International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 4, April 2014, PP 378-380
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

The Lorentz Force

Khalid Ahmed Zakaria
Faculty of Science, Department of Mathematics
Uuniversity Of Kordofan, El Obeid, Sudan
khalid4444@live.com

Norsarahaida S. Amin
Faculty of Science, Department of Mathematics
Universiti Teknologi Malaysiam, Malaysia
norsarahaida@utm.my

Abstract: *In this article, magnetohydrodynamics (MHD or Lorentz force is studied. The equations of MHD are derived in both case of the cylindrical coordinate and Cartesian coordinates. The Lorentz force is obtained in two directional flows and unidirectional flow.*

Keywords: *Magnetohydrodynamic force; Lorentz force.*

1. Introduction

Magnetohydrodynamics (MHD) is the study of the behavior of an electrically conducting fluid in the presence of a magnetic field.

The equations of MHD are the usual electromagnetic and hydrodynamics equations, modified to take account of the interaction between the motion and the magnetic field

2. Derivation of mhd Equations

The MHD body force or (Lorentz Force) is given in [1, 2, 3] by the following equation:

$$\rho\mathbf{f} = \mathbf{J}\times\mathbf{B}, \tag{1}$$

As in most of the problems involving conductors Maxwell's displacement currents are neglected, which is a valid approximation for non-relativistic phenomena typical of the response of an inertial liquid, so that the electric currents are regarded as flowing in closed circuits. Hence, the Maxwell's equations take the form:

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t},\ \nabla\cdot\mathbf{B} = 0,\ \nabla\times\mathbf{B} = \mu\mathbf{J},\ \nabla\cdot\mathbf{J} = 0, \tag{2}$$

Where, the generalized Ohm's law is:

$$\mathbf{J} = \sigma(\mathbf{E}\mp\mathbf{V}\times\mathbf{B}), \tag{3}$$

In the above equations, $\mathbf{E}$, $\mathbf{J}$, μ , σ and $\mathbf{B}$ are the electric field intensity, current density, the magnetic permeability, the electrical conductivity of the fluid and magnetic induction, respectively.

The total magnetic field $\mathbf{B}$ is $\mathbf{B} = \mathbf{B_0} + \mathbf{b}$, where $\mathbf{b}$ indicates the induced magnetic field as in [1].

It is assumed in [1, 2, 4] that:

- That there is no applied or polarization voltage so that $\mathbf{E} = 0$.
- The mechanical force $\mathbf{J}\times\mathbf{B}$ of electromagnetic origin is perpendicular to the magnetic field; it has no direct influence on the motion parallel to the field. [1].
- The magnetic field $\mathbf{B}$ is perpendicular to the velocity field $\mathbf{V}$ and the induced magnetic field $\mathbf{b}$ is negligible compared with the imposed field $\mathbf{B_0}$ so that the magnetic Reynolds number is small.

- The quantities σ, μ and ρ are so small throughout the flow field.

Under the above assumption, the Lorentz force (1) can be written in the following form:

$$\rho\mathbf{f} = \sigma(\mathbf{V}\times\mathbf{B})\times\mathbf{B} \tag{3}$$

Recall the following vector rule:

$$\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A} \tag{4}$$

Applying Equation (4) into Equation (3) results the following Equation:

$$\rho\mathbf{f} = -\sigma[\mathbf{B}\times(\mathbf{V}\times\mathbf{B})] \tag{5}$$

Also, recall the following vector relation:

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}.\mathbf{B}) \tag{6}$$

By invoking relation (6) into Equation (5), we can get the following Equation:

$$\rho\mathbf{f} = -\sigma[\mathbf{V}(\mathbf{B}.\mathbf{B}) - \mathbf{B}(\mathbf{B}.\mathbf{V})] \tag{7}$$

By assuming $\mathbf{B_0} = (0,0,B_0,)$ and $\mathbf{V} = (u,v,0)$, then Equation (7) reduces to the following Equation:

$$\rho\mathbf{f} = -\sigma B_0{}^2\mathbf{V} \tag{8}$$

Finally, Equation (8) can be written in the following form:

$$\rho\mathbf{f} = (-\sigma u B_0{}^2, -\sigma v B_0{}^2, 0) \tag{9}$$

Expression (9) is MHD body force or (The Lorentz Force) in two directional flows. In unidirectional flow where the velocity in the form $\mathbf{V} = (u,0,0)$, the MHD body force takes the following form:

$$\rho\mathbf{f} = (-\sigma u B_0{}^2, 0, 0) \tag{10}$$

Remark 1: Expression (9) is the same as the result obtained in [4, 5, 6].

For MHD flow in the cylindrical coordinates, we assume the velocity in the form:

$$\mathbf{V} = (U,0,W) \tag{11}$$

Where U and W are the velocity components in the radial and axial directions, respectively.

Consider the uniform vertical magnetic field $\mathbf{B_0}$ is applied in the radial direction, i.e. $\mathbf{B_0}$ can be written in the following form:

$$\mathbf{B_0} = (B_0,0,0) \tag{12}$$

Substituting expressions (11) and (12) into Equation (7) gives the following expression:

$$\rho\mathbf{f} = (0,0,-\sigma B_0{}^2 W) \tag{13}$$

Equation (13) is an MHD force (The Lorentz Force) in the cylindrical coordinates.

Remark 2: Equation (13) shows that a uniform magnetic field B_0 is applied in the transverse direction to the flow in the cylindrical coordinates. Equation (14) is the same as the result obtained in [7]

3. Conclusion

In the present study, The equation of magnetohydrodynamic (MHD) has been derived. The expressions of MHD equation are obtained for the cylindrical coordinate and Cartesian coordinates. The Lorentz force has been discussed for two directional flows as well as for unidirectional flow.

REFERENCES

[1] Cowling, T. G. Magnetohydrodynamics. New York: Interscience Publ.(1957).

[2] Wang, Y., Ali, N. and Hayat, T. Peristaltic motion of a magnetohydrodynamic generalized second-order fluid in an asymmetric channel. Numerical Numer Meth. Part. D .E, 27(2): 415-435 (2011).

[3] Romig, M. F. The Influence of Electric and Magnetic Fields on Heat Transfer to Electrically Conducting Fluids. In: Irvine, T. F. and Hartnett, J. P., eds. Adv. Heat Transfer. Elsevier. 1.52(267-354). (1964).

[4] Hayat, T., Javed, M. and Ali, N. MHD Peristaltic Transport of a Jeffery Fluid in a Channel With Compliant Walls and Porous Space. Transport Porous Med. 74: 259-274 (2008).

[5] Khan, M., Hayat, T. and Asghar, S. Exact solution for MHD flow of a generalized Oldroyd-B fluid with modified Darcy's law. Int. J. Eng. Sci.,. 44(56): 333 – 339 (2006)...

[6] Zakaria, K. A. and Amin, N. S. Peristaltic flow of a Magnetohydrodynamic Oldroyd-B fluid in an Asymmetric Channel. Int J Appl. Math. Mech.. 8: 18-37 , (2012)..

[7] Zakaria, K. A. and Amin, N. S. MHD Peristaltic Motion of a Maxwell Fluid in Axisymmetric Tube. J. Phys. Soc. Jpn..81(3): 034402 (2012).

AUTHOR'S BIOGRAPHY

Khalid Ahmed Zakaria obtained his B.Sc in Mathematics with distinction in 1987 from University of Khartoum, Sudan and his M.sc in Mathematics (First Class) in 2006 from University of Gazira, Sudan. He obtained his Ph.D in Mathematics at Universiti Teknologi Malaysia (UTM), Skudai, Malaysia.. He got best PHD student award based on excellent academic achievement from Universiti Teknologi Malaysia. His research interests are but not limited to:

1. Fluid Mechanics, MHD flow
2. Peristaltic motion, Physiological fluid.

Norsarahaida Saidina Amin obtained her B.Sc in Mathematics from University of Adelaide, Australia, in 1983. In 1984, she obtained her M.S in Applied Mathematics from Northwestern University, USA. She obtained her PhD in Applied Mathematics from University of East Anglia, England in 1989. Currently, she is the deputy dean Development Affairs at the Faculty of Science, Universiti Teknologi Malaysia (UTM), Skudai, Malaysia.. Her research interests include among others:

- Fluid Mechanics and Heat Transfer Biofluid Dynamics, Unsteady Boundary Layers, Low-gravity Transport Phenomena
- Mathematics Education for Engineers