International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 4, April 2014, PP 408-417
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Interval Edge Coloring of Gird Graphsand Equitable Interval Edge Coloring of Gird of Diamonds and Prism Graphs $Y_n, n > 2$

S.Sudha
Professor
Ramanujan Institute for Advanced Study in Mathematics
University of Madras
Chennai, India
subramaniansudha50@gmail.com

G.M.Raja
Ph.DResearch scholar
Ramanujan Institute for Advanced Study in Mathematics
University of Madras
Chennai, India
rajagopalmalliga@gmail.com

Abstract: *An arbitrary non-empty finite subset of consecutive integers is called an interval;an interval with a minimum element aand a maximum element bis defined by* $[a, b]$.The function$f: E(G) \to [1, k]$ *is called an interval edge coloring of a graph* G*, if all the k colors is used so that no two adjacent edges receive the same color and the set of colors defined on the edges incident to any vertex of* G *forms an interval. An interval edge coloring of a graph* G*is said to be equitable interval edge coloring if any two color classes of a graph* G*differ by at most one.Interval edge coloring of a grid graph* $G_{m,n}$*and equitable interval edge coloring gird of diamonds* $D_{m,n}$*, and prism graph* Y_n *are found in this paper.*

Keywords: *interval edge coloring, equitable interval edge coloring, gird graph, grid of diamonds, prism graph*

AMS Subject Classification: 05C15

1. INTRODUCTION

Kamalian [1] has discussed about the interval edge coloring of simple cycles in a cyclic way. He [2] has also obtained the results on one-sided interval edge coloring of bipartite graphs. Asratian and Kamalian [3, 4] have explained about the interval colorings of edges of a multigraph and they have also investigated the interval edge coloring of graphs. Kamalian [5] has obtained the interval coloring of complete bipartite graphs and trees.Kamalian [6] has also found the result on cyclically interval edge coloring of trees. In 2007 Gallian [7] gave the construction of the prism Y_n^m by considering the cartesian product of the cycle C_m and the pathP_n. In this paper, we have considered the prism graphs for Y_n^2 or simply written by Y_n by leaving 2. Lih [8] explained equitable coloring of graphs.The grid of diamonds graph was introduced by Sudha et al [9].Sudha et al [10] have found the equitable coloring of prisms and generalized Petersen graphs. In this paper, we have defined equitable interval edge coloring. We have also obtained the interval edge coloring of grid graphs $G_{m,n}$ for all $m > 1$ and $n > 1$ and equitable interval edge coloring of grid of diamonds graph $D_{m,n}$ for all m and $n$and prism graphs Y_n for all $n > 2$.

Definition 1. The cartesian product of two paths P_m and P_n is said to be the grid graph and is denoted by $G_{m,n}$.

Illustration 1.

The grid graph $G_{4,4}$ with the vertex set $V = \{V_{ij} \,/\, 1 \le i \le 4,\ 1 \le j \le 4\}$ and the edge set $E = \{v_{ij}v_{i+1j}\,, v_{ij}v_{ij+1} \,/\, v_iv_{i+1} \in E(P_4)\ and\ v_jv_{j+1} \in E(P_4)\}$ as shown in fig-1.

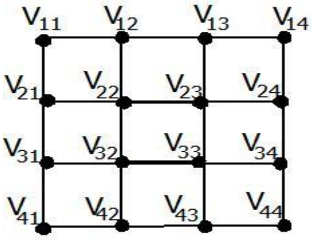

Figure 1

Deffinition 2. The grid of diamonds is the product of the paths P_{2m+1} and P_{2n+1} is denoted by $D_{m,n}$ and is defined with the vertex set

$V(P_{2m+1} \times P_{2n+1}) = \{(u_i, v_j) \;/\; if\ i + j\ is\ odd\}$and the edge set

$$E(P_{2m+1} \times P_{2n+1}) = \{(u_i, v_j)(u_k, v_l) \;/\; u_i u_k \in E(P_{2m+1}) and\ v_j v_l \in E(P_{2n+1})\}$$

where$u_i's$ for $1 \le i \le m$ are the vertices of the pathP_{2m+1} and $v_j's$ for $1 \le j \le m$ are the vertices of the pathP_{2n+1}. The grid of diamonds $D_{m,n}$ consists of $2mn + m + n$ vertices and 4mn edges.

Illustration 2.

The grid of diamonds $D_{3,3}$ is the product of the path P_7 with the path P_7 by and is as shown in fig-2.

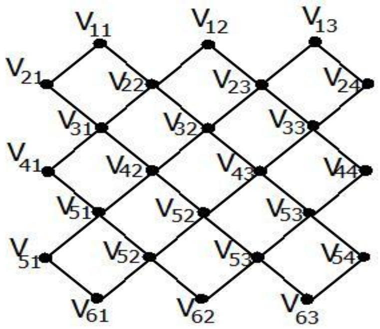

Figure 2.

Definition 3. The cartesian product of the cycle C_n and the path P_2 is said to be a prism graph and is denoted by Y_n.

Illutration 3.

Consider the prism graph Y_5 as per the definition 3, Y_5 is the cartesian product of cycle C_5 with the path P_2 as shown in fig-3.

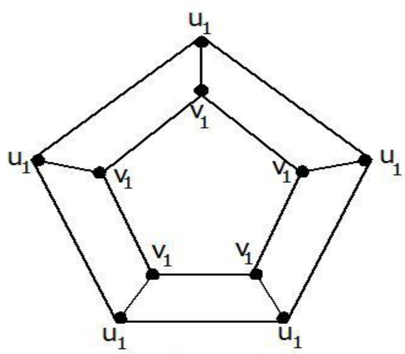

Figure 3.

Definition 4. An edge-coloring of a graph is the coloring of edge of the graph with the minimum number of colors without any two adjacent edges having the same color.

Definition 5. In edge-coloring of a graph the set of edges of same color are said to be in that color class. In k-edge coloring of a graph, there are k color classes. The color classes are represented by $C[1], C[2]$, ..., where 1,2, ... denote the colors.

Definition 6. An edge-coloring of a graph G with colors $1,2, \dots k$ is called an interval k-coloring if all the colors are used so that the colors of the edges incident to any vertex of G are distinct and are consecutive.

The smallest integer k for which the graph G is k-interval edge coloring is known as the chromatic number of interval edge coloring of G and is denoted by $\chi_{ie}(G)$.

Definition 7. A k-interval edge coloring of a graph is said to be equitable k-interval edge coloring if its edge set E is partitioned into k subsets $E_1, E_2, \dots E_k$ such that each E_i is an independent set and the condition $-1 \leq \mid E_i \mid - \mid E_j \mid \leq 1$ holds for all $1 \leq i \leq k$and $1 \leq j \leq k$.

The smallest integer k for which G is equitable interval edge coloring is known as the equitable chromatic number of interval edge coloring of G and is denoted by $\chi_{eie}(G)$.

Theorem 1. The grid graph $G_{m,n} m > 1, n > 1$ admits the interval edge coloring and $\chi_{ie}(G_{m,n}) = \Delta$.

Proof. The grid graph $G_{m,n} m > 1, n > 1$ is a cartesian product of the path P_m with the path P_n. We represent the vertices of the grid graph $G_{m,n}$ by v_{ij}, for $1 \leq i \leq m, 1 \leq j \leq n$ as shown in fig-4.

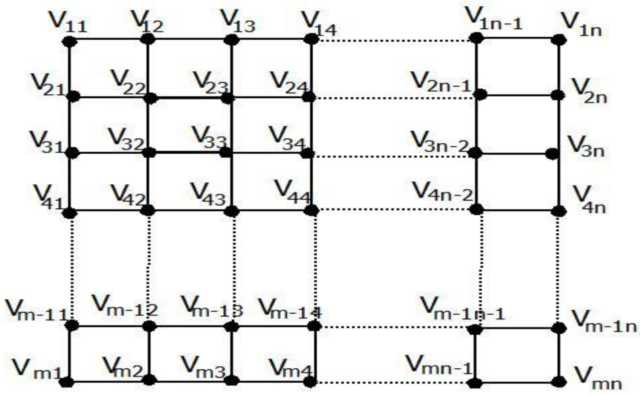

Figure 4

The maximum degree of the vertex in $G_{m,n}$is 4. Therefore by the definition of edge-coloring we require minimum 4 colors.

The function f is defined as the coloring from the edges of the grid graph $G_{m,n}$ to the set of colors (positive integers) $\{1,2,3,\dots\}$ as follows :

Case (i) : Let m and n be odd

for$1 \leq i \leq m-1,\ 1 \leq j \leq n-1$

$$f(v_{ij}v_{ij+1}) = \begin{cases} 3, & j \equiv 1 (mod\ 2) \\ 1, & j \equiv 0 (mod\ 2) \end{cases}$$

$$f(v_{mj}v_{mj+1}) = \begin{cases} 3, & j \equiv 1 (mod\ 2) \\ 2, & j \equiv 0 (mod\ 2) \end{cases}$$

$$f(v_{ij}v_{i+1j}) = \begin{cases} 2, & i \equiv 1 (mod\ 2) \\ 4, & i \equiv 0 (mod\ 2) \end{cases}$$

$$f(v_{in}v_{i+1n}) = \begin{cases} 2, & i \equiv 1 (mod\ 2) \\ 3, & i \equiv 0 (mod\ 2) \end{cases} \quad (1)$$

With this type of coloring, $G_{m,n}$satisfies the definition of interval edge coloring and $\chi_{ie}\ \left(G_{m,n}\right) = \Delta$.

Case (ii) : Let m be even and n be odd.

We define the function f as the coloring from the edges to the color set $\{1, 2, ...\}$ as follows :

for$1 \leq i \leq m,\ 1 \leq j \leq n-1$

$$f\left(v_{ij}v_{ij+1}\right) = \begin{cases} 1, & j \equiv 0(mod\ 2) \\ 3, & j \equiv 1(mod\ 2) \end{cases} \tag{2}$$

for$1 \leq i \leq m-1,\ 1 \leq j \leq n-1$

$$f\left(v_{ij}v_{i+1j}\right) = \begin{cases} 2, & i \equiv 1(mod\ 2) \\ 4, & i \equiv 0(mod\ 2) \end{cases}$$

$$f(v_{in}v_{i+1n}) = \begin{cases} 2, & i \equiv 1(mod\ 2) \\ 3, & i \equiv 0(mod\ 2) \end{cases} \tag{3}$$

The gird graph $G_{m,n}$ satisfies the definition of interval edge coloring for this type of coloring and its chromatic number of interval edge coloring is 4. We observe that the number of colors in the interval edge coloring and the maximum degree Δ are the same.

Case (iii) : Let m be odd and n be even.

We define the function f as the coloring from the edges to the color set $\{1, 2, ...\}$ as follows :

for$1 \leq i \leq m-1,\ 1 \leq j \leq n-1$

$$f\left(v_{ij}v_{ij+1}\right) = \begin{cases} 3, & j \equiv 1(mod\ 2) \\ 1, & j \equiv 0(mod\ 2) \end{cases}$$

$$f\left(v_{mj}v_{mj+1}\right) = \begin{cases} 3, & j \equiv 1(mod\ 2) \\ 2, & j \equiv 0(mod\ 2) \end{cases} \tag{4}$$

for$1 \leq i \leq m,\ 1 \leq j \leq n-1$

$$f\left(v_{ij}v_{i+1j}\right) = \begin{cases} 2, & i \equiv 1(mod\ 2) \\ 4, & i \equiv 0(mod\ 2) \end{cases} \tag{5}$$

With this type of coloring, the gird graph $G_{m,n}$satisfies the definition of interval edge coloring and the chromatic number of interval edge coloring is 4.

Case (iv) : Let m and n are even.

We define the function f as the coloring from the edges to the color set $\{1, 2, ...\}$ as follows :

for$1 \leq i \leq m,\ 1 \leq j \leq n-1$

$$f\left(v_{ij}v_{ij+1}\right) = \begin{cases} 3, & j \equiv 1(mod\ 2) \\ 1, & j \equiv 0(mod\ 2) \end{cases} \tag{6}$$

for$1 \leq i \leq m-1,\ 1 \leq j \leq n$

$$f\left(v_{ij}v_{i+1j}\right) = \begin{cases} 2, & i \equiv 1(mod\ 2) \\ 4, & i \equiv 0(mod\ 2) \end{cases} \tag{7}$$

The gird graph $G_{m,n}$ satisfies the definition of interval edge coloring and its chromatic number of interval edge coloring is 4.

Illutration 4.

Consider the grid graph $G_{4,5}$. We assign the colors 1, 2, 3 and 4 to the edges of $G_{4,5}$ by using case (ii) of the theorem 1 as shown in fig-5.

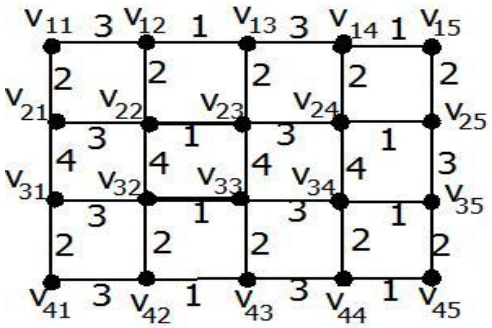

Figure 5.

This type of coloring the grid graph $G_{4,5}$ satisfies the definition of interval edge coloring.

Here $\chi_{ie}\ \ (G_{4,5}) = 4$

Illustration 5.

Consider the grid graph $G_{6,6}$. We assign the colors 1, 2, 3 and 4 to the edges of $G_{6,6}$ by using equation (5) as shown in fig-6.

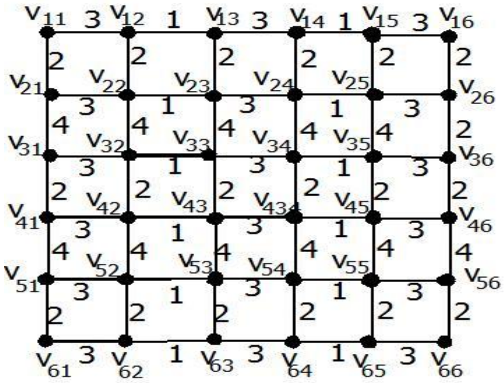

Figure 6

This type of coloring the grid graph $G_{6,6}$ satisfies the definition of interval edge coloring.

Here $\chi_{ie}(G_{6,6}) = 4$

Theorem 2. The grid of diamonds $D_{m,n}$ admits interval edge-coloring and its chromatic number of interval edge coloring is 4.

Proof : The grid of diamonds $D_{m,n}$ has mn diamonds with m and n colums. The vertices of $D_{m,n}$ are denoted by v_{ij},for $1 \leq i \leq 2m + 1$. In fig-7, j takes the value n if i is odd and j takes the value $n + 1$ if i is even as shown.

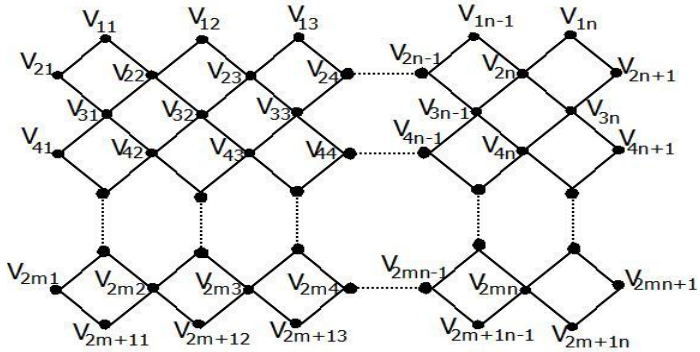

Figure 7.

The function f is defined as the coloring from the edges of $D_{m,n}$ to the set of colors (positive integers) $\{1,2,3,...\}$, $1 \leq i \leq 2m, 1 \leq j \leq 2n$ as follows :

for$j \equiv 1(mod\ 2)$

$$f(v_{ij}v_{i+1j}) = \begin{cases} 1, & i \equiv 1(mod\ 4) \\ 2, & i \equiv 2(mod\ 4) \\ 3, & i \equiv 3(mod\ 4) \\ 4, & i \equiv 0(mod\ 4) \end{cases} \quad (6)$$

$$f(v_{ij}v_{i+1j+1}) = \begin{cases} 2, & i \equiv 1(mod\ 4),\ j \neq n+1 \\ 4, & i \equiv 3(mod\ 4),\ j \neq n+1 \end{cases} \quad (7)$$

$$f(v_{ij+1}v_{i+1j}) = \begin{cases} 1, & i \equiv 2(mod\ 4),\ j \neq n+1 \\ 3, & i \equiv 0(mod\ 4),\ j \neq n+1 \end{cases} \quad (8)$$

for$j \equiv 0(mod\ 2)$

$$f(v_{ij}v_{i+1j}) = \begin{cases} 3, & i \equiv 1(mod\ 4) \\ 4, & i \equiv 2(mod\ 4) \\ 1, & i \equiv 3(mod\ 4) \\ 2, & i \equiv 0(mod\ 4) \end{cases} \quad (9)$$

$$f(v_{ij}v_{i+1j+1}) = \begin{cases} 4, & i \equiv 1(mod\ 4),\ j \neq n+1 \\ 2, & i \equiv 3(mod\ 4),\ j \neq n+1 \end{cases} \quad (10)$$

$$f(v_{ij+1}v_{i+1j}) = \begin{cases} 3, & i \equiv 2(mod\ 4),\ j \neq n+1 \\ 1, & i \equiv 0(mod\ 4),\ j \neq n+1 \end{cases} \quad (11)$$

With this type of coloring, the gird of diamonds $D_{m,n}$satisfies the definition of interval edge coloring and its chromatic number of interval edge coloring is 4 for $m > 1, n > 1$.

Illutration 6.

Consider the grid of diamonds $G_{3,2}$. We assign the colors 1, 2, 3 and 4 to the edges of $G_{3,2}$ by using theorem 2, as shown in fig-8.

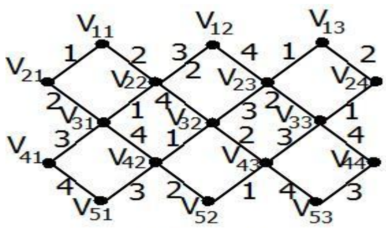

Figure 8.

This type of coloring, the grid of diamonds $G_{3,2}$ satisfies the definition of interval edge coloring. Here $\chi_{ie}(G_{3,2}) = 4$.

Corollary 1. The chromatic number of equitable interval edge coloring of grid of diamonds $D_{m,n}$ is 4 if the product mn is even.

Proof : Consider the grid of diamonds $D_{m,n}$, mn is even. If the product mn is even, then m and n belongs to any one of the following cases

(i) m is odd and n is even

(ii) m is even and n is odd

(iii) both m and n are even

By using theorem 2, all the above three cases the color classes $C[1], C[2], C[3]$ *and* $C[4]$ satisfy the condition $\left||C[i]| - |C[j]|\right| \leq 1$ for $1 \leq i \leq 4,\ 1 \leq j \leq 4$. Hence the grid of diamonds $D_{m,n}$ admits equitable interval edge coloring if the product mn is even, $\chi_{eie}(G_{m,n}) = \Delta$.

Illustration 7.

Consider the grid of diamonds $G_{4,3}$. We assign the colors 1, 2, 3 and 4 to the edges of $G_{4,3}$ by using theorem 2, as shown in fig-9.

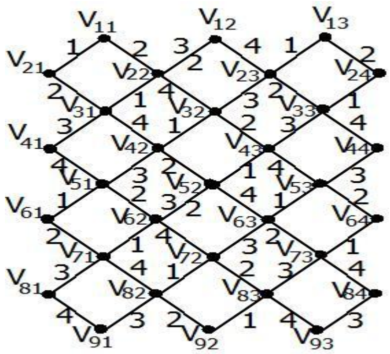

Figure 9.

Here $|C[1]| = |C[2]| = |C[3]| = |C[4]| = 12$. $\left||C[i]| - |C[j]|\right| \leq 1$ for $1 \leq i \leq 4$and $1 \leq j \leq 4$.

This type of coloring, the grid of diamonds $G_{4,3}$ satisfies the condition for equitable interval edge coloring. Here $\chi_{eie}(G_{4,3}) = 4$.

Theorem 3. The prism graph Y_n for $n > 2$admits equitable interval edge coloring and its chromatic number is 3.

Proof : The prism Y_n is the cartesian product of cycle C_n with the path P_2. It has $2n$ vertices and $3n$ edges. The outer vertices of Y_n are denoted by u_i, for $1 \leq i \leq n$ and the inner vertices of Y_n are denoted by v_i, for $1 \leq i \leq n$ as shown is fig-10.

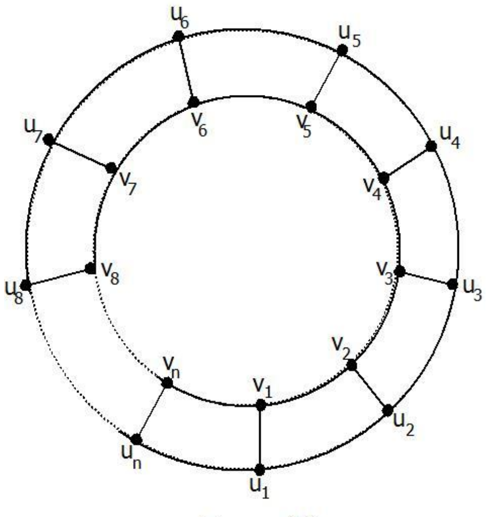

Figure 10.

Define the function $f: E(Y_n) \to \{1, 2, 3, \ldots\}$ such that no two adjacent edges receive the same color as follows :

Case (i) : Let n be even.

Define the function $f$as the coloring from the edges to the color set $\{1, 2, \ldots\}$ as follows :

for$1 \leq i \leq n - 1$,

$$f(u_i u_{i+1}) = \begin{cases} 3, & i \equiv 1 (mod\ 2) \\ 2, & i \equiv 0 (mod\ 2) \end{cases} \qquad f(u_n u_1) = 3$$

$$f(u_i v_i) = 1 \text{for all } i$$

for1 $\leq i \leq n-1$,

$$f(v_i v_{i+1}) = \begin{cases} 3, & i \equiv 1(mod\ 2) \\ 2, & i \equiv 0(mod\ 2) \end{cases}$$
$$f(v_n v_1) = 3$$

$f(u_i v_i) = 1$for all $i(12)$

With this type of coloring Y_n, Y_n satisfies the definition of interval edge coloring and we get $|C[1]| = |C[2]| = |C[3]| = n$.

Hence the prism Y_n admits equitable interval edge coloring and its chromatic number of interval edge coloring is 3,

Case (ii) : Let n be odd.

Define the function f for $1 \leq i \leq n-1$ as follows,

$$f(u_i u_{i+1}) = \begin{cases} 3, & i \equiv 1(mod\ 2) \\ 2, & i \equiv 0(mod\ 2) \end{cases}$$
$$f(u_n u_1) = 1$$

$f(u_i v_i) = 1$for$2 \leq i \leq n-1$,

$$f(u_1 v_1) = 2$$
$$f(u_n v_n) = 3$$

for1 $\leq i \leq n-1$

$$f(v_i v_{i+1}) = \begin{cases} 3, & i \equiv 1(mod\ 2) \\ 2, & i \equiv 0(mod\ 2) \end{cases}$$
$$f(v_n v_1) = 1(13)$$

With this type of coloring Y_n, Y_n satisfies the definition of interval edge coloring and we get $|C[1]| = |C[2]| = |C[3]| = n$. Hence the prism Y_n admits equitable interval edge coloring and its chromatic number of interval edge coloring is 3,

Illustration 8.

Consider the prism graph Y_8, and they we assign the colors 1, 2, and 3 to the edges of Y_8 by using equation (12) in theorem 2 of case (i), as shown in fig-11.

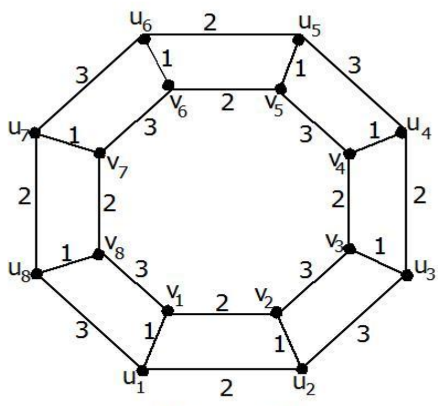

Figure 11.

Here $|C[1]| = |C[2]| = |C[3]| = 8$ and they satisfy the condition $\big||C[i]| - |C[j]|\big| \leq 1$, for $1 \leq i \leq 3$ and $1 \leq j \leq 3$. This type of coloring the prism graph Y_8 satisfies the condition for equitable interval edge coloring. Hence $\chi_{eie}(Y_8) = 3$.

Illustration 9.

Consider the prism graph Y_5, we assign the colors 1, 2, and 3 to the edges of Y_5 by using equation (13) in theorem 2 of case (ii), as shown in fig-12.

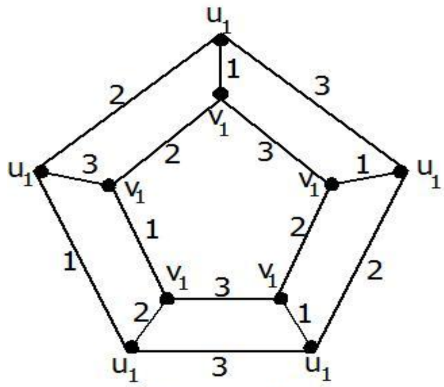

Figure 12.

Here $|C[1]| = |C[2]| = |C[3]| = 5$ and they satisfy the condition $\big||C[i]| - |C[j]|\big| \leq 1,$ for $1 \leq i \leq 3$ and $1 \leq j \leq 3$. This type of coloring the prism graph Y_5 satisfies the condition for equitable interval edge coloring. Hence $\chi_{eie}(Y_5) = 3$.

2. Conclusion

In this paper, we have proved that grid graphs $G_{m,n}$ for $m, n > 1$admit interval edge coloring. Further, we have considered the grid of diamonds $D_{m,n}$ for all m, n and prism graphs Y_n for all $n > 2$and proved it admits interval edge coloring.

References

[1] R. R. Kamalian, "On a Number of Colors in Cyclically Interval Edge Colorings of Simple Cycles," Open Journal of Discrete Mathematics, Vol. 3, 2013, pp. 43-48.

[2] R. R. Kamalian, "On one-sided interval Edge Colorings of Biregular Bipartite graphs," arXiv:1205.0130v1 [cs.DM] 1 May 2012.

[3] A. S. Asratianans R. R. Kamalian, "Interval Coloring of Edges of a Multigraph," Applied Mathematics, Vol. 5, Yerevan State University, 1987, pp. 25-34 (in Russian)

[4] A. S. Asratianans R. R. Kamalian, "Investigation of Interval Edge-Coloring of Graphs," Journal of Combinatorial Theory, Series B, Vol. 62, No. 1, 1994, pp. 34-43.

[5] R. R. Kamalian, "Interval Colorings of Complete Bipartite Graphs and Trees," Preprint of the Computing Centre of the Academy of Science of Armenia, 1989. (inRussian)

[6] R. R. Kamalian, "On a Number of Colors in Cyclically Interval Edge Colorings of Trees," Research Report LiTHMAT-R-2010/09-SE, Linkoping University, 2010.

[7] W. Mayer, Equitable coloring, American Mathematical Monthy 80 (1973) pp. 143-149.

[8] K. W. Lih, The Equitable coloring of graphs, handbook of combinatorial optimization, Vol-33, Kluwer Academic publishers, Boston MA, 1998.

[9] S. Sudha and V. Kanniga, "Star-in-coloring of benzenoidgraphs(coronene series) and grid graphs" (Communicated for publication).

[10] S. Sudha and G. M. Raja, "Equitable Coloring of Prisms and the Generalized Petersen Graphs," International Journal of Research in Engineering and Technology Vol. 2, 2014, pp. 105-112.

AUTHORS' BIOGRAPHY

Dr. S. Sudha has got 35 years of teaching and research experience. She is currently working as Professor of Mathematics at the Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai-600005. Her fields of interest are Computational Fluid Dynamics, Graph Theory, Fuzzy Graphs and Queueing Theory. She has published many articles in journals. She has also published some books.

G. M. Raja is a Ph.D. Research scholar at Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai-600005. He has published three articles in International Journals.