + \ldots$

$+ pa_{22} v_{2,0}(t)v_{2,1}(t) + p^2 a_{22} v_{2,1}^2(t) + p^3a_{22}v_{2,1}(t) v_{2,2}(t) + p^4 a_{22}v_{2,1}(t) v_{2,3}(t) + p^5 a_{22}v_{2,1}(t) v_{2,4}(t) + \ldots$

$+ p^2 a_{22} v_{2,0}(t)v_{2,2}(t) + p^3 a_{22} v_{2,1}(t)v_{2,2}(t) + p^4a_{22}v_{2,2}^2(t) + p^5 a_{22}v_{2,2}(t) v_{2,3}(t) + p^6 a_{22}v_{2,2}(t) v_{2,4}(t) + \ldots$

$+ p^3 a_{22} v_{2,0}(t)v_{2,3}(t) + p^4 a_{22} v_{2,1}(t)v_{2,3}(t) + p^5a_{22}v_{2,3}(t)v_{2,2}(t) + p^6 a_{22} v_{2,3}^2(t) + p^7 a_{22}v_{2,3}(t) v_{2,4}(t) + \ldots$

$+ p^4 a_{22} v_{2,0}(t)v_{2,4}(t) + p^5 a_{22} v_{2,1}(t)v_{2,4}(t) + p^6a_{22}v_{2,4}(t)v_{2,2}(t) + p^7 a_{22}v_{2,4}(t) v_{2,3}(t) + p^8 a_{22} v_{2,4}^2(t)$

$+ \ldots + a_{22}H_2] = 0$ (12)

By comparing the coefficients of various powers of p in equation (11) & (12),

Coefficient of p^1:-

$v_{1,1}^1(t) - a_1v_{1,0}(t) + a_{12}v_{1,0}(t)v_{2,0}(t) = 0$

$v_{2,1}^1(t) - a_2v_{2,0}(t) + a_{22}v_{2,0}^2(t) + a_{22}H_2 = 0$

Coefficient of p^2:-

$v_{1,2}{}^{1}(t) - a_1 v_{1,1}(t) + a_{12} v_{1,0}(t) v_{2,1}(t) + a_{12} v_{1,1}(t) v_{2,0}(t) = 0$

$v_{2,2}{}^{1}(t) - a_2 v_{2,1}(t) + a_{22} v_{2,0}(t) v_{2,1}(t) + a_{22} v_{2,0}(t) v_{2,1}(t) = 0$

Coefficient of p^3:-

$v_{1,3}{}^{1}(t) - a_1 v_{1,2}(t) + a_{12} v_{1,0}(t) v_{2,2}(t) + a_{12} v_{1,1}(t) v_{2,1}(t) + a_{12} v_{1,2}(t) v_{2,0}(t) = 0$

$v_{2,3}{}^{1}(t) - a_2 v_{2,2}(t) + a_{22} v_{2,0}(t) v_{2,2}(t) + a_{22} v_{2,1}{}^{2}(t) + a_{22} v_{2,2}(t) v_{2,0}(t) = 0$

Coefficient of p^4:-

$v_{1,4}{}^{1}(t) - a_1 v_{1,3}(t) + a_{12} v_{1,0}(t) v_{2,3}(t) + a_{12} v_{1,1}(t) v_{2,2}(t) + a_{12} v_{1,2}(t) v_{2,1}(t) + a_{12} v_{1,3}(t) v_{2,0}(t) = 0$

$v_{2,4}{}^{1}(t) - a_2 v_{2,3}(t) + a_{22} v_{2,0}(t) v_{2,3}(t) + 2a_{22} v_{2,1}(t) v_{2,2}(t) + a_{22} v_{2,3}(t) v_{2,0}(t) = 0$

Now $v_{1,0}(t) = c_1$, $v_{2,0}(t) = c_2$

$v_{1,1}(t) = a_1 \int_0^t v_{1,0}(t)\, dt - a_{12} \int_0^t v_{1,0}(t) v_{2,0}(t)\, dt$

$= (a_1 c - a_{12} c_1 c_2) t$

$\therefore$ $v_{1,1}(t) = (a_1 - a_{12} c_2)\, c_1 t$

$v_{2,1}(t) = a_2 \int_0^t v_{2,0}(t)\, dt - a_{22} \int_0^t v_{2,0}{}^{2}(t)\, dt - a_{22} H_2 \int_0^t 1\, dt$

$= (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2)\, t$

$\therefore$ $v_{2,1}(t) = (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2)\, t$

$v_{1,2}(t) = a_1 \int_0^t v_{1,1}(t)\, dt - a_{12} \int_0^t v_{1,0}(t) v_{2,1}(t)\, dt - a_{12} \int_0^t v_{1,1}(t) v_{2,0}(t)\, dt$

$= (a_1 - a_{12} c_2)(a_1 - a_{12} c_2)\, c_1 \frac{t^2}{2} - a_{12} c_1 (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2) \frac{t^2}{2}$

$\therefore$ $v_{1,2}(t) = [c_1 (a_1 - a_{12} c_2)^2 - a_{12} c_1 (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2)] \frac{t^2}{2}$

$v_{2,2}(t) = a_2 \int_0^t v_{2,1}(t)\, dt - 2a_{22} \int_0^t v_{2,0}(t) v_{2,1}(t)\, dt$

$= a_2 (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2) \frac{t^2}{2} - 2a_{22} c_2 (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2) \frac{t^2}{2}$

$\therefore$ $v_{2,2}(t) = (a_2 - 2a_{22} c_2)(a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2) \frac{t^2}{2}$

$v_{1,3}(t) = a_1 \int_0^t v_{1,2}(t)\, dt - a_{12} \int_0^t v_{1,0}(t) v_{2,2}(t)\, dt - a_{12} \int_0^t v_{1,1}(t) v_{2,1}(t)\, dt$

$- a_{12} \int_0^t v_{1,2}(t) v_{2,0}(t)\, dt$

$= (a_1 - a_{12} c_2)\, [c_1 (a_1 - a_{12} c_2)^2 - a_{12} c_1 (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2)] \frac{t^3}{6}$

$- a_{12} c_1 (a_2 - a_{22} c_2{}^{2} - a_{22} H_2)(a_2 - 2a_{22} c_2) \frac{t^3}{6}$

$- a_{12} c_1 (a_1 - a_{12} c_2)(a_2 - a_{22} c_2{}^{2} - a_{22} H_2) \frac{t^3}{3}$

$\therefore$ $v_{1,3}(t) = [(a_1 - a_{12} c_2)\, [c_1 (a_1 - a_{12} c_2)^2 - a_{12} c_1 (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2)]$

$- a_{12} c_1 (a_2 - a_{22} c_2{}^{2} - a_{22} H_2)\, [(a_2 - 2a_{22} c_2) + 2(a_1 - a_{12} c_2)]] \frac{t^3}{6}$

$v_{2,3}(t) = a_2 \int_0^t v_{2,2}(t)\, dt - 2a_{22} \int_0^t v_{2,0}(t) v_{2,2}(t)\, dt - a_{22} \int_0^t v_{2,1}{}^{2}(t)\, dt$

$= (a_2 - 2a_{22} c_2)^2 (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2) \frac{t^3}{6} - a_{22} (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2) \frac{t^3}{3}$

$\therefore$ $v_{2,3}(t) = (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2)\, [(a_2 - 2a_{22} c_2)^2 - 2\, a_{22} (a_2 c_2 - a_{22} c_2{}^{2} - a_{22} H_2)] \frac{t^3}{6}$

$v_{1,4}(t) = a_1\int_0^t v_{1,3}(t)\,dt - a_{12}\int_0^t v_{1,0}(t)v_{2,3}(t)\,dt - a_{12}\int_0^t v_{1,1}(t)v_{2,2}(t)\,dt$

$\quad - a_{12}\int_0^t v_{1,2}(t)v_{2,1}(t)\,dt - a_{12}\int_0^t v_{1,3}(t)v_{2,0}(t)\,dt$

$\quad = (a_1-a_{12}c_2)\,[(a_1-a_{12}c_2)\,[c_1(a_1-a_{12}c_2)^2 - a_{12}c_1(a_2c_2-a_{22}c_2^2-a_{22}H_2)]$

$\quad - a_{12}c_1(a_2c_2-a_{22}c_2^2-a_{22}H_2)\,[(a_2-2a_{22}c_2) + 2(a_1-a_{12}c_2)]]\,\frac{t^4}{24}$

$\quad - a_{12}c_1(a_2c_2-a_{22}c_2^2-a_{22}H_2)\,[(a_2-2a_{22}c_2)^2 - 2a_{22}(a_2c_2-a_{22}c_2^2-a_{22}H_2)]\,\frac{t^4}{24}$

$\quad - a_{12}c_1(a_1-a_{12}c_2)(a_2-2a_{22}c_2)(a_2c_2-a_{22}c_2^2-a_{22}H_2)\,\frac{t^4}{8}$

$\quad - a_{12}\,[c_1(a_1-a_{12}c_2)^2 - a_{12}c_1(a_2c_2-a_{22}c_2^2-a_{22}H_2)]\,(a_2c_2-a_{22}c_2^2-a_{22}H_2)\,\frac{t^4}{8}$

$\therefore\ v_{1,4}(t) = \{c_1[(a_1-a_{12}c_2)^2 - a_{12}c_1(a_2c_2-a_{22}c_2^2-a_{22}H_2)]$

$\quad [(a_1-a_{12}c_2)^2 + 3a_{12}(a_2c_2-a_{22}c_2^2-a_{22}H_2)]$

$\quad - (a_2c_2-a_{22}c_2^2-a_{22}H_2)[a_{12}c_2[(a_2-2a_{22}c_2) + 2(a_1-a_{12}c_2) + (a_2-2a_{22}c_2)^2$

$\quad - 2a_{22}(a_2c_2-a_{22}c_2^2-a_{22}H_2) + 3(a_1-a_{12}c_2)(a_2-2a_{22}c_2)]]\}\,\frac{t^4}{24}$

$v_{2,4}(t) = a_2\int_0^t v_{2,3}(t)\,dt - a_{22}c_2\int_0^t v_{2,3}(t)\,dt - 2a_{22}\int_0^t v_{2,1}(t)v_{2,2}(t)\,dt$

$\quad = (a_2-2a_{22}c_2)(a_2c_2-a_{22}c_2^2-a_{22}H_2)\,[(a_2-2a_{22}c_2)^2 - 2a_{22}c_2(a_2c_2-a_{22}c_2^2-a_{22}H_2)]\,\frac{t^4}{24}$

$\quad -2a_{22}(a_2-2a_{22}c_2)(a_2c_2-a_{22}c_2^2-a_{22}H_2)^2\,\frac{t^4}{8}$

$\therefore\ v_{2,4}(t) = (a_2-2a_{22}c_2)(a_2c_2-a_{22}c_2^2-a_{22}H_2)[(a_2-2a_{22}c_2)^2 - 8a_2(a_2c_2-a_{22}c_2^2-a_{22}H_2)]\,\frac{t^4}{24}$

When approximate terms are considered up to four terms, we get

$$N_1(t) = \lim_{p\to 1} v_1(t) = \sum_{x=0}^{4} v_{1,x}(t) = v_{1,0}(t) + v_{1,1}(t) + v_{1,2}(t) + v_{1,3}(t) + v_{1,4}(t)$$

$$N_1(t) = \lim_{p\to 1} v_2(t) = \sum_{x=0}^{4} v_{2,x}(t) = v_{2,0}(t) + v_{2,1}(t) + v_{2,2}(t) + v_{2,3}(t) + v_{2,4}(t)$$

The solutions are obtained by Homotopy analysis as follows,

$\therefore\ N_1(t) = c_1 + (a_1-a_{12}c_2)c_1t + [c_1(a_1-a_{12}c_2)^2 - a_{12}c_1(a_2c_2-a_{22}c_2^2-a_{22}H_2)]\,\frac{t^2}{2}$

$\quad + [(a_1-a_{12}c_2)\,[c_1(a_1-a_{12}c_2)^2 - a_{12}c_1(a_2c_2-a_{22}c_2^2-a_{22}H_2)]$

$\quad - a_{12}c_1(a_2-a_{22}c_2^2-a_{22}H_2)\,[(a_2-2a_{22}c_2) + 2(a_1 - a_{12}c_2)]]\,\frac{t^3}{6}$

$\quad + \{c_1[(a_1-a_{12}c_2)^2 - a_{12}c_1(a_2c_2-a_{22}c_2^2-a_{22}H_2)]$

$\quad [(a_1-a_{12}c_2)^2 + 3a_{12}(a_2c_2-a_{22}c_2^2-a_{22}H_2)]$

$\quad - (a_2c_2-a_{22}c_2^2-a_{22}H_2)[a_{12}c_2[(a_2-2a_{22}c_2) + 2(a_1-a_{12}c_2) + (a_2-2a_{22}c_2)^2$

$\quad - 2a_{22}(a_2c_2-a_{22}c_2^2-a_{22}H_2) + 3(a_1-a_{12}c_2)(a_2-2a_{22}c_2)]]\}\,\frac{t^4}{24}$

$\therefore\ N_2(t) = c_2 + (a_2c_2-a_{22}c_2^2-a_{22}H_2)\,t + (a_2-2a_{22}c_2)(a_2c_2-a_{22}c_2^2-a_{22}H_2)\,\frac{t^2}{2}$

$\quad + (a_2c_2-a_{22}c_2^2-a_{22}H_2)\,[(a_2-2a_{22}c_2)^2 - 2a_{22}(a_2c_2-a_{22}c_2^2-a_{22}H_2)]\,\frac{t^3}{6}$

$\quad + (a_2-2a_{22}c_2)(a_2c_2-a_{22}c_2^2-a_{22}H_2)[(a_2-2a_{22}c_2)^2 - 8a_2(a_2c_2-a_{22}c_2^2-a_{22}H_2)]\,\frac{t^4}{24}$

5. NUMERICLA ILLUSTATIONS

The nature of the ecological model is to be identified with a set of numerical solutions which can be illustrated in a course of specified time interval.

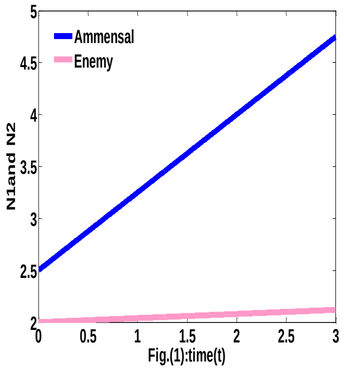
The fixed parameters are considered as $\mathbf{a_{12}=0.6, a_2=2.9, a_{22}= 0.4, H_2=0.9, c_1=2.5}$ **and** $\mathbf{c_2=2}$

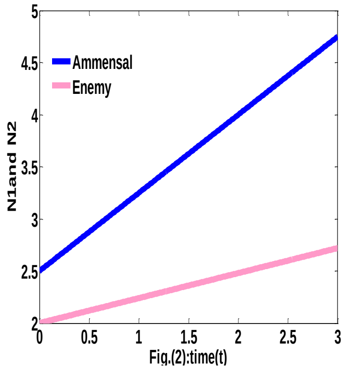
The varying variable is $\mathbf{a_1}$, **i.e** $\mathbf{a_1}$**= from 1.5 to 3.5**

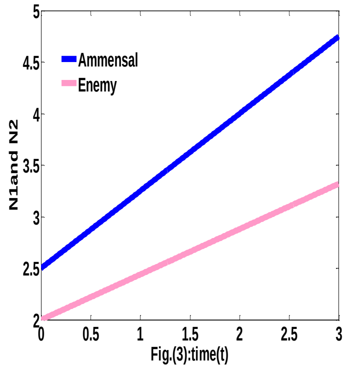
and then t* is derived(1.75,1.02,0.73,0.58,0.48,0.41,0.35,0.31,0.28,0.25,0.23,0.22,0.19,0.17,0.16)

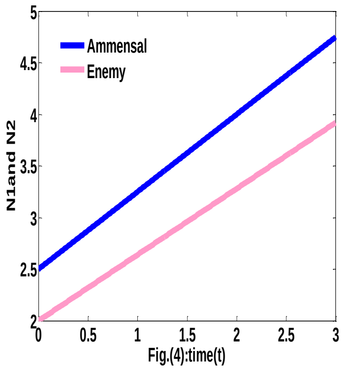
The obtained solutions are illustrated from Fig. (1) to Fig.(21).

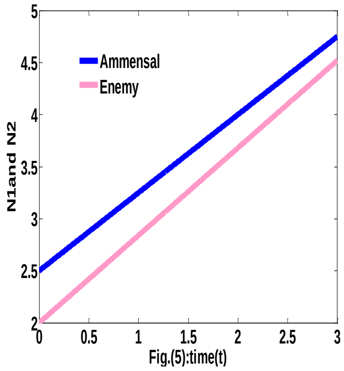
***Case(1):*In the case where natural growth rate of Ammensal Species is less than the growth rate of enemy species($\mathbf{a_1<a_2}$)-From Fig(1) to Fig(14)**

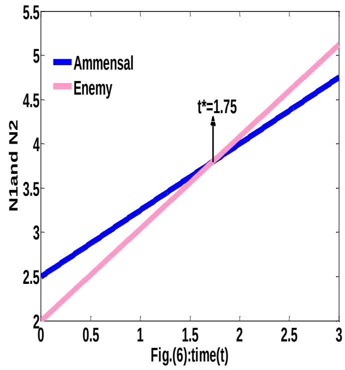


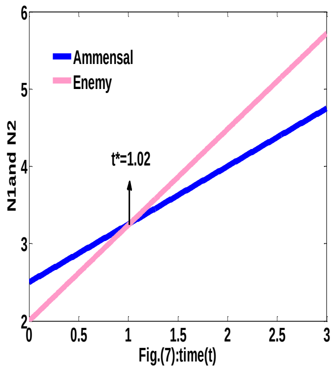


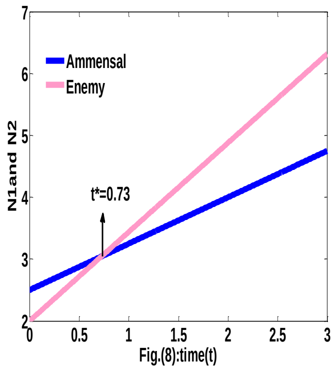


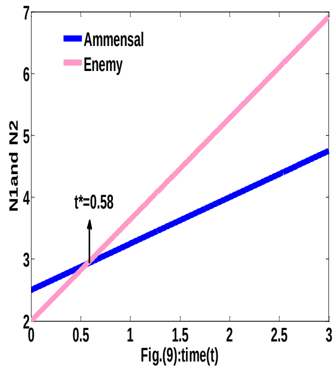


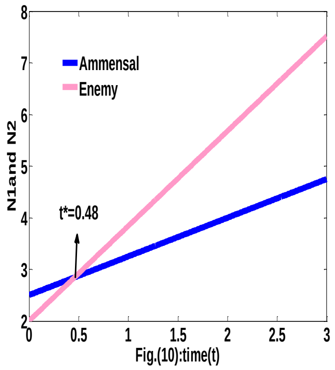


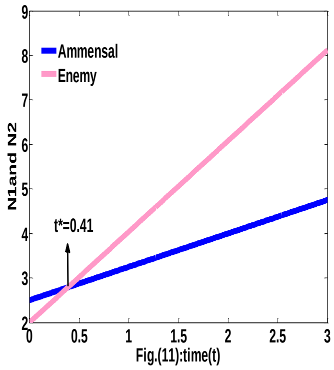


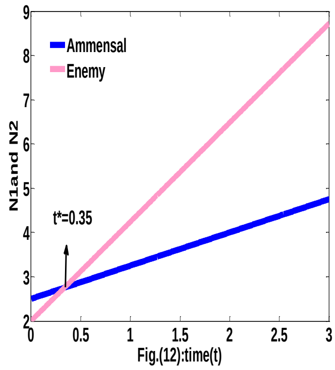


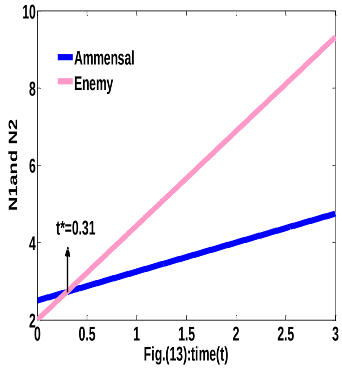


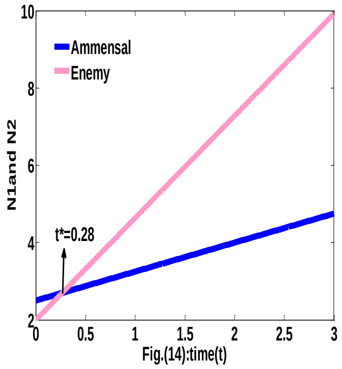


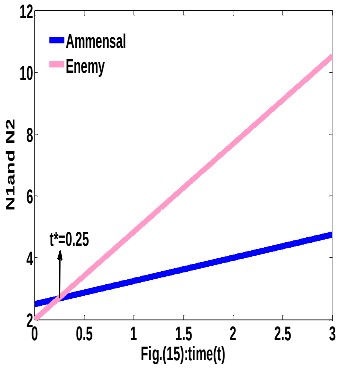




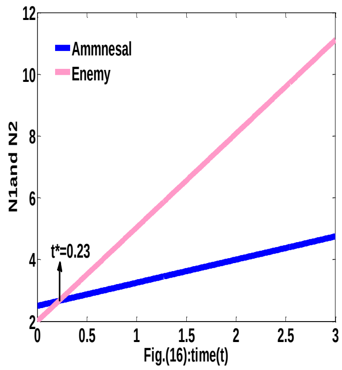


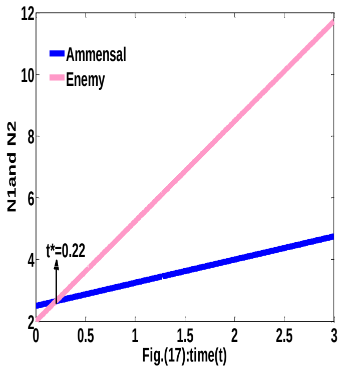


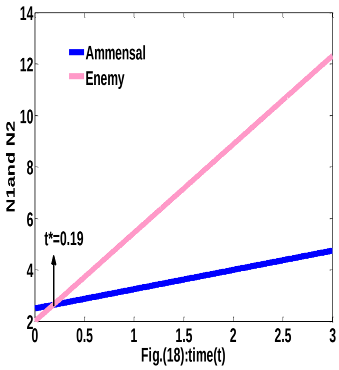


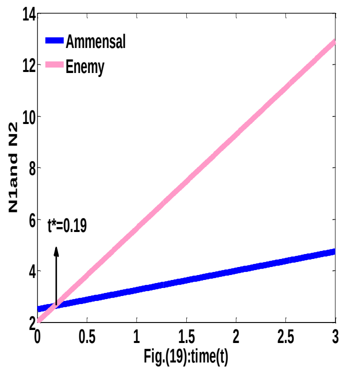


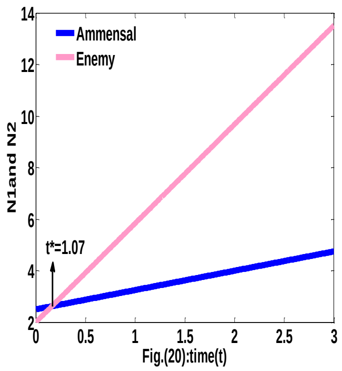
Case(2): **In this case where natural growth rate of Ammensal Species is greater than or equal to the growth rate of Enemy species($a_1 \geq a_2$) -From Fig(15) to Fig(21)**

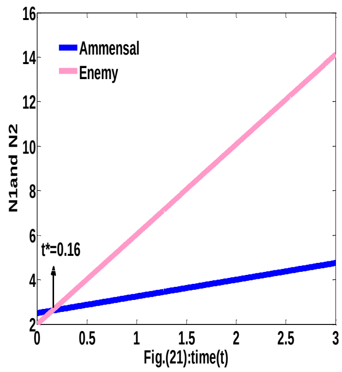












Conclusions:

Case(1):From Fig(1) to Fig(14)

The following results are obtained

(i).Ammensal species prevails over enemy species initially and there is no interaction between the species. The Enemy species has no sufficient growth rate. Afterwards, Ammensal falls down in it's growth rate by the influence of Enemy species.

(ii).Enemy species gradually reaches a position of having steady fast gain than the growth rate of Ammensal species. In the course of time, it is observed that the Enemy species has a steep raise after dominance reversal time (t*).

Case(2):From Fig(15) to Fig(21)

The obtained conclusions are specified as below:

(i).Ammensal species is dominant over the Enemy species up to time distinct(t*).After the dominance reversal time, the Enemy species commands Ammnesal species by pulling it to less growth rate in the course of time.

(ii).Ammensal Species acquires a considerable growth rate at initial stage and slowly arises. It eclipses the enemy species up to t*, after which the enemy species exceeds Ammensal species and the enemy species has a steep rise where as there is no appreciable growth in Ammensal Species.

6. Over All Conclusions

Ammensal model with harvesting for Ammensal species and unlimited resources for Enemy Species is formed by a couple of first order nonlinear differential equations. A series solution in a peculiar case of ecological Ammensalism is obtained by Homotopy Analysis. Some numerical solutions are utilized for analyzing various interactions between Ammensal species and enemy species

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References

[1]. Abbasbandy, S, The application of the Homotopy analysis method to nonlinear equations arising in heat transfer .phys. Lett. A.360,pp.109-113,2006.

[2]. Hilton, P.J, An introduction to Homotopy theory, Cambridge university press, Cambridge,1953.

[3]. Liao Shijun, Homotopy analysis method in nonlinear differential equations, Springer,pp.1-562,2012

[4]. Liao, S.J., The proposed Homotopy analysis technique for the solution of nonlinear problems. Ph.D dissertation,Shanghai Jiao Tong university,1992.

[5]. Liao, S.J, On the Homotopy analysis method for nonlinear problems.Appl.Math.Comput.147, pp.499-513,2004.

[6]. Liao, S.J:,On the relationship between the homotopy analysis method and Euler transform. Commun.Nonlinear Sci.Numer.Simulat.15,2003-2016.

[7]. Liao, S.J., Tan,Y.:A general approach to obtain series solutions of nonlinear differential equations.Stud.Appl.Math.119,pp.297-355,2007.

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