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The Magnetic Groups and Their Co Representations of the Quasi Crystals with Five Fold Symmetry

M.Radha Madhavi
Assistant professor
Department of Mathematics
Velammal Institute of Technology.
Chennai, India
mrmadhavi5@gmail.com

D.Bharathi
Professor
Department of Engineering Mathematics
A.U.College of Engineering,
Andhra University
Visakhapatnam, India
bharathidora57@yahoo.co.in

S. Uma Devi
Professor
Department of Engineering Mathematics
A.U.College of Engineering,
Andhra University
Visakhapatnam, India
umadevi_satagopam@yahoo.co.in

Abstract: *The magnetic point groups have their significance in the study of the macroscopic properties of magnetic crystals. Theory on Magnetic point groups derived from the 32 crystallographic point groups and their co representations already exist in literature. Since the discovery of quasi crystals great effort has been exerted to investigate their physical properties and physical property tensors. In this paper the magnetic groups of the pentagonal quasi crystals and their co representation are obtained to facilitate the study of large crystals.*

Keywords: *Magnetic groups, quasi crystals, Co representations.*

1. INTRODUCTION

The concept of symmetry was made by Shubnikov [11], Landau and Lipshitz[6] and several authors to explain the magnetic properties of crystals. The magnetic point groups have their significance in the study of the macroscopic properties of magnetic crystals. The magnetic point groups were derived by Shubinikov [11] And they have been tabulated by Tavger and Zaitsev [12]. According to the Landau's theory the occurrence of magnetic structure always involves comparatively weak interaction. Hence the crystal structure of a magnetic body is only a slight modification on that in the magnetic phase, which is usually changed into the magnetic phase when the temperature is reduced. If the magnetic crystal class of a body is specified, its macroscopic magnetic properties are qualitatively determined.

The most important of these is the presence or absence of a macroscopic magnetic moment, i.e of spontaneous magnetization in the absence of an external field. The magnetic moment M is a vector, behaving an axial vector under rotations and reflections by changing the sign under the operation R. We have 32 conventional crystallographic point groups together with 58 bi-coloured magnetic point groups constitutes 90 magnetic symmetry groups. Koptsik [5] discussed the magnetic symmetry groups in connection with the description of magnetic structure of crystals on the basis of Landau's theory of the second order phase transitions. The magnetic point groups are formed by half the elements of the generating point group and the other half elements in combination with the anti-symmetry operator θ, it can be represented as $M = H + \theta (G - H)$.

These groups shall have different irreducible representations from those of the generating point group due to the presence of the anti-symmetry operator θ and are called co representations of the magnetic symmetry group. Although this subject related to crystallographic groups was discussed by many authors [13], the co representations of all the 58 magnetic groups obtained from 32 crystallographic point groups were tabulated by Bradley and Cracknell [1] and some authors [2] tabulated the co representations of non crystallographic point group with eight fold symmetry $\bar{8}2m$.

Quasi crystals have a growing interest in anisotropy. Since quasi crystals possess positional and orientational long range order with non crystallographic rotational symmetries, they are fundamentally anisotropic and at the macroscopic level there should be some anisotropic physical properties. The discovery of quasi crystals, great effort has been exerted to investigate their physical properties. We know today that quasi crystal possess a number of really exotic physical properties. Some authors [3&10] have investigated non linear elasticity of the quasi crystals. Obviously, to determine the non linear elasticity tensor of a quasi crystal is very complicated, because on the one hand the tensor is of higher rank and on the other hand it comprises several parts associated with the phonon field and phason field and their coupling. Some authors [4] determined physical property tensors of quasi crystals using group theoretical methods and they also gave the symmetry group and basis function of the quasi crystals.

In 1996[7&8], a pentagonal quasi crystal (point group with five fold symmetry) was observed in an $Al_{70}\,CO_{15}\,Ni_{10}$ Tb alloy. The pentagonal quasi crystals represent intersecting immediate states between icosohedral and crystalline phases with anisotropic physical and mechanical properties, some researchers [9] focused on pentagonal quasi crystals to discuss the theory of structural quasi crystal to crystal transformation. In the present paper the magnetic groups and their co representations with five fold symmetry $\bar{5}, 52, 5m, \bar{5}2m, \overline{10}, \overline{10}2m$ are derived using the representation theory of point groups.

2. Derivation of Magnetic Groups of Quasi Crystals with Five Fold Symmetry

Let G be an ordinary point group and H be its halving sub group, then the magnetic group M derived from G is given by

$$M = H + \theta(G - H) \tag{1}$$

Where, θ is the anti symmetric operator.

If a crystal structure has a symmetric group G, then it can be said that the crystal transforms according to the identity representation of the group G. Similar is the case with magnetic crystals. It is possible to derive the magnetic groups by considering the one dimensional (1-d) alternating representations of the group G. Therefore, the number of magnetic groups of G is equal to the number of one-dimensional distinct real representations having character + 1 or -1 only. The magnetic groups and their co representations are discussed by taking $\bar{5}2m$ **(D_{5d})** as an example.

Table 1. *The character table for* $\bar{5}2m$ *(D5d)*

	E	**$2C_5$**	**$2(C_5)^2$**	**$5C'_2$**	**i**	**$2(S_{10})^3$**	**$2S_{10}$**	**$5\sigma_d$**
Γ_1	1	1	1	1	1	1	1	1
Γ_2	1	1	1	-1	1	1	1	-1
Γ_3	2	2cos(2π/5)	2cos(4π/5)	0	2	2cos(2π/5)	2cos(4π/5)	0
Γ_4	2	2cos(4π/5)	2cos(2π/5)	0	2	2cos(4π/5)	2cos(2π/5)	0
Γ_5	1	1	1	1	-1	-1	-1	-1
Γ_6	1	1	1	-1	-1	-1	-1	1
Γ_7	2	2cos(2π/5)	2cos(4π/5)	0	-2	-2cos(2π/5)	-2cos(4π/5)	0
Γ_8	2	2cos(4π/5)	2cos(2π/5)	0	-2	-2cos(4π/5)	-2cos(2π/5)	0

In table 1the point group $\bar{5}2m$ (D_{5d}) consists of 20 elements which are classified into eight conjugate classes and have eight irreducible representations out of which four are one dimensional and the remaining four are two dimensional representations . In the irreducible representation Γ_5 , the elements E, $2C_5$, $2C_5^2$, $5C_2^{'}$

(Represented by +1 character) forms the subgroup 52 (D5) while the remaining elements (represented by character -1) are multiplied by unitary operator θ. Thus the magnetic group of the $\bar{5}2m$ corresponding to Γ_5 is

$\bar{5}'2m' = \{E, 2C_5, 2C_5^2, 5C_2^{'}, \theta i, 2\theta S_{10}^3, 2\theta S_{10}, 5\theta\sigma_d\}$

In the similar way the magnetic group corresponding to Γ_6 and Γ_2 are

$\bar{5}'2'm = \{E, 2C_5, 2C_5^2, 5\sigma_d, \theta i, 2\theta S_{10}^3, 2\theta S_{10}, 5\theta C_2^{'}\}$

$\bar{5}2'm' = \{E, 2C_5, 2C_5^2, i, 2S_{10}^3, 2S_{10}, 5\theta C_2^{'}, 5\theta\sigma_d\}$ respectively.

Similarly, the magnetic groups for $\bar{5}$ **,** 52, 5m, $\overline{10}$ **&** $\overline{10}$ 2m corresponding to their alternating representations are

$\bar{5}'$ **= {E**, $C_5, C_5^2, C_5^3, C_5^4, \theta i, \theta S_{10}^7, \theta S_{10}^9, \theta S_{10}, \theta S_{10}^3\}$

$52' = \{E, 2C_5, 2C_5^2, 5\theta C_2^{'}\}$

$5m' = \{E, 2C_5, 2C_5^2, 5\theta\sigma_d\}$

$\overline{10}'$ **= {E**, $C_5, C_5^2, C_5^3, C_5^4, \theta\sigma_h, \theta S_5, \theta S_5^7, \theta S_5^3, \theta S_5\}$

$\overline{10}'2m' = \{E, 2C_5, 2C_5^2, 5C_2^{'}, \theta\sigma_h, 2\theta S_5^3, 2\theta S_5, 5\theta\sigma_v\}$

$\overline{10}'2'm = \{E, 2C_5, 2C_5^2, 5\sigma_v, \theta\sigma_h, 2\theta S_5^3, 2\theta S_5, 5\theta C_2^{'}\}$

$\overline{10}2'm' = \{E, 2C_5, 2C_5^2, \sigma_h, 2S_5^3, 2S_5, 5\theta C_2^{'}, 5\theta\sigma_v\}$.

3. METHOD TO FIND CO-REPRESENTATIONS OF THE MAGNETIC GROUPS

The co representations of the magnetic group M can be obtained from the irreducible representations of the sub group H of G. The elements of M are either R∈H or of the form B = RA where A is the co set representative of H in G. The Irreducible representations of H are classified into three cases using a simple test for characters.

That is

$$\sum_{B \in AH} X(B^2) = +|H| \quad \text{In case (a)}$$

$$= -|H| \quad \text{in case (b)} \qquad (2)$$

$$= 0 \quad \text{in case(c)}$$

In case (a) and (b), the irreducible representations Γ and $\bar{\Gamma}$ of H are equivalent and there exists a unitary matrix N such that

$$\Delta(R) = N\Delta^*(A^{-1}RA)N^{-1} \quad \text{For } R \in H \qquad (3)$$

And

$$NN^* = \pm\,\Delta(A^2) \qquad (4)$$

(+ sign in case ‘a’ and – sign in case ‘b’). For details of the proof, the reader may refer to Bradley and Cracknel [ ]. In case (c) the irreducible representations Γ and $\bar{\Gamma}$ are in equivalent. The co representations of the magnetic groups obtained from the irreducible representations of H are irreducible and are given by for $R \in H$, $B = RA \in HA$.

(i) $D(R) = \Delta(R)$ and $D(B) = \pm \Delta(BA^{-1}) N$ in case (a) (5)

(ii) $D(R) = \begin{bmatrix} \Delta(R) & 0 \\ 0 & \Delta(R) \end{bmatrix}$ and $D(B) = \begin{bmatrix} 0 & -\Delta(BA^{-1})N \\ \Delta(BA^{-1})N & 0 \end{bmatrix}$ in case (b) (6)

(iii) $D(R) = \begin{bmatrix} \Delta(R) & 0 \\ 0 & \overline{\Delta}(R) \end{bmatrix}$ and $D(B) = \begin{bmatrix} 0 & \Delta(BA) \\ \overline{\Delta}(BA^{-1}) & 0 \end{bmatrix}$ in case (c) (7)

Where, D refers to co representation matrix of M and Δ refers to irreducible representation matrix of H.

4. CO REPRESENTATIONS OF THE MAGNETIC GROUP $\bar{5}2m$ (D_{5D})

The point groups 52 and 5m are isomorphic groups having same class structure, multiplication table and irreducible representations except $5C_2^{'}$ in 52 is replaced by $5\sigma_d$ in 5m. The co set representative A is chosen to be θi for both 52 and 5m, and the co representations are independent of the choice of A.

Table 2. *The character tables 52 & 5m*

. 5m	E	$2C_5$	$2(C_5)^2$	$5\sigma_d$	Case
52	E	$2C_5$	$2(C_5)^2$	$5C'_2$	
Γ_1	1	1	1	1	a
Γ_2	1	1	1	-1	a
Γ_3	2	2cos(2π/5)	2cos(4π/5)	0	c
Γ_4	2	2cos(4π/5)	2cos(2π/5)	0	c

According to eq(2) Γ_1 & Γ_2 belongs to case (a) and Γ_3 & Γ_4 belongs to case (c). We find N using (3) satisfying $N N^* = 1 = +\Delta(A^2)$ for Γ_1 & Γ_2

We can choose N = 1 for Γ_1 & Γ_2

The co representations obtained from Γ_1 & Γ_2 of 52 (5m) are given by $D(R) = \Delta(R)$ and $D(B) = \pm \Delta(R) N$.

For Γ_3 & Γ_4

$$D(R) = \begin{bmatrix} \Delta(R) & 0 \\ 0 & \overline{\Delta}(R) \end{bmatrix}, D(B) = \begin{bmatrix} 0 & \Delta(BA) \\ \overline{\Delta}(BA^{-1}) & 0 \end{bmatrix}$$

Where B = RA and $\overline{\Delta}(R) = \Delta^*(A^{-1}RA)$

The co representations of the magnetic group $\bar{5}'2m'$ ($\bar{5}'2'm$) and $\bar{5}2'm'$ are tabulated in table 3 and 4 respectively. Similarly we can tabulate the co representations of the magnetic groups $\bar{5}$, 52, 5m, $\overline{10}$ & $\overline{10}$ 2m and they are tabulated in table 5.

Table 3. *The co representations of the Magnetic group* $\bar{5}'2m'\ (\bar{5}'2'm)$

Reps. → Elts.↓	$D\Gamma_1$	$D\Gamma_2$	$D\Gamma_3$	$D\Gamma_4$
E	1	1	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
C_5	1	1	$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a^4 & 0 & 0 \\ 0 & 0 & a^* & 0 \\ 0 & 0 & 0 & a^{4*} \end{pmatrix}$	$\begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & a^3 & 0 & 0 \\ 0 & 0 & a^{2*} & 0 \\ 0 & 0 & 0 & a^{3*} \end{pmatrix}$
C_5^2	1	1	$\begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & a^3 & 0 & 0 \\ 0 & 0 & a^{2*} & 0 \\ 0 & 0 & 0 & a^{3*} \end{pmatrix}$	$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a^4 & 0 & 0 \\ 0 & 0 & a^* & 0 \\ 0 & 0 & 0 & a^{4*} \end{pmatrix}$
C_5^3	1	1	$\begin{pmatrix} a^3 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & a^{3*} & 0 \\ 0 & 0 & 0 & a^{2*} \end{pmatrix}$	$\begin{pmatrix} a^4 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a^{4*} & 0 \\ 0 & 0 & 0 & a^* \end{pmatrix}$
C_5^4	1	1	$\begin{pmatrix} a^4 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a^{4*} & 0 \\ 0 & 0 & 0 & a^* \end{pmatrix}$	$\begin{pmatrix} a^3 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & a^{3*} & 0 \\ 0 & 0 & 0 & a^{2*} \end{pmatrix}$
C_2	1	-1	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

C_2'	1	-1	$\begin{pmatrix} 0 & a^2 & 0 & 0 \\ a^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^{2*} \\ 0 & 0 & a^{3*} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & a^4 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & a^{4*} \\ 0 & 0 & a^{*} & 0 \end{pmatrix}$
C_2''	1	-1	$\begin{pmatrix} 0 & a^4 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & 0 & 0 & a^{4*} \\ 0 & 0 & a^{*} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & a^2 & 0 & 0 \\ a^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^{2*} \\ 0 & 0 & a^{3*} & 0 \end{pmatrix}$
C_2'''	1	-1	$\begin{pmatrix} 0 & a & 0 & 0 \\ a^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^{*} \\ 0 & 0 & a^{4*} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & a^3 & 0 & 0 \\ a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^{3*} \\ 0 & 0 & a^{2*} & 0 \end{pmatrix}$
C_2^{IV}	1	-1	$\begin{pmatrix} 0 & a^3 & 0 & 0 \\ a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^{3*} \\ 0 & 0 & a^{2*} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & a & 0 & 0 \\ a^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & a^{*} \\ 0 & 0 & a^{4*} & 0 \end{pmatrix}$
θi	±1	±1	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$
θS_{10}^7	±1	±1	$\begin{pmatrix} 0 & 0 & a & 0 \\ 0 & 0 & 0 & a^4 \\ a^{*} & 0 & 0 & 0 \\ 0 & a^{4*} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^3 \\ a^{2*} & 0 & 0 & 0 \\ 0 & a^{3*} & 0 & 0 \end{pmatrix}$
θS_{10}^9	±1	±1	$\begin{pmatrix} 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^3 \\ a^{2*} & 0 & 0 & 0 \\ 0 & a^{3*} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & a & 0 \\ 0 & 0 & 0 & a^4 \\ a^{*} & 0 & 0 & 0 \\ 0 & a^{4*} & 0 & 0 \end{pmatrix}$
θS_{10}	±1	±1	$\begin{pmatrix} 0 & 0 & a^3 & 0 \\ 0 & 0 & 0 & a^2 \\ a^{3*} & 0 & 0 & 0 \\ 0 & a^{2*} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & a^4 & 0 \\ 0 & 0 & 0 & a \\ a^{4*} & 0 & 0 & 0 \\ 0 & a^{*} & 0 & 0 \end{pmatrix}$

θS_{10}^{3}	±1	±1	$\begin{pmatrix} 0 & 0 & a^{4} & 0 \\ 0 & 0 & 0 & a \\ a^{4*} & 0 & 0 & 0 \\ 0 & a^{*} & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & a^{3} & 0 \\ 0 & 0 & 0 & a^{2} \\ a^{3*} & 0 & 0 & 0 \\ 0 & a^{2*} & 0 & 0 \end{pmatrix}$
$\theta\sigma_d$	±1	∓ 1	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$
$\theta\sigma_d^{'}$	±1	∓ 1	$\begin{pmatrix} 0 & 0 & 0 & a^{2} \\ 0 & 0 & a^{3} & 0 \\ 0 & a^{2*} & 0 & 0 \\ a^{3*} & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & a^{4} \\ 0 & 0 & a & 0 \\ 0 & a^{4*} & 0 & 0 \\ a^{*} & 0 & 0 & 0 \end{pmatrix}$
$\theta\sigma_d^{''}$	±1	∓ 1	$\begin{pmatrix} 0 & 0 & 0 & a^{4} \\ 0 & 0 & a & 0 \\ 0 & a^{4*} & 0 & 0 \\ a^{*} & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & a^{2} \\ 0 & 0 & a^{3} & 0 \\ 0 & a^{2*} & 0 & 0 \\ a^{3*} & 0 & 0 & 0 \end{pmatrix}$
$\theta\sigma_d^{'''}$	±1	∓ 1	$\begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & a^{4} & 0 \\ 0 & a^{*} & 0 & 0 \\ a^{4*} & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & a^{3} \\ 0 & 0 & a^{2} & 0 \\ 0 & a^{3*} & 0 & 0 \\ a^{2*} & 0 & 0 & 0 \end{pmatrix}$
$\theta\sigma_d^{IV}$	±1	∓ 1	$\begin{pmatrix} 0 & 0 & 0 & a^{3} \\ 0 & 0 & a^{2} & 0 \\ 0 & a^{3*} & 0 & 0 \\ a^{2*} & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & a^{4} & 0 \\ 0 & a^{*} & 0 & 0 \\ a^{4*} & 0 & 0 & 0 \end{pmatrix}$

Where $a = e^{i2\pi/5}$, $a^2 = e^{i4\pi/5}$, $a^3 = e^{i6\pi/5}$, $a^4 = e^{i8\pi/5}$

Table 4. *The co representations of the Magnetic group* $\bar{5}2'm'$

Reps. → Elts.↓	$D\Gamma_1$	$D\Gamma_2$	$D\Gamma_3$	$D\Gamma_4$	$D\Gamma_5$	$D\Gamma_6$	$D\Gamma_7$	$D\Gamma_8$	$D\Gamma_9$	$D\Gamma_{10}$
E	1	1	1	1	1	1	1	1	1	1
C_5	1	a	a^{-1}	a^{2}	a^{-2}	1	a	a^{-1}	a^{2}	a^{-2}
C_5^{2}	1	a^{2}	a^{-2}	a^{-1}	a	1	a^{2}	a^{-2}	a^{-1}	a

C_5^3	1	a^{-2}	a^2	a	a^{-1}	1	a^{-2}	a^2	a	a^{-1}
C_5^4	1	a^{-1}	a	a^{-2}	a^2	1	a^{-1}	A	a^{-2}	a^2
I	1	1	1	1	1	-1	-1	-1	-1	-1
S_{10}^7	1	a	a^{-1}	a^2	a^{-2}	-1	-a	$-a^{-1}$	$-a^2$	$-a^{-2}$
S_{10}^9	1	a^2	a^{-2}	a^{-1}	a	-1	$-a^2$	$-a^{-2}$	$-a^{-1}$	-a
S_{10}	1	a^{-2}	a^2	a	a^{-1}	-1	$-a^{-2}$	$-a^2$	-a	$-a^{-1}$
S_{10}^3	1	a^{-1}	a	a^{-2}	a^2	-1	$-a^{-1}$	-a	$-a^{-2}$	$-a^2$
θC_2	±1	±1	±1	±1	±1	±1	±1	±1	±1	±1
$\theta C_2'$	±1	±a	$\pm a^{-1}$	$\pm a^2$	$\pm a^{-2}$	±1	±a	$\pm a^{-1}$	$\pm a^2$	$\pm a^{-2}$
$\theta C_2''$	±1	$\pm a^2$	$\pm a^{-2}$	$\pm a^{-1}$	±a	±1	$\pm a^2$	$\pm a^{-2}$	$\pm a^{-1}$	±a
$\theta C_2'''$	±1	$\pm a^{-2}$	$\pm a^2$	±a	$\pm a^{-1}$	±1	$\pm a^{-2}$	$\pm a^2$	±a	$\pm a^{-1}$
θC_2^{IV}	±1	$\pm a^{-1}$	±a	$\pm a^{-2}$	$\pm a^2$	±1	$\pm a^{-1}$	±a	$\pm a^{-2}$	$\pm a^2$
$\theta\sigma_d$	±1	±1	±1	±1	±1	∓ 1	∓ 1	∓ 1	∓ 1	∓ 1
$\theta\sigma_d'$	±1	±a	$\pm a^{-1}$	$\pm a^2$	$\pm a^{-2}$	∓ 1	∓ a	$\mp a^{-1}$	$\mp a^2$	$\mp a^{-2}$
$\theta\sigma_d''$	±1	$\pm a^2$	$\pm a^{-2}$	$\pm a^{-1}$	±a	∓ 1	$\mp a^2$	$\mp a^{-2}$	$\mp a^{-1}$	∓ a
$\theta\sigma_d'''$	±1	$\pm a^{-2}$	$\pm a^2$	±a	$\pm a^{-1}$	∓ 1	$\mp a^{-2}$	$\mp a^2$	∓ a	$\mp a^{-1}$
$\theta\sigma_d^{IV}$	±1	$\pm a^{-1}$	±a	$\pm a^{-2}$	$\pm a^2$	∓ 1	$\mp a^{-1}$	∓ a	$\mp a^{-2}$	$\mp a^{2}$

Where $a = ei2^{\pi/5}$

Table 5. *The co representations of the magnetic group with pentagonal symmetry*

M H	5m 5	52 5	$\bar{5}$ 5
Rep of H Co rep of M N	A E_1 $\overline{E}_1$ E_2 $\overline{E}_2$ a c c c c α - - - -	A E_1 $\overline{E}_1$ E_2 $\overline{E}_2$ a c c c c α - - - -	A E_1 $\overline{E}_1$ E_2 $\overline{E}_2$ a c c c c α - - - -

M H	$\bar{5}'\,2'm$ 5m	$\bar{5}'\,2m'$ 52
Rep of H Co rep of M N	A_1 A_2 E_1 E_2 a a c c α α - -	A_1 A_2 E_1 E_2 a a c c α α - -

M H	$\bar{5}2'm'$ $\bar{5}$
Rep of H Co rep of M N	A_g E_{1g} $\overline{E}_{1g}$ E_{2g} $\overline{E}_{2g}$ A_u E_{1u} $\overline{E}_{1u}$ E_{2u} $\overline{E}_{2u}$ a a a a a a a a a a α α α α α α α α α α

M H	$\overline{10}'\,2'm$ 5m	$\overline{10}'\,2m'$ 52
Rep of H Co rep of M N	A_1 A_2 E_1 E_2 a a c c α α - -	A_1 A_2 E_1 E_2 a a c c α α - -

M H	$\overline{10}\,2'm'$ $\overline{10}$
Rep of H Co rep of M N	A^1 E_1^1 $\overline{E}_1^1$ E_2^1 $\overline{E}_2^1$ A^{11} E_1^{11} $\overline{E}_1^{11}$ E_2^{11} $\overline{E}_2^{11}$ a a a a a a a a a a α α α α α α α α α α

M H	$\bar{5}\bar{3}m$ 235
Rep of H Co rep of M N	A T_1 T_2 G H a a a a a α β β γ ε

Where $\alpha = 1$ $\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\gamma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\varepsilon = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

5. Conclusion

The magnetic point groups have their significance in the study of the macroscopic properties of magnetic crystals. The magnetic groups of the pentagonal quasi crystals and their co representation are obtained to facilitate the study physical properties of large crystals.

References

[1] D.Bharathi & A.Prasad Journal of Indian Acad. Math., Vol.27,No.2,299-306, (2005).

[2] Duquenses J-Y and Perin B, Phys.Rev.Lett. , 85, 4301(2000).

[3] Gong Ping et al. , Chinese Physics, Vol. 15, No. 9, 2065-2079(2006),.

[4] Kopsik.V.K, Shubnikovski Gruppy. , Moscow Univ. Press(1966).

[5] Li X Z , Y U Rc et al , Phil. Mag. Lett., 73, 255(1996) .

[6] Li X Z and Hiragak, J.Mater.Res,11, 1891, (1996).

[7] Rochal S B and Kozinking Y A , Phys.Rev B, 72, 24210(2005).

[8] Richer M and Trebin H-R J.Phys.,A35 6953 (2002),

[9] Tavger.B.A and Zaitsev V.M, Soviet Physics.JETP, 3, 430 (1956).

[10] Hubnikov.A.V, Symmetry and Anti-Symmetry of finite figures, USSR Academy of Sciences, Moscow (1951).

[11] Bradley, C.J and Crackenell, A.P, The Mathematical theory of Symmetry in Solids (Clarendon Press, Oxford), 1972. Ch.4.

[12] Landu,L.D and Lipshitz,E.M, Electrodynamics of continuous media, Pregaman Press, Oxford. (1960).

[13] Wianer. P, Group theory and its applications to the quantum mechanics of atomic spectra, Academic Press, New York (1959).

[14] Yi-Jian et al , Acta. Cryst., A46, 772-776 (1990).

Authors' Biography

M.Radha Madhavi, She completed her Ph.D in 2013 from Andhra University, Visakhapatnam, A.P. She has 3 published papers. Presently she is working as assistant professor in Velammal institute of technology.

D.Bharathi, She is working as professor in the Department of Engineering Mathematics, A.U.College of Engineering, Andhra University,Visakhapatnam. She produced one Ph.D and several M.Phli's.

S. Uma Devi, She is working as professor in the Department of Engineering Mathematics, A.U.College of Engineering,Andhra University,Visakhapatnam. She produced 3 Ph.D and several M.Phli's