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Common Fixed Point Results for Hybrid Pairs of Occasionally Weakly Compatible Mappings Defined on b-Metric Space

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Abstract: *The aim of this paper is to obtain some common fixed point theorems for hybrid pairs of single and multi-valued occasionally weakly compatible mappings using a symmetric δ derived from an ordinary symmetric d in b-metric space.*

Keywords: *Occasionally weakly compatible mappings, single and multi—valued maps, common fixed point theorem, b- metric space.*

2000 Mathematics Subject Classification: 47H10; 54H25.

1 INTRODUCTION

The study of fixed point theorems, involving four single-valued maps, began with the assumption that all of the maps are commuted. Sessa [8] weakened the condition of commutativity to that of pairwise weakly commuting. Jungck generalized the notion of weak commutativity to that of pairwise compatible [5] and then pairwise weakly compatible maps [6]. Jungck and Rhoades [7] introduced the concept of occasionally weakly compatible maps.

Abbas and Rhoades [1] generalized the concept of weak compatibility in the setting of single and multi-valued maps by introducing the notion of occasionally weakly compatible (owc).

The concept of $b-metric\,space$ was introduced by Czerwik [3]. Several papers deal with fixed point theory for single and multi- valued maps in $b-metric\,space$.

The aim of this paper is to obtain some common fixed point theorems for owc maps to hybrid pairs of single and multi-valued maps using a symmetric δ derived from an ordinary symmetric d in $b-metric\,space$.

2 PRELIMINARY NOTES

Definition2.1.[2] Let (X,d) denotes a metric space, $x \in X$ and $A \subseteq X, D(x,A) = \inf\{d(x,a): a \in A\}$ and $CB(X)$ is the class of all nonempty closed and bounded subsets of X. For every $A,B \in CB(X)$

$\delta(A,B) = \sup\{d(a,b): a \in A, b \in B\}$.

We appeal to the fact that $\delta(A,B) = 0 \;\; iff \;\; A = B = \{x\}$ for $A,B \in CB(X)$. If $a \in X$, we write $\delta(a,B)$ for $\delta(\{a\},B)$.

Definition2.2.[3] Let X be a nonempty set and $s \geq 1$ a given real number. A function $d: X \times X \longrightarrow R_+$ (nonnegative real numbers) is called a $b-metric$ provided that, for all $x,y,z \in X$,

$(bi)\;\; d(x,y) = 0 \;\; iff \;\; x = y,$

$(bii)\;\; d(x,y) = d(y,x),$

$(biii)\ d(x,z) \leq s[d(x,y) + d(y,z)]$.

The pair (X,d) is called $b-metric\,space$ with parameter s.

It is clear that the definition of $b-metric\,space$ is an extension of usual metric space. Also, if we consider $s=1$ in above definition, then we obtain definition of usual metric space.

Definition2.3.[1] Maps $f: X \longrightarrow X$ and $T: X \longrightarrow CB(X)$ are said to be occasionally weakly compatible (owc) if and only if there exist some point x in X such that $fx \in Tx$ and $fTx \subseteq Tfx$.

3 Main Results

Theorem3.1 Let (X,d) be a b-metric. Let $f,g: X \longrightarrow X$ and $F,G: X \longrightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{g,G\}$ are owc and satisfy inequality

$$\delta(Fx,Gy) \leq max\left\{d(fx,gy), \delta(gy,Fx)\left[\frac{1+\delta(fx,Fx)}{1+\delta(gy,Gy)}\right], \delta(fx,Gy)\left[\frac{1+\delta(gy,Gy)}{1+\delta(fx,Fx)}\right]\right\} \tag{3.1}$$

for all $x,y \in X$. Then $f,g,F\ \&\ G$ have a unique common fixed point in X.

Proof: By hypothesis there exist points $x,y \in X$ such that $fx \in Fx, fFx \subseteq Ffx$ and $gy \in Gy, gGy \subseteq Ggy$.

As $fx \in Fx$ so $ffx \subseteq fFx \subseteq Ffx$, $gy \in Gy$ so $ggy \subseteq gGy \subseteq Ggy$ and hence $d(f^2x, g^2y) \leq \delta(Ffx, Ggy)$. First we show that $gy = fx$. Suppose not. Then condition (3.1) implies that

$$\begin{aligned}
&\delta(Ffx,Ggy)\\
&\leq max\left\{d(ffx,ggy), \delta(ggy,Ffx)\left[\frac{1+\delta(ffx,Ffx)}{1+\delta(ggy,Ggy)}\right], \delta(ffx,Ggy)\left[\frac{1+\delta(ggy,Ggy)}{1+\delta(ffx,Ffx)}\right]\right\}\\
&\quad \leq max\left\{d(f^2x,g^2y), \delta(g^2y,Ffx)\left[\frac{1+\delta(ffx,Ffx)}{1+\delta(ggy,Ggy)}\right], \delta(ffx,Ggy)\left[\frac{1+\delta(ggy,Ggy)}{1+\delta(ffx,Ffx)}\right]\right\}\\
&\quad \leq max\{d(f^2x,g^2y), \delta(g^2y,Ffx), \delta(f^2x,Ggy)\}\\
&\quad \leq max\{d(f^2x,g^2y), \delta(Ffx,Ggy), \delta(Ffx,Ggy)\}\\
&\quad = \delta(Ffx,Ggy),
\end{aligned}$$

a contradiction, and hence $gy = fx$. Obviously $d(fx, g^2y) \leq \delta(Fx, Gfx)$. Next we claim that $x = fx$. If not, then condition (3.1) implies that

$$\begin{aligned}
\delta(Fx,Gfx) &\leq max\left\{d(fx,gfx), \delta(gfx,Fx)\left[\frac{1+\delta(fx,Fx)}{1+\delta(gfx,Gfx)}\right], \delta(fx,Gy)\left[\frac{1+\delta(gfx,Gfx)}{1+\delta(fx,Fx)}\right]\right\}\\
&\leq max\left\{d(fx,ggy), \delta(ggy,Fx)\left[\frac{1+\delta(fx,Fx)}{1+\delta(ggy,Ggy)}\right], \delta(gy,Gy)\left[\frac{1+\delta(ggy,Ggy)}{1+\delta(fx,Fx)}\right]\right\}\\
&\leq max\{d(fx,g^2y), \delta(Ggy,Fx), 0\}\\
&\leq max\{\delta(Fx,Gfx), \delta(Gfx,Fx), 0\}\\
&= \delta(Fx,Gfx),
\end{aligned}$$

which is again a contradiction. Similarly we can prove $y = gy$. Thus f,g,F,G have a common fixed point. Uniqueness follows from (3.1).

Theorem3.2 Let (X,d) be a b-metric. Let $f,g: X \longrightarrow X$ and $F,G: X \longrightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{g,G\}$ are owc and satisfy inequality

$$\delta(Fx,Gy) \le max\left\{d(fx,gy),\left(1-\frac{\delta(gy,Gy)+\delta(fx,Gy)}{\delta(fx,Fx)+\delta(gy,Fx)}\right)\right\} \tag{3.2}$$

for all $x, y \in X$. Then f, g, F & G have a unique common fixed point in X.

Proof: By hypothesis there exist points $x, y \in X$ such that $fx \in Fx, fFx \subseteq Ffx$ and $gy \in Gy, gGy \subseteq Ggy$.

As $fx \in Fx$ so $ffx \subseteq fFx \subseteq Ffx$, $gy \in Gy$ so $ggy \subseteq gGy \subseteq Ggy$ and hence $d(f^2x, g^2y) \le \delta(Ffx, Ggy)$. First we show that $gy = fx$. Suppose not. Then condition (3.2) implies that

$$\delta(Ffx,Ggy) \le max\left\{d(ffx,ggy),\left(1-\frac{\delta(ggy,Ggy)+\delta(ffx,Ggy)}{\delta(ffx,Ffx)+\delta(ggy,Ffx)}\right)\right\}$$

$$\le max\left\{d(f^2x,g^2y),\left(1-\frac{\delta(ggy,Ggy)+\delta(f^2x,Ggy)}{\delta(ffx,Ffx)+\delta(g^2y,Ffx)}\right)\right\}$$

$$\le max\left\{\delta(Ffx,Ggy),\left(1-\frac{\delta(Ffx,Ggy)}{\delta(Ggy,Ffx)}\right)\right\}$$

$$= \delta(Ffx,Ggy),$$

a contradiction, and hence $gy = fx$. Obviously $d(fx, g^2y) \le \delta(Fx, Gfx)$. Next we claim that $x = fx$. If not, then condition (3.2) implies that

$$\delta(Fx,Gfx) \le max\left\{d(fx,gfx),\left(1-\frac{\delta(gfx,Gfx)+\delta(fx,Gfx)}{\delta(fx,Fx)+\delta(gfx,Fx)}\right)\right\}$$

$$\le max\left\{d(fx,ggy),\left(1-\frac{\delta(ggy,Ggy)+\delta(gy,Ggy)}{\delta(fx,Fx)+\delta(ggy,Fx)}\right)\right\}$$

$$\le max\left\{d(fx,g^2y),\left(1-\frac{\delta(g^2y,Ggy)+\delta(gy,Ggy)}{\delta(fx,Fx)+\delta(ggy,Fx)}\right)\right\}$$

$$\le max\left\{\delta(Fx,Gfx),\left(1-\frac{\delta(fx,Gfx)}{\delta(gfx,Fx)}\right)\right\}$$

$$\le max\left\{\delta(Fx,Gfx),\left(1-\frac{\delta(Fx,Gfx)}{\delta(Gfx,Fx)}\right)\right\}$$

$$= \delta(Fx,Gfx),$$

which is again a contradiction. Similarly we can prove $y = gy$. Thus f, g, F, G have a common fixed point. Uniqueness follows from (3.2).

Theorem3.3 Let (X, d) be a b-metric. Let $f, g: X \longrightarrow X$ and $F, G: X \longrightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f, F\}$ and $\{g, G\}$ are owc and satisfy inequality

$$\delta(Fx,Gy) \le \alpha d(fx,gy) + \beta\delta(fx,Gy) + \gamma\delta(gy,Fx) \tag{3.3}$$

for all $x, y \in X, \alpha, \beta, \gamma > 0$ & $(\alpha + \beta + \gamma) = 1$.Then f, g, F & G have a unique common fixed point in X.

Proof: By hypothesis there exist points $x, y \in X$ such that $fx \in Fx, fFx \subseteq Ffx$ and $gy \in Gy, gGy \subseteq Ggy$.

As $fx \in Fx$ so $ffx \subseteq fFx \subseteq Ffx$, $gy \in Gy$ so $ggy \subseteq gGy \subseteq Ggy$ and hence $d(f^2x, g^2y) \le \delta(Ffx, Ggy)$. First we show that $gy = fx$. Suppose not. Then condition (3.3) implies that

$$\delta(Ffx,Ggy) \le \alpha d(f^2x,g^2y) + \beta\delta(ffx,Ggy) + \gamma\delta(ggy,Ffx)$$

$$\leq \alpha\delta(Ffx, Ggy) + \beta\delta(Ffx, Ggy) + \gamma\delta(Ggy, Ffx)$$
$$\leq (\alpha + \beta + \gamma)\delta(Ffx, Ggy)$$
$$= \delta(Ffx, Ggy),$$

a contradiction, and hence $gy = fx$. Obviously $d(fx, g^2y) \leq \delta(Fx, Gfx)$. Next we claim that $x = fx$. If not, then condition (3.3) implies that

$$\delta(Fx, Gfx) \leq \alpha d(fx, gfx) + \beta\delta(fx, Gfx) + \gamma\delta(gfx, Fx)$$
$$\leq \alpha d(fx, ggy) + \beta\delta(Fx, Gfx) + \gamma\delta(ggy, Fx)$$
$$\leq \alpha d(fx, g^2y) + \beta\delta(Fx, Gfx) + \gamma\delta(Ggy, Fx)$$
$$\leq \alpha\delta(Fx, Gfx) + \beta\delta(Fx, Gfx) + \gamma\delta(Gfx, Fx)$$
$$\leq (\alpha + \beta + \gamma)\delta(Fx, Gfx)$$
$$= \delta(Fx, Gfx)$$

which is again a contradiction. Similarly we can prove $y = gy$. Thus f, g, F, G have a common fixed point. Uniqueness follows from (3.3).

Theorem3.4 Let (X, d) be a b-metric. Let $f, g: X \to X$ and $F, G: X \to CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f, F\}$ and $\{g, G\}$ are owc and satisfy inequality

$$\delta(Fx, Gy) \leq \alpha d(fx, gy) + (1-\alpha)\left[\frac{\delta(fx,Fx)+\delta(gy,Gy)+\delta(fx,Gy)+\delta(gy,Fx)}{2}\right] \tag{3.4}$$

for all $x, y \in X$ & $\alpha \in [0,1)$. Then f, g, F & G have a unique common fixed point in X.

Proof: By hypothesis there exist points $x, y \in X$ such that $fx \in Fx, fFx \subseteq Ffx$ and $gy \in Gy, gGy \subseteq Ggy$.

As $fx \in Fx$ so $ffx \subseteq fFx \subseteq Ffx$, $gy \in Gy$ so $ggy \subseteq gGy \subseteq Ggy$ and hence $d(f^2x, g^2y) \leq \delta(Ffx, Ggy)$. First we show that $gy = fx$. Suppose not. Then condition (3.4) implies that

$$\delta(Ffx, Ggy) \leq \alpha d(ffx, ggy) + (1-\alpha)\left[\frac{\delta(ffx, Ffx) + \delta(ggy, Ggy) + \delta(ffx, Ggy) + \delta(ggy, Ffx)}{2}\right]$$
$$\leq \alpha d(f^2x, g^2y) + (1-\alpha)\left[\frac{\delta(f^2x,Ffx)+\delta(g^2y,Ggy)+\delta(Ffx,Ggy)+\delta(Ggy,Ffx)}{2}\right]$$
$$= \delta(Ffx, Ggy),$$

a contradiction, and hence $gy = fx$. Obviously $d(fx, g^2y) \leq \delta(Fx, Gfx)$. Next we claim that $x = fx$. If not, then condition (3.4) implies that

$$\delta(Fx, Gfx) \leq \alpha d(fx, gfx) + (1-\alpha)\left[\frac{\delta(fx, Fx) + \delta(gfx, Gfx) + \delta(fx, Gfx) + \delta(gfx, Fx)}{2}\right]$$
$$= \delta(Fx, Gfx),$$

which is again a contradiction. Similarly we can prove $y = gy$. Thus f, g, F, G have a common fixed point. Uniqueness follows from (3.4).

Corollary3.5 Let (X, d) be a b-metric. Let $f, g: X \to X$ and $F, G: X \to CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f, F\}$ and $\{g, G\}$ are owc and satisfy inequality

$$\delta(Fx,Gy) \le \alpha\left[\frac{d(fx,gy)+\delta(fx,Gy)\delta(fx,Fx)+\delta(gy,Fx)\delta(gy,Gy)}{1+\delta(fx,Fx)+\delta(gy,Gy)}\right] \tag{3.5}$$

for all $x,y \in X$ & $\alpha \in [0,1)$. Then f,g,F & G have a unique common fixed point in X.

Theorem3.6 Let (X,d) be a b-metric. Let $f,g: X \rightarrow X$ and $F,G: X \rightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{g,G\}$ are owc and satisfy inequality

$$\delta(Fx,Gy) \le max\left\{d(fx,gy),\delta(fx,Fx),\delta(gy,Gy),\frac{1}{2}[\delta(fx,Gy)+\delta(gy,Fx)]\right\} \tag{3.6}$$

for all $x,y \in X$. Then f,g,F & G have a unique common fixed point in X.

Proof: By hypothesis there exist points $x,y \in X$ such that $fx \in Fx, fFx \subseteq Ffx$ and $gy \in Gy, gGy \subseteq Ggy$.

As $fx \in Fx$ so $ffx \subseteq fFx \subseteq Ffx$, $gy \in Gy$ so $ggy \subseteq gGy \subseteq Ggy$ and hence $d(f^2x,g^2y) \le \delta(Ffx,Ggy)$. First we show that $gy = fx$. Suppose not. Then condition (3.6) implies that

$$\delta(Ffx,Ggy) \le max\left\{d(f^2x,g^2y),\delta(f^2x,Ffx),\delta(g^2y,Ggy),\frac{1}{2}[\delta(f^2x,Ggy)+\delta(g^2y,Ffx)]\right\}$$
$$\le max\{\delta(Ffx,Ggy),0,0,\delta(Ffx,Ggy)\}$$
$$= \delta(Ffx,Ggy),$$

a contradiction, and hence $gy = fx$. Obviously $d(fx,g^2y) \le \delta(Fx,Gfx)$. Next we claim that $x = fx$. If not, then condition (3.6) implies that

$$\delta(Fx,Gfx) \le max\left\{d(fx,gfx),\delta(fx,Fx),\delta(gfx,Gfx),\frac{1}{2}[\delta(fx,Gfx)+\delta(gfx,Fx)]\right\}$$
$$\le max\left\{d(fx,g^2y),\delta(fx,Fx),\delta(gfx,Gfx),\frac{1}{2}[\delta(gy,Ggy)+\delta(g^2y,Fx)]\right\}$$
$$\le max\left\{\delta(Fx,Gfx),0,0,\frac{1}{2}[\delta(fx,Gfx)+\delta(gfx,Fx)]\right\}$$
$$\le max\left\{\delta(Fx,Gfx),0,0,\frac{1}{2}[\delta(fx,Gfx)+\delta(Gfx,Fx)]\right\}$$
$$= \delta(Fx,Gfx),$$

which is again a contradiction. Similarly we can prove $y = gy$. Thus f,g,F,G have a common fixed point. Uniqueness follows from (3.6).

Corollary3.7 Let (X,d) be a b-metric. Let $f,g: X \rightarrow X$ and $F,G: X \rightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{g,G\}$ are owc and satisfy inequality

$$\delta(Fx,Gy) \le max\{d(fx,gy),\delta(fx,Fx),\delta(fx,Gy),\delta(gy,Gy),\delta(gy,Fx)]\} \tag{3.7}$$

for all $x,y \in X$. Then f,g,F & G have a unique common fixed point in X.

Proof: Clearly the result immediately follows from Theorem 3.1.

Corollary3.8 Let (X,d) be a b-metric. Let $f: X \rightarrow X$ and $F: X \rightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{g,G\}$ are owc and satisfy inequality

$$\delta(Fx,Gy) \le max\left\{d(fx,gy),\frac{1}{2}[\delta(fx,Fx)+\delta(fy+Fy)],\frac{1}{2}[\delta(fx,Fx)+\delta(fx,Fy)]\right\} \tag{3.8}$$

for all $x,y \in X$. Then f & F have a unique common fixed point in X.

Theorem3.9 Let (X,d) be a b-metric space. Let $f,g:X\longrightarrow X$ and $F,G:X\longrightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{g,G\}$ are owc and satisfy inequality

$$\delta^p(Fx,Gy)\le max\{ad(fx,gy)\delta^{p-1}(fx,Fx),ad(fx,gy)\delta^{p-1}(gy,Gy),a\delta(fx,Fx)\delta^{p-1}(gy,Gy),c\delta^{p-1}(fx,Gy)+\delta(gy,Fx)]\} \tag{3.9}$$

for all $x,y\in X$, with $a\ge 0, 0<c<1$ and $p\ge 2$. Then f,g,F & G have a unique common fixed point in X.

Proof: By hypothesis there exist points $x,y\in X$ such that $fx\in Fx, fFx\subseteq Ffx$ and $gy\in Gy, gGy\subseteq Ggy$.

As $fx\in Fx$ so $ffx\subseteq fFx\subseteq Ffx$, $gy\in Gy$ so $ggy\subseteq gGy\subseteq Ggy$ and hence $d(f^2x,g^2y)\le\delta(Ffx,Ggy)$. First we show that $gy=fx$. Suppose not. Then condition (3.9) implies that

$$\delta^p(Ffx,Ggy)$$
$$\le max\{ad(ffx,ggy)\delta^{p-1}(ffx,Ffx),ad(ffx,ggy)\delta^{p-1}(ggy,Ggy),a\delta(ffx,Ffx)\delta^{p-1}(ggy,Ggy),c\delta^{p-1}(ffx,Ggy)$$
$$+\delta(ggy,Ffx)]\}$$
$$\le\delta^p(Ffx,Ggy),$$

a contradiction, and hence $gy=fx$. Obviously $d(fx,g^2y)\le\delta(Fx,Gfx)$. Next we claim that $x=fx$. If not, then condition (3.9) implies that

$$\delta^p(Fx,Ggy)$$
$$\le max\{ad(fx,ggy)\delta^{p-1}(fx,Fx),ad(fx,ggy)\delta^{p-1}(ggy,Ggy),a\delta(fx,Fx)\delta^{p-1}(ggy,Ggy),c\delta^{p-1}(fx,Ggy)$$
$$+\delta(ggy,Fx)]\}$$
$$\le\delta^p(Fx,Ggy),$$

which is again a contradiction. Similarly we can prove $y=gy$. Thus f,g,F,G have a common fixed point. Uniqueness follows from (3.9).

Corollary3.10 Let (X,d) be a b-metric space. Let $f:X\longrightarrow X$ and $F:X\longrightarrow CB(X)$ be single and multi-valued maps, respectively such that the pair $\{f,F\}$ is owc and satisfy inequality

$$\delta^p(Fx,Fy)\le max\{ad(fx,fy)\delta^{p-1}(fx,Fx),ad(fx,fy)\delta^{p-1}(fy,Fy),a\delta(fx,Fx)\delta^{p-1}(fy,Fy),c\delta^{p-1}(fx,Fy)+\delta(fy,Fx)]\} \tag{3.10}$$

for all $x,y\in X$, with $a\ge 0, 0<c<1$ and $p\ge 2$. Then f,g,F & G have a unique common fixed point in X.

If we put in above Theorem $f=g$ and $F=G$, we obtain the following result.

Corollary3.11 Let (X,d) be a b-metric space with parameter $s\ge 1$. Let $f:X\longrightarrow X$ and $F,G:X\longrightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{f,G\}$ are owc and satisfy inequality

$$\delta^p(Fx,Gy)\le max\{ad(fx,fy)\delta^{p-1}(fx,Fx),ad(fx,fy)\delta^{p-1}(fy,Gy),a\delta(fx,Fx)\delta^{p-1}(fy,Gy),c\delta^{p-1}(fx,Gy)+\delta(fy,Fx)]\} \tag{3.11}$$

for all $x,y\in X$, with $a\ge 0, 0<c<1$ and $p\ge 2$. Then f,g,F & G have a unique common fixed point in X.

Now, letting $f=g$ we get the next corollary.

Corollary3.12 Let (X,d) be a b-metric space with parameter $s \geq 1$. Let $f,g:X \longrightarrow X$ and $F,G:X \longrightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{g,G\}$ are owc and satisfy inequality

$$\delta^p(Fx,Gy) \leq \left[ad^p(fx,gy) + (1-\alpha)\max\{\delta^p(fx,gy),\delta^p(gy,Gy),\delta^{\frac{p}{2}}(gy,Fx).\delta^{\frac{p}{2}}(fx,Gy)\}\right] \tag{3.12}$$

for all $x,y \in X$, with $a \geq 0, 0 < c < 1$ and $p \geq 2$. Then f,g,F & G have a unique common fixed point in X.

We can also prove the result with contractive modulus i.e.,

A function $\varphi:[0,\infty) \to [0,\infty)$ is said to have a contractive modulus if $\varphi(0) = 0$ & $\varphi(t) < t$ for $t > 0$.

Corollary3.13 Let (X,d) be a b-metric. Let $f,g:X \longrightarrow X$ and $F,G:X \longrightarrow CB(X)$ be single and multi-valued maps, respectively such that the pairs $\{f,F\}$ and $\{g,G\}$ are owc and satisfy inequality

$$\delta(Fx,Gy) \leq max\{\varphi(d(fx,gy)),\varphi(\delta(fx,Fx)),\varphi(\delta(fx,Gy)),\varphi(\delta(gy,Gy)),\varphi(\delta(gy,Fx))\} \tag{3.13}$$

for all $x,y \in X$. Then f,g,F & G have a unique common fixed point in X.

4 Conclusion

We prove some common fixed point results for occasionally weakly compatible mappings in hybrid pairs of single and multi-valued maps using a symmetric δ derived from an ordinary symmetric d in $b-metric\ space$.Our result generalize the result of various authors present in $b-metric\ space$.

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