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Radiation Effect on Heat and Mass Transfer in MHD Flow of a Casson Fluid over a Stretching Surface

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Abstract: *The effect of thermal radiation on heat and mass transfer in MHD flow of a Casson fluid over a porous stretching sheet is investigated. The governing non-linear partial differential equations for the flow are solved by employing the Runge-Kutta fourth order method along with shooting technique. The velocity, temperature and concentration profiles for different governing parameters are illustrated graphically. It is observed that increase in the yield stress of the fluid results in the reduction of velocity while the temperature and concentration are enhanced. The numerical values of the skin friction coefficient, the Nusselt number and the Sherwood number for different values of physical parameters are analyzed.*

Keywords: *Casson fluid, MHD flow, Heat and Mass transfer, Radiation.*

1. Introduction

Heat and mass transfer finds applications in variety of engineering process such as migration of moisture in heat exchanger devices, petroleum reservoirs, chemical catalytic reactors, studies on heat and mass transfer include the works of Gebhart and Pera [1] on vertical plates, Chen and Yuh [2] on inclined plates. In fluid dynamics the effects of external magnetic field on magnetohydrodynamic (MHD) flow over a stretching sheet are very important due to its applications in many engineering problems, such as glass manufacturing, geophysics, paper production, and purification of crude oil. The flow due to stretching of a flat surface was first investigated by Crane [3]. Pavlov [4] studied the effect of external magnetic field on the MHD flow over a stretching sheet. Andersson [5] discussed the MHD flow of viscous fluid on a stretching sheet and Mukhopadhyay et al. [6] presented the MHD flow and heat transfer over a stretching sheet with variable fluid viscosity. The process of suction and blowing has wide applications in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. It is also well known that suction or injection of fluid through the surface, as in mass transfer cooling can significantly modify the flow field. This will affect the rate of heat transfer in forced, free and mixed convection. There are several numerical studies on the effects of uniform suction or injection on the boundary layer flow and heat transfer, by Watanabe and Kawakami [7], Pop and Watanabe [8]. The most important non-Newtonian fluid possessing a yield value is the Casson fluid, which has significant applications in polymer processing industries and biomechanics. Casson fluid is a shear thinning liquid which has an infinite viscosity at a zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear. Casson's constitute equation represents a nonlinear relationship between stress and rate of strain and has been found to be accurately applicable to silicon suspensions, suspensions of bentonite in water and lithographic varnishes used for printing inks [9]-[11]. Eldabe(12) considered the heat transfer of a Casson fluid between two rotating cylinders. The flow of Casson fluid in a tube was studied by Dash et al.(13) .Many authors [14]-[16] studied the flow or/and heat transfer of a Casson fluid in different geometries. Recently Haritha et al.(17) studied effect of thermal radiation on heat and mass transfer in MHD flow of a Casson fluid over a stretching surface with variable thermal conductivity.

In view of the above discussion, in this paper the radiation effect on heat and mass transfer in MHD flow of a Casson fluid over a stretching surface is studied. The governing equations of the flow are solved numerically.

2. FORMULATION OF THE PROBLEM

We consider the laminar boundary layer two-dimensional flow of heat and mass transfer of an incompressible, conducting non-Newtonian Casson fluid over a porous stretching sheet at y = 0. We choose the Cartesian coordinate system such that x- axis is parallel to the surface and y-axis perpendicular to the surface. The fluid occupies the half space y > 0. The flow is taken to be steady and the magnetic Reynolds number is considered to be very small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. The heat and mass transfer phenomenon with radiation is considered. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is taken as (Eldabe and Silwa, 1995)

$$\tau_{ij} = \begin{cases} 2\left(\mu_\beta + P_y/\sqrt{2\pi}\right)e_{ij,}\pi > \pi_c \\ 2\left(\mu_\beta + P_y/\sqrt{2\pi_c}\right)e_{ij,}\pi < \pi_c \end{cases}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} denotes the (i, j)th component of the deformation rate, π the product of the component of deformation rate with itself, π_c a critical value of this product based on the non-newtonian model, μ_B the plastic dynamic viscosity of the non-Newtonian fluid and p_y the yield stress of the fluid. The governing equations of the steady boundary layer flow of the Casson fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B_0^2 u}{\rho}\right)u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_1}{\rho c_p}(T - T_\infty) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Where (u, v) are the velocity components in x- and y- directions respectively, υ is the kinematic viscosity, $\beta = \mu_B\sqrt{2\pi_c}/p_y$ is the non-Newtonian Casson parameter, K is the thermal conductivity, ρ is the density, B_0 is the magnetic induction, T and T_∞ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream respectively, while C is the concentration, σ is the electrical conductivity of the fluid, c_p the specific heat at constant pressure, q_r the radiation heat flux, Q_1 the heat generation constant, D the coefficient of mass diffusivity.

The appropriate boundary conditions for the problem are given by

$$\begin{aligned} &u = u_w, v = -v_0, T = T_w, C = C_w \ at \ y = 0 \\ &u \to 0, T \to T_\infty, C \to C_\infty \quad as \quad y \to \infty \end{aligned} \tag{5}$$

Where u_w = cx is stretching velocity of the sheet with c(>0) being the stretching constant. v_0 is the suction velocity, T_w is the temperature of the sheet.

The radiative heat flux term is simplified by using Rosseland approximation as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial(T^4)}{\partial y} \tag{6}$$

Where q_r represents the radiative heat flux in the y-direction, σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. We assume that the temperature difference with in the flow is sufficiently small such that T^4 may be expressed as a linear function of the temperature .This is accomplished by expanding T^4 in a Taylors series about T_∞ and neglecting higher order terms so that

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^3 \tag{7}$$

Introducing the similarity variables

$$\eta = y\sqrt{\frac{c}{\nu}},\ f(\eta) = \frac{\psi}{x\sqrt{c\nu}},\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$
$$M = \frac{\sigma B_0^2}{\rho c},\ \Pr = \frac{\nu\rho c_p}{k},\ \alpha = \frac{Q_0}{\rho c_p}, Sc = \frac{\nu}{D},\ Nr = \frac{4\sigma^* T_\infty^3}{k^* k} \tag{8}$$

Where ψ is the stream function with $u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x}$ and η is the stream variable.

using equation (8), the equations (2), (3), and (4) reduce to

$$(1 + \frac{1}{\beta})f''' + ff'' - f'^2 - Mf' = 0 \tag{9}$$

$$(1 + Nr)\theta'' + \Pr\alpha\theta + \Pr f\theta' = 0 \tag{10}$$

$$\phi'' + Scf\phi' = 0 \tag{11}$$

The corresponding boundary conditions are

$$\begin{aligned} &f = f_w,\ f' = 1,\ \theta = 1,\ \phi = 1 \quad at \quad \eta = 0 \\ &f' = 0,\ \theta = 0,\ \phi = 0 \quad as \quad \eta \to \infty \end{aligned} \tag{12}$$

Where $f_w = \frac{v_0}{\sqrt{c\nu}}$ is the suction parameter.

Knowing the velocity field, temperature field, concentration field the Skin- friction, Nusselt number and Sherwood number, can be calculated by

$$(\mathrm{Re}_x)^{\frac{1}{2}} c_f = (1 + \frac{1}{\beta})f''(0),\qquad (\mathrm{Re}_x)^{-\frac{1}{2}}\ Nu = -\theta'(0)\ \ and\ \ (\mathrm{Re}_x)^{-\frac{1}{2}} Sh = -\phi'(0)$$

Where $\mathrm{Re}_x = \frac{xu_w(x)}{\nu}$ (stretching Reynold's number).

3. RESULTS AND DISCUSSIONS

In this paper the effect of radiation and magnetic field on the heat and mass transfer of a Casson fluid flow in the presence of temperature dependent heat source past a stretching surface is studied. The velocity, temperature and concentration profiles have been discussed for variations in the flow parameters casson parameter β, Hartmann number M, Suction parameter fw, heat source parameter α and radiation parameter Nr.

Fig(1) shows that the Casson parameter decreases both the velocity and the boundary layer thickness. In fig.(2) the effect of suction is shown. In the presence of suction the velocity decreases and increasing values of suction parameter results in further reduction in velocity and hence the thickness of boundary layer decreases. In fig.(3) the effect of magnetic field on velocity is presented. The presence of magnetic field reduces the velocity and thus leads to reduction in the boundary layer thickness as the Lorentz force increases the thickness of boundary layer decreases further. This is accordance with the fact that Lorentz force acts as a retarding force. In fig.(3) the effect of suction is shown. In the presence of suction the velocity decreases and increasing values of suction parameter results in further reduction in velocity and hence the thickness of boundary layer decreases.

The temperature profiles are presented in figures (4-9) . From fig(4) it is observed the presence of temperature dependant heat source facilitates for enhancement of temperature distribution throughout thermal boundary layer. The increase in temperature dependant heat source parameter results in the enhancement of temperature. From (fig.5) it is observed that the temperature decreases from its peak value throughout the thermal boundary layer and approaches zero value far downstream. The presence of yield stress increases the temperature, further increase in the yield stress results in the increase of the thickness of boundary layer. The presence of (fig.6) suction is very significant on the behavior of temperature. When suction parameter is 1 the temperature is reduced and thus there is a significant reduction in thickness of thermal boundary layer, when Suction parameter is 3 there is a threefold reduction in the thickness of boundary layer to that of impermeable case. fig.(7) shows that with the increase of magnetic field the temperature also increases. From fig.(8) it is noticed that the presence of radiation increases the temperature and hence leads to an increase in thickness of thermal boundary layer. From fig.(9) it is observed that increasing values of Prandtl number leads to significant reduction of temperature. This is due to the fact that larger Prandtl number indicates low thermal conductivity.

The concentration profiles are presented in figures (10-12). The presence of yield stress (fig.10) increases the concentration however, its effect is not very significant. Fig.(11) shows the presence of Suction parameter decreases the concentration. Increase in suction parameter leads to a significant reduction in concentration. The effect of Schmidt's number is shown in fig.(12). As the molecular diffusion increases the concentration is significantly reduced.

For selected values of magnetic field and suction parameter it is noticed that the presence of Casson parameter decreases the wall shear stress due to the fact that the presence of Casson parameter and increase in the value of β reduce velocities and hence the skin friction coefficient reduces. Similarly the Nusselt number and Sherwood number reduce. The presence of magnetic field and increasing values of magnetic field enhance the skin friction while Nusselt number and Sherwood number reduce. The presence of suction increase the wall shear, Nusselt number and Sherwood number. The presence of radiation and increasing values of radiation parameter reduce the values of Nusselt number which is in tune with the fact that the radiation parameter increases the thickness of boundary layer. For selected values of Nr and α increasing values of Prandtl number increase the Nusselt number and a similar feature is observed in Nusselt number for increasing values of heat source parameter. The Sherwood number is observed to increase for an increase in the values of Schmidt's number.

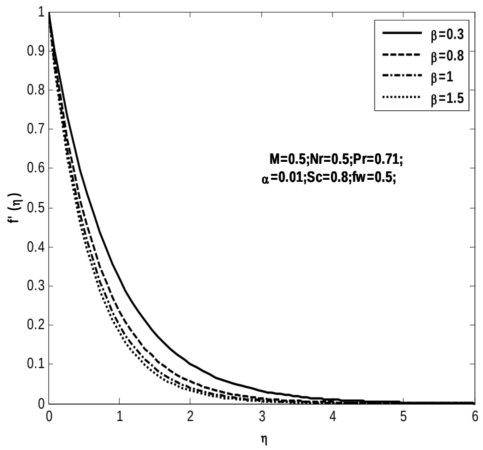


Fig.1 *Variation of velocity with* β

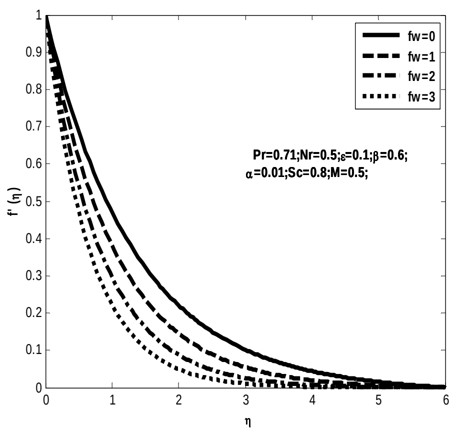


Fig.2 *Variation of velocity with fw*

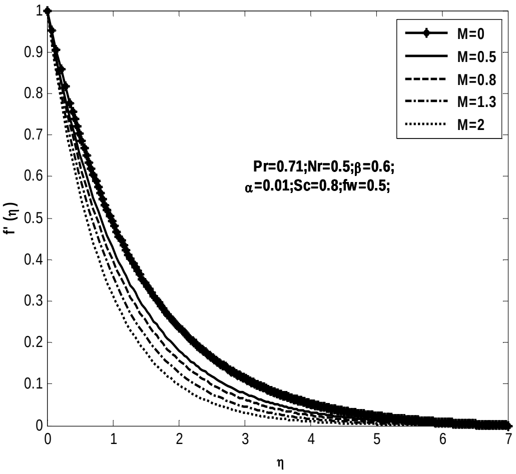


Fig.3 *Variation of velocity with M*

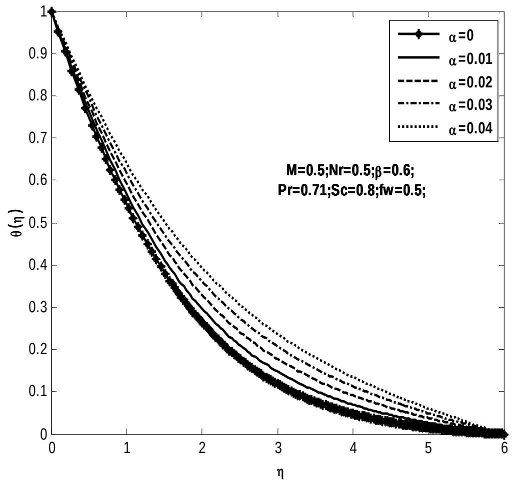


Fig.4 *Variation of temperature with* α

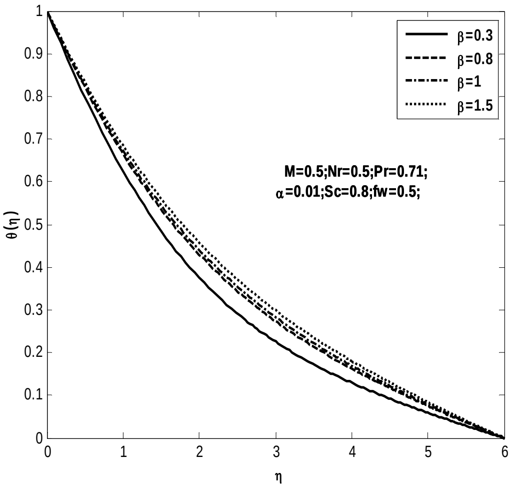


Fig.5 *Variation of temperature with* β

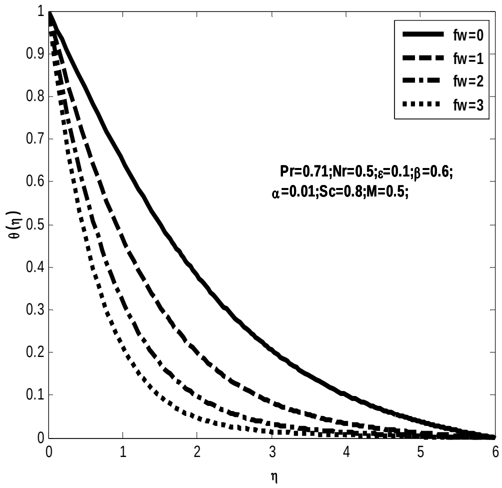
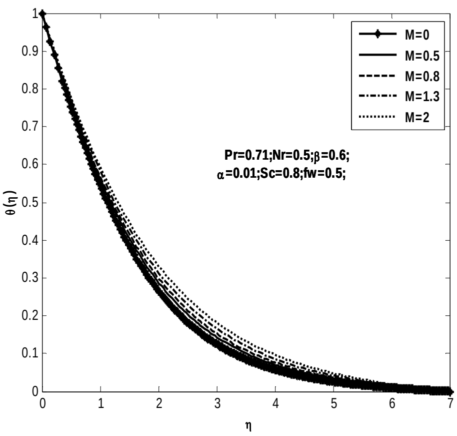


Fig.6 *Variation of temperature with fw*



. Fig. 7 Variation of temperature with M

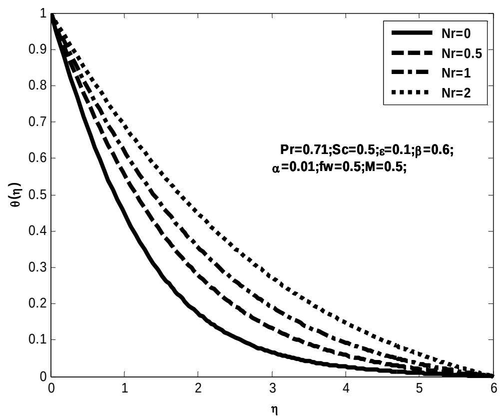


Fig. 8 Variation of temperature with Nr

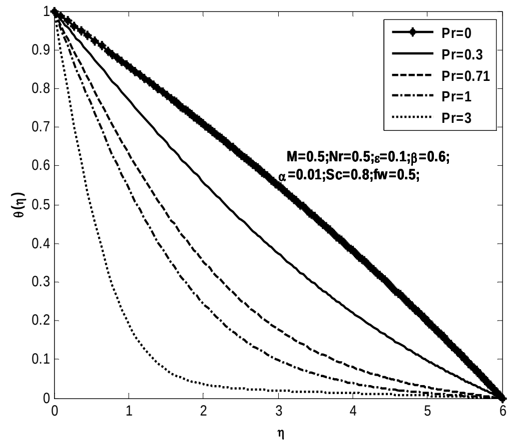


Fig. 9 Variation of temperature with Pr

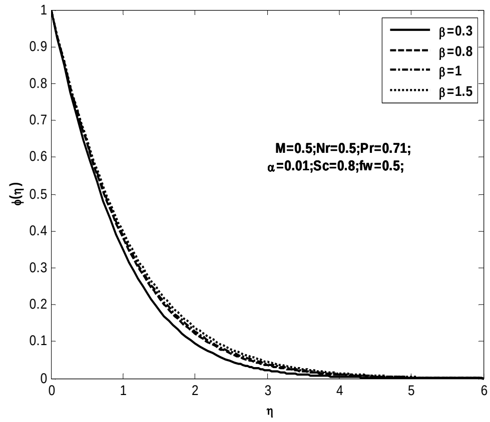


Fig. 10 Variation of concentration with β

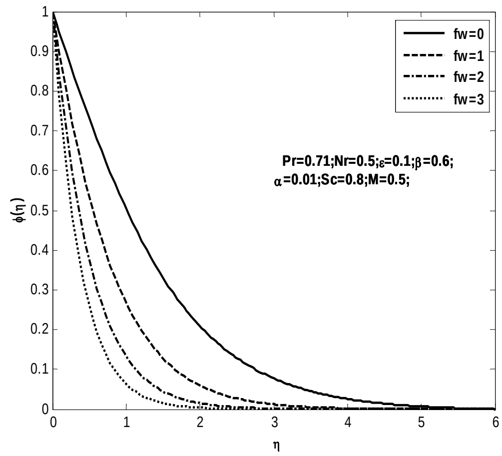


Fig. 11 Variation of concentration with fw

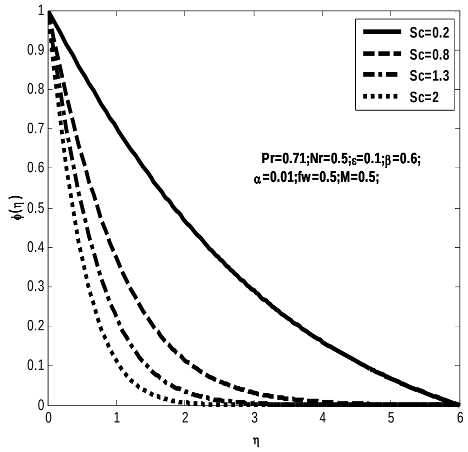


Fig. 12 Variation of concentration with Sc

β	M	fw	$-(1+1/\beta)f''(0)$	$-\theta(0)$	$-\phi'(0)$
0.3	0.5	0.5	1.85532	0.65798	0.95295
0.8			1.65938	0.55175	0.84586
1			1.35436	0.53263	0.82409
1.5			1.05144	0.51560	0.80376
0.8	0		0.92102	0.50352	0.78871
	0.5		1.12988	0.56328	0.85859
	0.8		1.48849	0.54411	0.83729
	1.3		1.73061	0.53459	0.82636
	2.0		2.03298	0.52113	0.81044
	0.5	0	0.96115	0.50594	0.79176
		1	1.25587	0.38467	0.55233
		2	1.54321	0.72351	1.15578
		3	2.06418	1.12098	1.85208

Nr	Pr	α	$-\theta'(0)$
0	0.71	0.01	0.78211
0.5			0.59932
1			0.48004
2			0.39730
0.5	0		0.20461
	0.3		0.41226
	0.71		0.63419

	1		0.82339
	3		1.53117
		0	0.58147
		0.01	0.56416
		0.02	0.52247
		0.03	0.49934
		0.04	0.46229

Sc	$-\phi'(0)$
0.2	0.33988
0.8	0.85721
1.3	1.31154
2	1.76213

4. CONCLUSIONS

i. Increase in the Casson parameter β, Hartmann number M and suction parameter fw decrease the velocity.

ii. The temperature and concentration increases with increase of Casson parameter β.

iii. The temperature and concentration decreases with increase of suction parameter fw.

iv. With the increase of radiation parameter Nr and temperature dependant heat source parameter α the temperature increases.

v. The temperature decreases by increasing the Prandtl number Pr also an increase in Schmidt's number Sc causes decrease in the concentration profile and the boundary layer thickness.

REFERENCES

[1] Gebhart B; Pera L, The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. Int. J Heat Mass Transfer 14: 2025-2050 (1971).

[2] Chen T S; Yuh C F, Combined heat and mass transfer in mixed convection along vertical and inclined plates. Int J Heat Mass Transfer 23:527-537.

[3] Crane,L.J. "Flow past a stretching plate",ZAMP,21,pp.645-655(1970).

[4] K. B. Pavlov, "Magnetohydrodynamic flow of an incompressible viscous fluid caused by the deformation of a plane surface," Magnetohydrodynamics, vol. 10, pp. 146–148, 1974.

[5] H. I. Andersson, "MHD flow of a viscoelastic fluid past a stretching surface," Acta Mechanica, vol. 95, no. 1–4, pp. 227– 230, 1992.

[6] S. Mukhopadhyay, G. C. Layek, and S. A. Samad, "Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity," International Journal of Heat and Mass Transfer, vol. 48, no. 21-22, pp. 4460–4466, 2005.

[7] T. Watanabe, H. Kawakami, Trans. Japan Soc. Mech. Eng. 55 (1989) 3365–3369.

[8] I. Pop and T. Watanabe, TechnischeMechanik 13 (1992) 49–54.

[9] Casson, N., A flow equation for pigment oil-suspensions of the printing ink type", in Rheolgy of Disperse Systems (C.C. Mill, ed.), P. 84. Pergamon Press, London, 1959.

[10] Tamamashi, B., Consideration of certain hemorheological phenomena from the stand-point of surface chemistry, in Hemorheology (A.L. Copley, ed.), P. 89, Pergamon Press, London, 1968.

[11] Walawander, W.P., Chen, T.Y. and Cala, D.F., An aproximate Casson fluid model for tube flow of blood, Biorheology, 12 (1975), 111.

[12] Eldabe,N.T.M. and Salwa, M.G.E., Heat transfer of MHD non-Newtonian Casson fluid flow between two rotating cylinders. J. Phys. Soc.Japan,64,p.41(1995).

[13] Dash,R.K.,Mehta,R.K. and Jayarama, G., Casson fluid flow in a pipe filled with homogeneous porous medium. Int.J.Eng.Sci.,34,p.1145(1996).

[14] Batra, R.L. and Jena, B., Flow of a Casson fluid in a slightly curved tube, Int. J. Engng. Sci.,29 (1991), 1245.

[15] Das, B. and Batra, R.L., Secondary flow of a Casson fluid in a slightly curved tube, Int. J. Non-Linear Mechanics, 28 (5) (1993), 567.

[16] Sayed Ahmed, M.E. and Attia, H.A., Magnetohydrodynamic flow and heat transfer of a non-Newtonian fluid in an eccentric annulus, Can. J.Phys., 76 (1998), 391.

[17] Haritha, A. and Sarojamma, G., Effect of thermal radiation on heat and mass transfer in MHD flow of a Casson fluid over a stretching surface with variable thermal conductivity, "Int J. of Applied Mathematics", vol.29, issue 01,pp. 1287-1293.