International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 5, May 2014, PP 453-459
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Five Dimensional Bianchi Type-I (KasnerForm)Cosmological Models

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Abstract: *The spatially homogeneous and anisotropic five dimensional Bianchi type-I in Kasner form have been considered in general relativity. The exact solutions of the Einstein's field equations with variable gravitational and cosmological constant have been obtained.A law of variation of scale factor which yields a time dependent deceleration parameter has been used. The gravitational constant* $G(t)$*, cosmological constant* $\Lambda(t)$ *and some physical quantities are also obtained.*

Keywords: *Five dimensionalBianchi type-I space-time in Kasner form, Gravitational constant, Cosmological constant.*

1. Introduction

In Einstein's field equations (with c = 1), there are two constants, gravitational constant (G) and cosmological constant (Λ).*G* has many consequences in astrophysics. *G* plays the role of coupling constant between geometry and matter where as Λ corresponds to universal repulsion. Dirac [1] introduced time variation of the gravitational constant *G*. E Garcio-Berro*et al.* [2] and Belinchon [3] have studied the theories of gravity in which *G* varies with time and cosmological models based on them. Tripathi [4] has obtained Bianchi type-I universe with decaying vacuum energy and time varying gravitational constant.

In the context of quantum field theory, a cosmological term corresponds to the energy density of vacuum. The cosmological term, which is measure of the energy of empty space, provides a repulsive force opposing the gravitational pull between the galaxies. Recently, it is observed that smallness of cosmological constant impose problems in cosmology and elementary particle physics theory. In the absence of any interaction with matter or radiation, the cosmological constant remains a constant. In the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. Recent observations by Perlmutter*et al*. [5,6,7,] and Riess*et al*. [8,9] strongly favor a significant and a positive value of cosmological constant Λ . Kalita*et al*. [10], Chawla*et al*. [11] and Pradhan*et al.* [12] have studied the time dependent cosmological constant in different context. The observations of large-scale cosmic microwave background suggest that our physical universe is expanding isotropic and homogeneous models with a positive cosmological constant. At early stages of evolution of the universe, cosmological models which are spatially homogeneous but anisotropic have significant roles in description of the universe. Ladke [13] has studied the Bianchi type-I (Kasner form) cosmological model in $f(R)$ theory of gravity. Recently, Pradhan*et al*. [14] have studied the Bianchi type-I transit cosmological models with time dependent gravitational and cosmological constants.

Higher dimensional cosmological models play a vital role in many aspects of early stage of cosmological problems. The study of higher dimensional space-time provides an idea that our universe is much smaller at early stage of evolution as observed today. Many researchers inspired to enter in to the field of higher dimension theory to explore the knowledge of the universe. Wesson [15,16] have studied several aspects of five dimensional space-time in variable mass theory and bimetric theory of relativity. Lorentz and Petzold[17], Ibanez and Verdaguer[18], Reddy and Venkateswara[19], Adhav*et. al.*[20]have studied the multi dimensional cosmological model in general relativity and in other alternative theories of gravitation.

Motivated by the above research, in this paper, considering the five dimensional space-time metric of the spatially homogeneous and anisotropic Bianchi type-I in Kasner form, the exact solutions of the Einstein's field equations with variable gravitational and cosmological constant have been obtained by considering lawof variation of scale factor which yields a time dependent deceleration parameter. Gravitational constant (G) and cosmological constant(Λ) are also obtained. The physical behavior of the model has been discussed.

2. METRIC AND FIELD EQUATIONS

A fivedimensional space-time metric of the spatially homogeneous and anisotropic Bianchi type-I in Kasner formis given by

$$ds^2 = -dt^2 + t^{2q_1}dx^2 + t^{2q_2}dy^2 + t^{2q_3}dz^2 + t^{2q_4}du^2, \tag{1}$$

where q_1, q_2, q_3 and q_4 are constant.

Einstein field equations with time dependent G and Λ are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}, \tag{2}$$

where R_{ij} is the Ricci tensor; R is the Ricci scalar, G is the gravitational constant and Λ is the cosmological constant.

For a perfect fluid, the stress energy-momentum tensor T_{ij} is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \tag{3}$$

where $\rho =$ matter density, $p =$ thermodynamic pressure, $u^i =$ fluid four-velocity vector of the fluid which satisfies

$$u^i u_j = 1, \tag{4}$$

In a commoving coordinate system,equation (2) for the metric (1) and (3) give

$$\frac{(t^{q_1})^{\cdot\cdot}}{t^{q_1}} + \frac{(t^{q_2})^{\cdot\cdot}}{t^{q_2}} + \frac{(t^{q_1})^{\cdot}}{t^{q_1}}\frac{(t^{q_2})^{\cdot}}{t^{q_2}} = -8\pi G(t)p + \Lambda(t), \tag{5}$$

$$\frac{(t^{q_2})^{\cdot\cdot}}{t^{q_2}} + \frac{(t^{q_3})^{\cdot\cdot}}{t^{q_3}} + \frac{(t^{q_2})^{\cdot}}{t^{q_2}}\frac{(t^{q_3})^{\cdot}}{t^{q_3}} = -8\pi G(t)p + \Lambda(t)t), \tag{6}$$

$$\frac{(t^{q_3})^{\cdot\cdot}}{t^{q_3}} + \frac{(t^{q_4})^{\cdot\cdot}}{t^{q_4}} + \frac{(t^{q_3})^{\cdot}}{t^{q_3}}\frac{(t^{q_4})^{\cdot}}{t^{q_4}} = -8\pi G(t)p + \Lambda(t)t), \tag{7}$$

$$\frac{(t^{q_1})^{\cdot\cdot}}{t^{q_1}} + \frac{(t^{q_4})^{\cdot\cdot}}{t^{q_4}} + \frac{(t^{q_1})^{\cdot}}{t^{q_1}}\frac{(t^{q_4})^{\cdot}}{t^{q_4}} = -8\pi G(t)p + \Lambda(t), \tag{8}$$

$$\frac{(t^{q_1})^{\cdot}}{t^{q_1}}\frac{(t^{q_2})^{\cdot}}{t^{q_2}} + \frac{(t^{q_2})^{\cdot}}{t^{q_2}}\frac{(t^{q_3})^{\cdot}}{t^{q_3}} + \frac{(t^{q_3})^{\cdot}}{t^{q_3}}\frac{(t^{q_4})^{\cdot}}{t^{q_4}} + \frac{(t^{q_4})^{\cdot}}{t^{q_4}}\frac{(t^{q_1})^{\cdot}}{t^{q_1}} = 8\pi G(t)p + \Lambda(t), \tag{9}$$

The spatial volume *V*for the model is

$V = t^S$,(10)

where $S = q_1 + q_2 + q_3 + q_4$.

We define a mean scale factor of the model as

$$a = V^{1/4} = t^{S/4}. \tag{11}$$

The generalized mean Hubble parameter H isgiven by

$$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4), \tag{12}$$

where $H_1 = \frac{(t^{q_1})^{\cdot}}{t^{q_1}}$, $H_2 = \frac{(t^{q_2})^{\cdot}}{t^{q_2}}$, $H_3 = \frac{(t^{q_3})^{\cdot}}{t^{q_3}}$, $H_4 = \frac{(t^{q_4})^{\cdot}}{t^{q_4}}$ are the directional Hubble parameters inthe directions of x, y, z and u respectively and dot (˙) denotes derivative with respect to cosmic time *t*.

Now H becomes

$$H = \frac{1}{4}\left[\frac{(t^{q_1})^{\cdot}}{t^{q_1}} + \frac{(t^{q_2})^{\cdot}}{t^{q_2}} + \frac{(t^{q_3})^{\cdot}}{t^{q_3}} + \frac{(t^{q_4})^{\cdot}}{t^{q_4}}\right]. \tag{13}$$

The physical parameters that are of cosmological importance are:

The expansion scalar θ is given by

$$\theta = 4H = \left[\frac{(t^{q_1})^{\cdot}}{t^{q_1}} + \frac{(t^{q_2})^{\cdot}}{t^{q_2}} + \frac{(t^{q_3})^{\cdot}}{t^{q_3}} + \frac{(t^{q_4})^{\cdot}}{t^{q_4}}\right]. \tag{14}$$

The mean anisotropy parameter Δ is given by

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2. \tag{15}$$

The shear scalar σ is defined as

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{(t^{q_1})^{\cdot}}{t^{q_1}}\right)^2 + \left(\frac{(t^{q_2})^{\cdot}}{t^{q_2}}\right)^2 + \left(\frac{(t^{q_3})^{\cdot}}{t^{q_3}}\right)^2 + \left(\frac{(t^{q_4})^{\cdot}}{t^{q_4}}\right)^2\right] - \frac{\theta^2}{8}. \tag{16}$$

We define a deceleration parameter$q$ as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right). \tag{17}$$

3. Solutions of the Field Equations

The field equations (5)-(9) from a system of five equations with eight unknowns parameters $q_1, q_2, q_3, q_4, G, p, \rho$ and Λ .

Now, we assume a power-law form of the gravitational constant *G*.

$$G \propto a^m \text{, i.e. } G = h a^m , \tag{18}$$

where h is the constant of proportionality and$m$ is a positive constant.

Also we assume equation of state for perfect fluid

$$p = \gamma\rho , \tag{19}$$

where $\gamma(0 \le \gamma \le 1)$ is a constant.

Now, subtracting equations (6) from (5), (7) from (6), (8) from (7),and (5) from (8), and solving we get

$$\frac{t^{q_1}}{t^{q_2}} = d_1 \exp\left(k_1 \int a^{-4} dt\right), \tag{20}$$

$$\frac{t^{q_2}}{t^{q_3}} = d_2 \exp\left(k_2 \int a^{-4} dt\right), \tag{21}$$

$$\frac{t^{q_3}}{t^{q_4}} = d_3 \exp\left(k_3 \int a^{-4} dt\right), \tag{22}$$

$$\frac{t^{q_4}}{t^{q_1}} = d_4 \exp\left(k_4 \int a^{-4} dt\right), \tag{23}$$

where d_1 , d_2 , d_3 , d_4 and k_1 , k_2 , k_3 , k_4 are constants of integration.

After solving equations (20)-(23), we get

$$t^{q_1} = r_1 a \exp\left(m_1 \int a^{-4} dt\right), \tag{24}$$

$$t^{q_2} = r_2 a \exp\left(m_2 \int a^{-4} dt\right), \tag{25}$$

$$t^{q_3} = r_3 a \exp\left(m_3 \int a^{-4} dt\right), \tag{26}$$

$$t^{q_4} = r_4 a \exp\left(m_4 \int a^{-4} dt\right), \tag{27}$$

where $r_1 = \left(d_1^2 d_2 d_4\right)^{\frac{1}{4}}$, $r_2 = \left(d_1^{-2} d_2 d_4\right)^{\frac{1}{4}}$, $r_3 = \left(d_1^{-2} d_2^{-3} d_4\right)^{\frac{1}{4}}$, $r_4 = \left(d_1^2 d_4^{-3} d_2\right)^{\frac{1}{4}}$ (28)

and

$$m_1 = \frac{2k_1 + k_2 + k_4}{4}, \quad m_2 = \frac{k_2 + k_4 - 2k_1}{4}, \quad m_3 = -\frac{\left(k_4 - 2k_1 + 3k_2\right)}{4}, \quad m_4 = -\frac{\left(2k_1 + k_2 - 3k_4\right)}{4}. \tag{29}$$

Here r_1 , r_2 , r_3 , r_4 and m_1 , m_2 , m_3 , m_4 satisfy the relation

$$r_1 r_2 r_3 r_4 = 1 \quad , \quad m_1 + m_2 + m_3 + m_4 = 0 . \tag{30}$$

Now we can obtain the metric function as a function of t if mean scale factor is known

Here we consider $a = \sqrt{t^n e^t}$ (31)

If we put $n = 0$ in above equation then $a = \sqrt{e^t}$ i.e. an exponential law of variation of scale factor.This choice of scale factor yield a time-dependent deceleration parameter.

Using equation (31) in equation (24)-(27)), we get

$$t^{q_1} = r_1 \left(t^n e^t\right)^{1/2} \exp\left[m_1 \int \left(t^n e^t\right)^{-2} dt\right], \tag{32}$$

$$t^{q_2} = r_2 \left(t^n e^t\right)^{1/2} \exp\left[m_2 \int \left(t^n e^t\right)^{-2} dt\right], \tag{33}$$

$$t^{q_3} = r_3 \left(t^n e^t\right)^{1/2} \exp\left[m_3 \int \left(t^n e^t\right)^{-2} dt\right], \tag{34}$$

$$t^{q_4} = r_4 \left(t^n e^t\right)^{1/2} \exp\left[m_4 \int \left(t^n e^t\right)^{-2} dt\right], \tag{35}$$

Solving these equations, we obtain

$$t^{q_1} = r_1 \left(t^n e^t\right)^{1/2} \exp\left[m_1 F(t)\right], \tag{36}$$

$$t^{q_2} = r_2 \left(t^n e^t\right)^{1/2} \exp\left[m_2 F(t)\right], \tag{37}$$

$$t^{q_3} = r_3 \left(t^n e^t\right)^{1/2} \exp\left[m_3 F(t)\right], \tag{38}$$

$$t^{q_4} = r_4 \left(t^n e^t\right)^{1/2} \exp\left[m_4 F(t)\right], \tag{39}$$

where $F(t) = \int \left(t^n e^t\right)^{-2} dt = -2^{2n-1}\, 1\text{-}2\overline{|n, 2t}\ .$ (40)

Hence space-time metric reduced to

$$ds^2 = -dt^2 + \left(t^n e^t\right)\left[r_1^2 \exp\left(2m_1 F(t)dx^2\right) + r_2^2 \exp\left(2m_2 F(t)dy^2\right) + r_3^2 \exp\left(2m_3 F(t)dz^2\right) + r_4^2 \exp\left(2m_4 F(t)du^2\right)\right] \tag{41}$$

Using equations (36)-(39), the directional Hubble parameters in the directions of x , y z and u-axes are found to be

$$H_1 = \frac{1}{2}\left(1 + \frac{n}{t}\right) + m_1 \left(t^n e^t\right)^{-2}, \tag{42}$$

$$H_2 = \frac{1}{2}\left(1 + \frac{n}{t}\right) + m_2 \left(t^n e^t\right)^{-2}, \tag{43}$$

$$H_3 = \frac{1}{2}\left(1 + \frac{n}{t}\right) + m_3 \left(t^n e^t\right)^{-2}. \tag{44}$$

$$H_4 = \frac{1}{2}\left(1 + \frac{n}{t}\right) + m_4 \left(t^n e^t\right)^{-2}. \tag{45}$$

The Mean Hubble parameter is given by

$$H = \frac{1}{2}\left(1 + \frac{n}{t}\right). \tag{46}$$

Spatial volume V is given by

$$V = \left(t^n e^t\right)^2. \tag{47}$$

The expansion scalar $\theta = 4H$ is given by

$$\theta = 2\left(1 + \frac{n}{t}\right). \tag{48}$$

The mean anisotropy parameter Δ is given by

$$\Delta = \left(m_1^2 + m_2^2 + m_3^2 + m_4^2\right)\left(1 + \frac{n}{t}\right)^{-2} \left(t^n e^t\right)^{-4}. \tag{49}$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\left(m_1^2 + m_2^2 + m_3^2 + m_4^2\right)\left(t^n e^t\right)^{-4}. \tag{50}$$

The deceleration parameter q is computed as

$$q = \frac{2n}{(n+1)^2} - 1. \tag{51}$$

Using equation (18) and (31), we obtain

$$G = h\left(t^n e^t\right)^{m/2}. \tag{52}$$

The cosmological constant $\Lambda(t)$ is computed as

$$\Lambda = \frac{1}{2}\left(1 + \frac{n}{t}\right)^2 + \frac{1}{(1+\gamma)}\left[\beta_2 \left(t^n e^t\right)^{-4} - nt^{-3}\right], \tag{53}$$

where $\beta_2 = m_1^2 + m_2^2 + m_1 m_2 + \gamma\left(m_1 m_2 + m_2 m_3 + m_3 m_4 + m_4 m_1\right)$.

4. Discussion and Conclusion

i) From the equation (47), it is observed that as $t = 0$, the spatial volume vanishes.

ii) From equation (48), it is observed that, the expansion scalar θ start with infinite value at $t = 0$ and then becomes constant after some finite time.

iii) From equation (49), it is observed that time $t \to \infty$, $\Delta \to 0$. Thus this model has transition from initial anisotropy to isotropy at present epoch.

iv) From the equation (51), we observe that $q > 0$ for $t < \sqrt{2n} - n$ and $q < 0$ for $t > \sqrt{2n} - n$. It is observed that for $0 < n < 2$, this model evolves from deceleration to acceleration phase.

v) From equation (52), we observe that G is an increasing function of time i.e.As $t \to 0$, $G \to 0$ whereas for $t \to \infty$, $G \to \infty$.

vi) It is interesting to note that all the results obtained by Pradhan*et al.* [13] can be reproduced from these results by giving appropriate values to the functions concerned.

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