International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 9, September 2014, PP 764-769
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Vertex BIMAGIC Total Labeling for BISTAR $B_{n,n}$

N. Murugesan
Assistant Professor,
Department of Mathematics,
Government Arts College,
Coimbatore
nmgson@yahoo.co.in

R. Senthil Amutha
Assistant Professor,
Department of Mathematics,
Sree Saraswathi Thyagaraja College,
Pollachi
rsamutha@rediffmail.com

Abstract: *A vertex bimagic total labeling on a graph with v vertices and e edges is a one - to - one map taking the vertices and edges onto the integers 1, 2, 3, …v + e with the property that the sum of the label on the vertex and the labels of its incident edges is one of the constants k_1 or k_2, independent of the choice of the vertex. In this paper we have discussed that bistar $B_{n,n}$ are vertex bimagic total labeling for odd n>1 and even n>2.*

Keywords: *Vertex magic total labeling, Vertex bimagic total labeling, magic constants, bistar $B_{n,n}$ pendant vertices, centres*

1. Introduction

A graph G(V,E) has vertex set V = V(G) and edge set E = E(G) and e = | E | and v = | V |.

A general reference for the graph theoretic notions is D. B West [1] . Graph labelings were first introduced in the mid sixties. A labeling of a graph G(V,E) is a mapping from the set of vertices, edges, or both vertices and edges to the set of labels. Based on the domain we distinguish vertex labelings, edge labelings and total labeling.

There are basically two types of labelings of graph, based on assignment of some numbers to the elements of graph and assignment of qualitative nature of the elements of graph named as Quantitative Labelings and Qualitative Labelings respectively. Labeling of graphs subject to certain conditions gave raise to enormous work which is listed by G.J.Gallian [2]. Magic labelings were introduced by Sedl´aˇcek in 1963 [3]. In [4] Kotzig and Rosa defined magic labeling which was called as edge magic labeling by Ringel and Llado [5]. Based on edge magic labeling of graphs, super magic strength of simple graphs like bistar $B_{n,n}$, $(2n+1)\ P_2$, P_{n^2},, $W_n=C_n+k_1$ are discussed in [6].

W. D. Wallis and others [7][8], introduced Edge-magic total labelings that generalize the idea of a magic square and can be referred for a discussion of magic labelings and a standardization of the terminology. J. Basker Babujee & V.Vishnu Priya have introduced (1,1) edge bimagic labeing in their paper "Edge Bimagic labeling in certain types of graphs obtained by some standard graphs" [9]. Also V.Vishnu Priya, K.Manimegalai & J. Basker Babujee [10] have proved edge bimagic labeling for some trees like $B_{m,n}$, $k_{1,n,n}$,Y_{n+1} , J. Basker Babujee has himself introduced (1,1) vertex bimagic labeling in [11]. For simplicity in this paper we name it as vertex bimagic total labeling and discuss that bistar $B_{n,n}$ are vertex bimagic total labeling for odd n>1 and even n>2.

2. Vertex Magic Total Labeling

In general, by labeling a graph we mean an injective map defined from the set of vertices to the set of natural numbers, the same is extended to the set of edges. The labeling from the set of vertices and edges to the set of natural numbers, such that the sum of labels of vertex and the edges incident to that vertex is a constant. Let $G = \{V, E, f\}$ be a simple graph with $v = |V|$ and $e = |E|$. A one to one and onto mapping f from $V \cup E$ to the finite subset $\{1,2,\ldots v+e\}$ of natural numbers such that for every vertex x, $f(x) + f(xy_i) = k$, where y_i's are vertices adjacent to x, is called as a vertex magic total labeling and denoted by VMTL. The constant k is called magic constant.

3. VERTEX BIMAGIC TOTAL LABELING

A graph with vertex magic total labeling with two constants k_1 *or* k_2 is called a vertex bimagic total labeling and denoted by VBMTL. The constants k_1 and k_2 are called magic constants.

3.1. Example

Consider a Path P_4 .This is a graph with 4 vertices and 3 edges as given below.

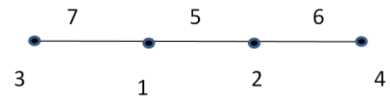

Fig.1. *Path* P_4

In this graph the integers from 1 to 7 are used such that, at the vertex labeled 3,4 the magic constant is 10 and at vertices labeled 1,2 the magic constant is 13.

This graph has vertex bimagic total labeling with constant $k_1 = 10, k_2 = 13$.

4. VERTEX BIMAGIC TOTAL LABELING FOR BISTAR $B_{n,n}$

4.1. Definition

$B_{n,n}$ is the bistar obtained from two disjoint copies of $K_{1,n}$ by joining the centre vertices through an edge.

In a bistar there are $2n+2$ vertices and $2n+1$ edges, altogether there are $4n+3$ elements. There are totally $2n$ pendant vertices and 2 centre vertices. The degree of the pendant vertices are 1 and the degree of the central vertices are $n+1$ obtained from both the n edges of $K_{1,n}$ and the common edge of the centres.

4.2. Theorem

The bistar, $B_{n,n}$ is vertex bimagic total labeling, for odd $n>1$.

Proof:

Let the vertex set be $V=\{u,v,v_1,v_2,\ldots,v_{2n}\}$ where u and v are the centre vertices of the two stars $K_{1,n}$ where n >1 and odd. u_i*'s* and v_i*'s* are the vertices of the pendant vertices of the two stars $K_{1,n}$ and the edge set is $E(G)=\{e_1,e_2,\ldots,e_{2n+1}\}$.

We can define the bijection as $f: VUE \rightarrow \{1,2,\ldots 4n+3\}$ by

$f(u) = 1;\ f(v) = 2$

The vertex labeling is defined as

$$
\begin{aligned}
f(u_i) &= 4n + 6 - 4i && \text{for} \quad 1 \le i \le \tfrac{n+1}{2}\\
&= 6n + 7 - 4i && \text{for} \quad \tfrac{n+3}{2} \le i \le n\\
f(v_i) &= 4n + 7 - 4i && \text{for} \quad 1 \le i \le \tfrac{n+1}{2}\\
&= 6n + 6 - 4i && \text{for} \quad \tfrac{n+3}{2} \le i \le n
\end{aligned}
$$

and the edge labeling is defined as

$$
\begin{aligned}
f(uv) &= 2n+3\\
f(uu_i) &= 4i && \text{for} \quad 1 \le i \le \tfrac{n+1}{2}\\
&= 4i - 2n - 1 && \text{for} \quad \tfrac{n+3}{2} \le i \le n\\
f(vv_i) &= 4i - 1 && \text{for} \quad 1 \le i \le \tfrac{n+1}{2}\\
&= 4i - 2n && \text{for} \quad \tfrac{n+3}{2} \le i \le n
\end{aligned}
$$

Then the constants k_1 and k_2 of the vertex bimagic total labeling is obtained as are obtained below,

To find k_1:

$$2nk_1 = \sum_{i=1}^{n} f(u_i) + \sum_{i=1}^{n} f(v_i) + \sum_{i=1}^{n} f(uu_i) + \sum_{i=1}^{n} f(vv_i)$$

$$= \sum_{i=1}^{\frac{n+1}{2}}(4n+6-4i) + \sum_{i=\frac{n+3}{2}}^{n}(6n+7-4i) + \sum_{i=1}^{\frac{n+1}{2}}(4n+7-4i) + \sum_{i=\frac{n+3}{2}}^{n}(6n+6-4i) + \sum_{i=1}^{\frac{n+1}{2}}(4i)$$

$$+ \sum_{i=\frac{n+3}{2}}^{n}(4i-2n-1) + \sum_{i=1}^{\frac{n+1}{2}}(4i-1) + \sum_{i=\frac{n+3}{2}}^{n}(4i-2n)$$

$$= \sum_{i=1}^{\frac{n+1}{2}}(4n+6-4i+4n+7-4i+4i+4i-1)$$

$$+ \sum_{i=\frac{n+3}{2}}^{n}(6n+7-4i+6n+6-4i+4i-2n-1+4i-2n)$$

$$= \sum_{i=1}^{\frac{n+1}{2}}(8n+12) + \sum_{i=\frac{n+3}{2}}^{n}(8n+12) = \sum_{i=1}^{n}(8n+12) = (8n+12)n = 2n(4n+6)$$

$(i.e)$ $2nk_1 = 2n(4n+6)$

$k_1 = (4n+6)$

Though

$$2k_2 = \left[f(u) + f(uv) + \sum_{i=1}^{n} f(uu_i)\right] + \left[f(v) + f(uv) + \sum_{i=1}^{n} f(vv_i)\right]$$

it is sufficient to find k_2 as below

To find k_2:

(i) $k_2 = f(u) + f(uv) + \sum_{i=1}^{n} f(uu_i)$ or

(ii) $k_2 = f(v) + f(uv) + \sum_{i=1}^{n} f(vv_i)$

Consider

(i) $k_2 = f(u) + f(uv) + \sum_{i=1}^{n} f(uu_i)$

$$= 1 + (2n+3) + \sum_{i=1}^{\frac{n+1}{2}}(4i) + \sum_{i=\frac{n+3}{2}}^{n}(4i-2n-1)$$

$$= (2n+4) + \sum_{i=1}^{\frac{n+1}{2}}(4i) + \sum_{i=\frac{n+3}{2}}^{n}(4i) - \sum_{i=\frac{n+3}{2}}^{n}(2n+1)$$

$$= (2n+4) + \sum_{i=1}^{n}(4i) - (2n+1)\left(n - \left(\frac{n+1}{2}\right)\right)$$

$$= (2n+4) + 4n\left(\frac{n+1}{2}\right) - (2n+1)\left(n - \left(\frac{n+1}{2}\right)\right)$$

$$= (2n+4) + 2n(n+1) - (2n+1)\left(\frac{n-1}{2}\right) = \frac{1}{2}(2n^2+9n+9)$$

$(i.e)$ $k_2 = \frac{1}{2}(2n^2+9n+9)$

Consider

(ii) $k_2 = f(v) + f(uv) + \sum_{i=1}^{n} f(vv_i)$

$$= 2 + (2n+3) + \sum_{i=1}^{\frac{n+1}{2}}(4i-1) + \sum_{i=\frac{n+3}{2}}^{n}(4i-2n)$$

$$= (2n+5) + \sum_{i=1}^{\frac{n+1}{2}}(4i) - \left(\frac{n+1}{2}\right) + \sum_{i=\frac{n+3}{2}}^{n}(4i) - \sum_{i=\frac{n+3}{2}}^{n}(2n)$$

$$= (2n+5) + \sum_{i=1}^{n}(4i) - \left(\frac{n+1}{2}\right) - 2n\left(n - \left(\frac{n+1}{2}\right)\right)$$

$$= (2n+5) + 4n\left(\frac{n+1}{2}\right) - \left(\frac{n+1}{2}\right) - 2n\left(\frac{n-1}{2}\right)$$

$$= \frac{1}{2}(2n^2 + 9n + 9)$$

$$(i.e)\ k_2 = \frac{1}{2}(2n^2 + 9n + 9)$$

Hence the bistar, $B_{n,n}$ is vertex bimagic total labeling, for odd $n>1$.

4.3. Remark

The edge labels for v_i's of the above case may also be given as

$f(vv_i) = 4i + 3 \quad ; \quad 1 \le i \le \frac{n+1}{2}$

$\qquad\quad = 4i - 2n - 2 \quad ; \quad \frac{n+3}{2} \le i \le n$

or as below

$f(vv_i) = 4i - 1 \quad ; \quad i = 1,3,\ldots n$

$\qquad\quad = 2i + 2 \quad ; \quad i = 2,4,\ldots n-1$

4.4. Remark

When $n=1$, the labeling becomes VMTL. So, the VBMTL is defined for odd $n>1$

4.5. Remark

k_1 follows an arithmetic sequence $6n+4$, $6n$, $6n-4$, $6n-8$,... for all odd n values as $n = 1,3,\ldots$ at the pendant vertices. k_2 follows an arithmetic sequence nk_1, $\frac{n-1}{2}k_1$, $\frac{n-2}{2}k_1$, ... for all odd n values as $n = 1,3,\ldots$ at the centre vertices. k_2 also follows the relation $8m_r + r + 1$ where $m_r = 1,3,6,\ldots$with $m_r = m_{r-1} + r$, $r = 1,2,3,\ldots$

4.6. Example

Consider the bistar, $B_{3,3}$,

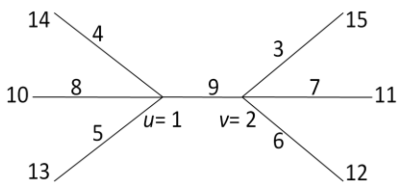

Fig 2. *Bistar, $B_{3,3}$*

Here k_1=18 at the pendant vertices and k_2 =27 at the centres.

The bistar, $B_{3,3}$ is vertex bimagic total labeling.

The same labeling is not possible for even values of n in an bistar $B_{n,n}$ because the difference between the respective values of u_i's and v_i's are such that 3,1 and 1,3 alternatively.(i.e) Along u_i's the values are 10,13,14 where the difference between these numbers is such that 13-10 = 3,

14 -13 =1 and similarly along v_i's the values are 11,12,15 where the difference between these numbers is such that 12-11=1, 15-12 = 3. So to complete one cycle (1+3= 3+1 = 4) any two difference needs three values and when 2 more differences are included 5 values are required. Hence only on adding 2 values every time in n, starting with 1, this labeling becomes impossible for any even n. So a new function for bistar $B_{n,n}$, for even n is given.

4.7. Theorem

The bistar, $B_{n,n}$ is vertex bimagic total labeling, for even $n>2$.

Proof:

Let the vertex set be $V=\{u,v,v_1,v_2,\ldots,v_{2n}\}$ where u & v are the centre vertices of the two stars $K_{1,n}$ and n >1 is odd. v_i's and v_j's are the vertices of the pedant vertices of the two stars $K_{1,n}$ and the edge set is $E(G)=\{e_1,e_2,\ldots,e_{2n+1}\}$.

We can define the bijection as $f{:}V \cup E \rightarrow \{1,2,\ldots 4n+3\}$ by *f(u) = 1; f(v) = 3*

the vertex labeling is defined as

$$f(u_i) = 4(n+1) - 2i \quad \text{for } 1 \le i \le \frac{n}{2}$$
$$= 4(n+1) - 2i + 1 \quad \text{for } \frac{n}{2}+1 \le i \le n$$
$$f(v_i) = 4(n+1) - 2i + 1 \quad \text{for } 1 \le i \le \frac{n}{2}$$
$$= 4(n+1) - 2i \quad \text{for } \frac{n}{2}+1 \le i \le n$$

and the edge labeling is defined as

$$f(uv) = 2$$
$$f(uu_i) = i+5 \quad \text{for } i = 1,2$$
$$= 2i+3 \quad \text{for } 3 \le i \le \frac{n}{2}$$
$$= 2i+2 \quad \text{for } 1 \le r \le \frac{n}{2}$$
$$f(vv_i) = i+3 \quad \text{for } i = 1,2$$
$$= 2i+2 \quad \text{for } 3 \le i \le \frac{n}{2}$$
$$= 2i+3 \quad \text{for } 1 \le r \le \frac{n}{2}$$

Then the constants k_1 and k_2 of the vertex bimagic total labeling are obtained as below,

To find k_1:

$$2nk_1 = \sum_{i=1}^{n} f(u_i) + \sum_{i=1}^{n} f(v_i) + \sum_{i=1}^{n} f(uu_i) + \sum_{i=1}^{n} f(vv_i)$$

$$= \sum_{i=1}^{\frac{n}{2}} (4(n+1) - 2i) + \sum_{i=\frac{n}{2}+1}^{n} (4(n+1) - 2i + 1) + \sum_{i=1}^{\frac{n}{2}} (4(n+1) - 2i + 1)$$
$$+ \sum_{i=\frac{n}{2}+1}^{n} (4(n+1) - 2i) + \sum_{i=1}^{2} (i+5) + \sum_{i=3}^{\frac{n}{2}} (2i+3) + \sum_{i=\frac{n}{2}+1}^{n} (2i+2) + \sum_{i=1}^{2} (i+3)$$
$$+ \sum_{i=3}^{\frac{n}{2}} (2i+2) + \sum_{i=\frac{n}{2}+1}^{n} (2i+3)$$

$$= \sum_{i=1}^{\frac{n}{2}} (4(n+1) - 2i) + \sum_{i=\frac{n}{2}+1}^{n} (4(n+1) - 2i) + \left(n - \frac{n}{2}\right) + \sum_{i=1}^{\frac{n}{2}} (4(n+1) - 2i) + \frac{n}{2}$$
$$+ \sum_{i=\frac{n}{2}+1}^{n} (4(n+1) - 2i) + \sum_{i=1}^{2} (i+5+i+3) + \sum_{i=3}^{n} (2i+2) + \sum_{i=3}^{n} (2i+3)$$

$$= 2\sum_{i=1}^{n} (4(n+1) - 2i) + n + \sum_{i=1}^{2} (2i+8) + \sum_{i=3}^{n} (2i+2+2i+3)$$

$$= 8(n+1)n - 4\sum_{i=1}^{n} (i) + n + 8(2) + 2(3) + \sum_{i=3}^{n} (4i+5)$$

$$= 8(n+1)n - 2n(n+1) + n + 22 + 4\left(\frac{n(n+1)}{2} - 3\right) + 5(n-2) = n(8n+14)$$

$$= 2n(4n+7)$$

$(i.e)\ 2nk_1 = 2n(4n+7)$

$k_1 = 4n+7$

To find k_2:

$$k_2 = f(u) + f(uv) + \sum_{i=1}^{n} f(uu_i) = 1 + 2 + \sum_{i=1}^{2} (i+5) + \sum_{i=3}^{\frac{n}{2}} (2i+3) + \sum_{i=\frac{n}{2}+1}^{n} (2i+2)$$

$$= 3 + 5(2) + 3 + \sum_{i=3}^{n}(2i + 2) + \left(\frac{n}{2} - 2\right) = 16 + 2\left(\frac{n(n+1)}{2} - 3\right) + 2(n - 2) + \left(\frac{n}{2} - 2\right)$$
$$= n^2 + 7\frac{n}{2} + 4 = \frac{1}{2}(2n^2 + 7n + 8)$$

$(i.e)\ k_2 = \dfrac{1}{2}(2n^2 + 7n + 8)$

In a similar manner k_2 can be obtained for v_i*'s.*

4.8. Example

The bistar $B_{4,4}$ is considered here.

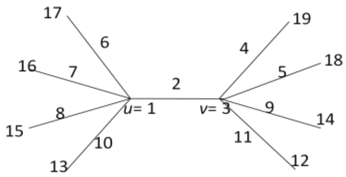

Fig. 3. *Bistar,* $B_{4,4}$

Here k_1=23at the pendant vertices and k_2 =34 at the centres.

The bistar, $B_{4,4}$ is vertex bimagic total labeling.

5. Conclusion

Vertex bimagic total labeling on n- bistar $B_{n,n}$ for odd n>1 and even n>2 have been discussed in this paper. Further VBMTL can be studied for many other graphs like regular graphs, complete graphs etc.

References

[1] D. B. West , An Introduction to Graph Theory, Prentice-Hall (2001).

[2] J. A. Gallian, A dynamic survey of graph labeling, Electronic J. Combinatorics 5 (2011), #DS6, 1-246

[3] J. Sedl´a˘cek, On magic graphs, Math. Slov., 26 (1976), 329–335.

[4] A.Kotzig and A.Rosa, Magic valuations of finite graphs, Canad. Math. Bull. 13(4), (1970), 451-461.

[5] G.Ringel and A.S. Llado, Another tree conjecture, Bull. ICA, 18,(1996),83-85.

[6] D. G. Akka and Nanda S. Warad, Super magic strength of a graph , Indian J.Pure Appl. Math. ,41(4), August 2010, 557-568.

[7] W.D. Wallis, Magic Graphs, Birkh¨auser, Boston MA 2001.

[8] W. D. Wallis, E.T. Baskoro, Mirka Miller and Slamin ,Edge-magic total labeling, Australasian Journal of Combinatorics 22(2000). pp.177-190

[9] J. Basker Babujee and V.Vishnu Priya, Edge Bimagic labeling in graphs, Acta Ciencia Indica, Vol. XXXIM, No. 3, 741(2005).

[10] V.Vishnu Priya, K.Manimegalai & J. Basker Babujee, Edge bimagic labeling for some trees, Acta ciencia Indica, Vol XXXVM.No.2,569(2009)

[11] J. Basker Babujee, Bimagic labeling in path graphs, The Mathematics Education, Volume 38, No.1, P. 12-16(2004).

Authors' Biography

N. Murugesan, Assistant Professor, Department of Mathematics, Government Arts College, Coimbatore is a renowned researcher in the field of Automata theory and Graph theory with 18 years of experience in teaching field, a number of publications both in National and International Journals

R. Senthil Amutha, Assistant Professor, Department of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi is a researcher in the field of graph theory and has 15 years of experience in teaching field, with publications in International Journals