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Solution of Linear System of Partial Differential Equations by Legendre Multiwavelet Andchebyshev Multiwavelet

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Abstract: *In this work the Legendre multiwavelet and Chebyshev multiwavelet basis with considering the standard Galerkin method has been applied to give the approximate solution for linear first order system of partial differential equations (PDE's). The properties ofthe Legendre multiwaveletand Chebyshev multiwavelet are presented. These properties together with the standard Galerkin method are then utilized to reduce linear first order system of PDE's to the solution of an algebraic system. Numerical results and comparison with exact solution are given to demonstrate the applicability and efficiency of the method.*

Keywords: *Legendre multiwavelet,Chebyshev multiwavelet, system of partial differential equations, Galerkin method.*

1. Introduction

In 1807, Joseph Fourier developed a method for representing a signal with a series of coefficients based on an analysis function. He laid the mathematical basis from which the wavelet theory is developed. The first to mention wavelets was Alfred Haar in 1909 in his PhD thesis. In the 1930's, Paul Levy found the scale-varying Haar basis function superior to Fourier basis functions. The transformation method of decomposing a signal into wavelet coefficients and reconstructing the original signal again is derived in 1981 by Jean Morlet and Alex Grossman. In 1986, Stephane Mallat and Yves Meyer developed a multiresolution analysis using wavelets. They mentioned the scaling function of wavelets for the first time; it allowed researchers and mathematicians to construct their own family of wavelets using the derived criteria. Around 1998, Ingrid Daubechies used the theory of multiresolution wavelet analysis to construct her own family of wavelets. Her set of wavelet orthonormal basis functions have become the cornerstone of wavelet applications today.Wavelet analysis can be performed in several ways, a continuous wavelet transform, a discretized continuous wavelet transform and a true discrete wavelet transform. The application of wavelet analysis becomes more widely spread as the analysis technique becomes more generally known. The fields of application vary from science, engineering, medicine to finance. Types of wavelets are Haar Wavelets (orthogonal in L_2, compact Support, scaling function is symmetric wavelet function is antisymmetric, Infinite support in frequency domain), Shannon Wavelet (orthogonal, localized in frequency domain, easy to calculate, infinite support and slow decay), Meyer Wavelets (Fourier transform of father function) and Daubishes wavelets (orthogonal in L_2, compact support, zero moments of father-function).Studying systems of partial differential equations (PDEs) is very important. Such systems arise in many areas of mathematics, engineering and physical sciences. These equations are often too complicated to be solved exactly and even if an exact solution is obtained, the required calculations may be too complicated. Very recently, many powerful methods have been presented, such as the Adomian decomposition method[1-5], the homotopy perturbation method (HPM) [6-9], and the differential transform method[10-13]. The application of Legendre wavelets for solving differential, integral and integro-differential equations is thoroughly considered in [14-20]. Chebyshev wavelet used to solve integral and integro-differential equations in [21-23]. In this paper we are dealing with the numerical approximation of the following second order system of linear partial differential equations

$$u_t = f(u,v), \quad v_t = g(u,v), \qquad x \in [0,1], t \in [0,1], \tag{1}$$

with initial condition

$$u(x,0) = f_1(x), \qquad v(x,0) = g_1(x), \quad x \in [0,1]. \tag{2}$$

The aim of this workis to present two numerical methods(Legendre and Chebyshev multiwavelet) for approximating the solutionof a linear first order system of partial differential equations (PDE's). These methods consists of expanding the solution by Legendre multiwaveletand Chebyshev multiwavelet with unknown coefficients. The properties of Legendre multiwavetlet and Chebyshev multiwavelet together with the Galerkin method are then utilized to evaluate the unknown coefficients and find an approximate solution to Eqs. (1).The article is organized as follows: In Section II, we describe the basic formulation of wavelets and Legendre multiwavelet and Chebyshev multiwavelet required for our subsequent development. Section III is devoted to the solution of Eq. (1) by using Legendre multiwavelet and Chebyshev multiwavelet Galerkin approximation. In Section IV, wereport our numerical finding and demonstrate the accuracy of the proposed scheme by considering numerical examples. Section V consists of a brief summary.

2. Properties of Legendre Multiwavelets and Chebyshev Multiwavelets

2.1 Wavelets

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously we have the following family of continuous wavelets [15,21,22]

$$\psi_{a,b}(t) = |a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right), \quad a,b \in \mathbb{R} \ , a \neq 0. \tag{3}$$

If we restrict the parameters a and b to discrete values as $a = {a_0}^{-k}$, $b = nb_0{a_0}^{-k}$, $a_0 > 1, b_0 > 0$ and $n, k \in \mathbb{N}$ we have the following family of discrete wavelets:

$$\psi_{k,n}(t) = |a|^{\frac{k}{2}}\psi\left({a_0}^k t - nb_0\right), \tag{4}$$

where $\psi_{k,n}(t)$ form a wavelet basis for $L^2(R)$. In particular, when $a_0 = 2$ and $b_0 = 1$ then $\psi_{k,n}(t)$ forms an orthonormal basis [4, 5, 6,9,10].

2.2 Legendre Multiwavelets [15, 21, 24, 25]

Legendre multiwavelets $\psi_{nm}(t) = \psi(k,n,m,t)$have four arguments; $n, n = 0,1,2,\ldots,2^k - 1, k$ can assume any positive integer, m is the order for Legendre polynomials and t is the normalized time. They are defined on the interval [0,1]:

$$\psi_{nm}(t) = \begin{cases} \sqrt{2m+1}\,2^{\frac{k}{2}}P_m\left(2^k t - n\right), & \text{for } \frac{n}{2^k} \le t \le \frac{n+1}{2^k} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where $m = 0,1,\ldots,M-1$, M nonnegative integer and $n = 0,1,2,\ldots,2^k - 1$. The coefficient $\sqrt{2m+1}$ is for orthonormality, $P_m(t)$ are the well-known shifted Legendre polynomials of order m which are defined on the interval [0, 1], and can be determined with the aid of the following recurrence formula:

$$P_0(t) = 1, \quad P_1(t) = 2t-1, \quad P_{m+1}(t) = \left(\frac{2m+1}{m+1}\right)(2t-1)P_m(t) - \left(\frac{m}{m+1}\right)P_{m-1}(t), m = 1,2,3,\ldots. \tag{6}$$

Also the two-dimensional Legendre multiwavelet are defined as [10]:

$$\psi_{n_1m_1n_2m_2}(x,t) = \begin{cases} A\,P_{m_1}\left(2^{k_1}x - n_1\right)P_{m_2}\left(2^{k_2}t - n_2\right), & \text{for } \frac{n_1}{2^{k_1}} \le x \le \frac{n_1+1}{2^{k_1}}, \\ & \quad \frac{n_2}{2^{k_2}} \le t \le \frac{n_2+1}{2^{k_2}} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where $A=\sqrt{(2m_1+1)(2m_2+1)2^{\frac{k_1+k_2}{2}}}$, n_1 and n_2 are defined similarly to n, k_1 and k_2 can assume any positive integer, m_1 and m_2 are the order for Legendre polynomials and $\psi_{n_1m_1n_2m_2}(x,t)$ forms a basis for $L^2([0,1]\times[0,1])$.

2.3 Chebyshev Multiwavelets

The second Chebyshev wavelets $\psi_{nm}(t)=\psi(k,n,m,t)$involve four arguments, n = 1,…, 2^{k-1} , k is assumed any positive integer, m is the degree of thesecond Chebyshev polynomials and t is the normalizedtime. They are defined on the interval [0, 1] as [21-23]:

$$\psi_{n,m}(t)=\begin{cases}2^{\frac{k}{2}}\widetilde{U}_m\left(2^k t-2n+1\right), & \frac{n-1}{2^{k-1}}\le t<\frac{n}{2^{k-1}}\ , \\ 0, & \text{otherwise,}\end{cases} \tag{8}$$

where : $\widetilde{U}_m(t)=\sqrt{\frac{2}{\pi}}U_m(t),\qquad m=0,1,\dots,M-1,$

$$U_0(t)=1,\qquad U_0(t)=2t,\ \ U_{m+1}(t)=2tU_m(t)-U_{m-1}(t),\ \ m=1,2,\dots\ . \tag{9}$$

Also the two-dimensional Chebyshev multiwavelet are defined as:

$$\psi_{n_1m_1n_2m_2}(x,t)=\begin{cases}2^{\frac{k_1+k_2}{2}}\widetilde{U}_{m_1}\left(2^{k_1}x-2n_1-1\right)\widetilde{U}_{m_2}\left(2^{k_2}t-2n_2-1\right), \text{for } \frac{n_1-1}{2^{k_1-1}}\le x\le\frac{n_1}{2^{k_1-1}}, \\ \qquad \frac{n_2-1}{2^{k_2-1}}\le t\le\frac{n_2}{2^{k_2-1}}, \\ 0, \qquad \text{otherwise,}\end{cases} \tag{10}$$

n_1 and n_2 are defined similarly to n, k_1 and k_2 can assume any positive integer, m_1 and m_2 are the order for chebyshev polynomials and $\psi_{n_1m_1n_2m_2}(x,t)$ forms a basis for $L^2([0,1]\times[0,1])$.

2.4 Function Approximation

A functionf(x, t) defined over [0,1]×[0,1]can be expand as :

$$f(x,t)=\sum_{n=1}^{\infty}\sum_{i=0}^{\infty}\sum_{l=1}^{\infty}\sum_{j=0}^{\infty}c_{n,i,l,j}\,\psi_{n,i}(x)\psi_{l,j}(t). \tag{11}$$

If the infinite series in equation (14) is truncated, then equation (14) can be written as :

$$f(x,t)=\sum_{n=1}^{2^{k_1-1}}\sum_{i=0}^{N}\sum_{l=1}^{2^{k_2-1}}\sum_{j=0}^{M}c_{n,i,l,j}\,\psi_{n,i}(x)\psi_{l,j}(t)=\Psi^T(x)F\,\Psi(t) \tag{12}$$

Where $\Psi(x)$ and $\Psi(t)$ are $2^{k_1}(M+1)\times 1$ and $2^{k_2}(N+1)\times 1$ matrices, respectively given by

$$\Psi(x)=\left[\psi_{10}(x),\dots,\psi_{1M_1}(x),\dots,\psi_{20}(x),\dots,\psi_{2M_1}(x),\dots,\psi_{(2^{k_1}-1)0}(x),\dots,\psi_{(2^{k_1}-1)M}(x)\right],$$

$$\Psi(t)=\left[\psi_{10}(t),\dots,\psi_{1M_1}(t),\dots,\psi_{20}(t),\dots,\psi_{2M_1}(t),\dots,\psi_{(2^{k_1}-1)0}(t),\dots,\psi_{(2^{k_1}-1)N}(t)\right] \tag{13}$$

Also, F is a$2^{k_1}(M_1+1)\times 2^{k_2}(M_2+1)$ matrix whose elements can be calculated from

$$\int_0^1\int_0^1\psi_{ni}(x)\phi_{lj}(t)f(x,t)dtdx, \tag{14}$$

with, $n=0,1,\dots,2^{k_1}-1, i=0,\dots,M, l=0,1,\dots,2^{k_2}-1, j=0,\dots,N.$

3. Solution of Fist Order System of Linear Partial Differential Equations

we are dealing with the numerical approximation of the first order system of linear partial differential equations (1) With initial condition (2),

Let

$$F(u,v) = u_t - f(u,v), \quad G(u,v) = v_t - g(u,v). \tag{15}$$

A Galerkin approximation to (18) is constructed as follows. The approximation u_{NM} and v_{NM} are sought in the form of the truncated series [25]:

$$u_{NM}(x,t) = \sum_{n=1}^{2^{k_1}} \sum_{i=0}^{N} \sum_{l=1}^{2^{k_2}} \sum_{j=0}^{M} t\, a_{n,i,l,j}\, \psi_{n,i}(x)\psi_{l,j}(t) + w_1(x,0),$$

$$v_{NM}(x,t) = \sum_{n=1}^{2^{k_1}} \sum_{i=0}^{N} \sum_{l=1}^{2^{k_2}} \sum_{j=0}^{M} t\, b_{n,i,l,j}\, \psi_{n,i}(x)\psi_{l,j}(t) + w_2(x,0), \tag{16}$$

where$w_1(x,0) = f_1(x)$, $w_2(x,0) = g_1(x)$, and $\psi_{i,j}$ are Legendre or Chebyshev multiwavelet basis. Now we have $u_{NM}(x,0) = f_1(x), v_{NM}(x,0) = g_1(x)$.This approximation provides greater flexibility in which to impose initial conditions. The expansion coefficient$c_{n,i,l,j}$ are determined by Galerkin equations:

$$\langle F(u_{NM}), \psi_{n,i}\psi_{l,j}\rangle = 0, \quad \langle G(v_{NM}), \psi_{n,i}\psi_{l,j}\rangle = 0, \tag{17}$$

where $\langle . \rangle$ denotes inner product defined as

$$\langle F(u_{NM}), \psi_{n,i}\psi_{l,j}\rangle = \int_0^1 \int_0^1 F(u_{NM})(x,t)\psi_{ni}(x)\psi_{lj}(t)dtdx,$$

$$\langle G(u_{NM}), \psi_{n,i}\psi_{l,j}\rangle = \int_0^1 \int_0^1 G(v_{NM})(x,t)\psi_{ni}(x)\psi_{lj}(t)dtdx. \tag{18}$$

Galerkin equations (17) gives a system of $2^{k_1-1}(N+1) \times 2^{k_2-1}(M+1)$linear equations which can be solved for the elements of $a_{n,i,l,j}$, $b_{n,i,l,j}$, $i = 0,1, \dots N$, $j = 0,1, \dots, M, n = 1,2, \dots, 2^{k_1}, l = 1,2, \dots, 2^{k_2}$ using suitable method and get the approximate solution (19).

4. Illustrative Examples

Example 1. We consider another linear system of PDEs [1,10]

$$u_t + v_x = 0,$$

$$v_t + u_x = 0, \tag{19}$$

With the initial conditions

$$u(x,0) = e^x, \quad and\ v(x,0) = e^{-x}, \tag{20}$$

We applied the Legendre multiwavelets and Chebyshev multiwavelet methods at $k_1 = k_2 = 0$ and $M = N = 3$

and solved Eq. (14). The exact solution,$u(x,t) = e^x \cosh(t) + e^{-x}\sinh(t)$, $v(x,t) = e^{-x}\cosh(t) - e^x \sinh(t)$, Fig. (1a) and (1b) shows the exact , Legendre multiwavelets and Chebyshev multiwavelet solution of u(x,t) and v(x,t) respectively, figure (1c) show the the exact, Legendre multiwavelets and Chebyshev multiwavelet solution of u(x,t) and v(x,t)at t=0.1 and $0 \le x \le 1$, table 1 show the absolute error obtained by , Legendre multiwavelets and Chebyshev multiwavelet of u(x,t) and v(x,t) .

Example 2. We consider another linear system of PDEs [1,10]

$$u_t + u_x - 2v = 0,$$

$$v_t + u_x + 2u = 0, \tag{21}$$

With the initial conditions

$$u(x,0) = \sin(x), \quad and\ v(x,0) = \cos(x), \tag{22}$$

We applied the Legendre multiwavelets and Chebyshev multiwavelet methods at $k_1 = k_2 = 0$ and $M = N = 3$ and solved Eq. (14). The exact solution, $u(x,t) = \sin(x+t)$, $v(x,t) = \cos(x+t)$, Fig. (2a) and (2b) shows the exact , Legendre multiwavelets and Chebyshev multiwavelet solution of u(x,t) and v(x,t) respectively, figure (2c) show the the exact , Legendre multiwavelets and Chebyshev multiwavelet solution of u(x,t) and v(x,t)at t=0.1 and $0 \le x \le 1$, table 2 show the absolute error obtained by, Legendre multiwavelets and Chebyshev multiwavelet of u(x,t) and v(x,t) .

Example3. We consider another linear system of PDEs

$$u_t - v_x + u + v = 0,$$

$$v_t - u_x + u + v = 0, \tag{23}$$

With the initial conditions

$$u(x,0) = \sinh(x), \quad and\ v(x,0) = \cosh(x), \tag{24}$$

We applied the Legendre multiwavelets and Chebyshev multiwavelet methods at $k_1 = k_2 = 0$ and $M = N = 3$ and solved Eq. (14). The exact solution, $u(x,t) = \sin(x+t)$, $v(x,t) = \cos(x+t)$, Fig. (3a) and (3b) shows the exact, Legendre multiwavelets and Chebyshev multiwavelet solution of u(x,t) and v(x,t) respectively, figure (3c) show the the exact , Legendre multiwavelets and Chebyshev multiwavelet solution of u(x,t) and v(x,t)at t=0.1 and $0 \le x \le 1$, table 3 show the absolute error obtained by, Legendre multiwavelets and Chebyshev multiwavelet of u(x,t) and v(x,t) .

Example4. We consider another linear system of PDE s

$$u_t - v_x - u + v + 2 = 0,$$

$$v_t + u_x - u + v + 2 = 0, \tag{25}$$

With the initial conditions

$$u(x,0) = 1 + e^x, \quad and\ v(x,0) = -1 + e^x, \tag{26}$$

We applied the Legendre multiwavelets and Chebyshev multiwavelet methods at $k_1 = k_2 = 0$ and $M = N = 3$ and solved Eq. (25). The exact solution, $u(x,t) = \sin(x+t)$, $v(x,t) = \cos(x+t)$, Fig. (4a) and (4b) shows the exact, Legendre multiwavelets and Chebyshev multiwavelet solution of u(x,t) and v(x,t) respectively, figure (4c) show the the exact, Legendre multiwavelets and Chebyshev multiwavelet solution of u(x,t) and v(x,t)at t=0.1 and $0 \le x \le 1$, table 4 show the absolute error obtained by, Legendre multiwavelets and Chebyshev multiwavelet of u(x,t) and v(x,t) .

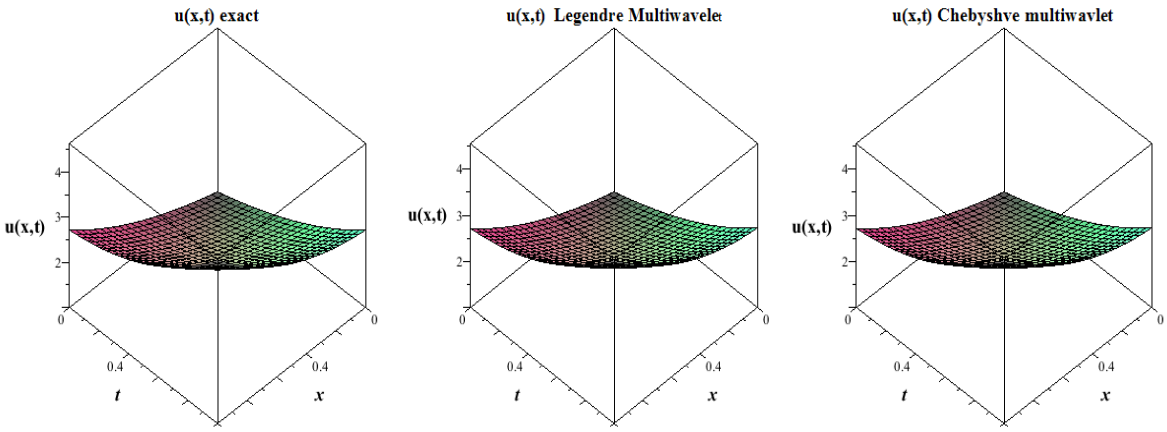

Fig. 1a *Exact and approximate solution of u(x, t) by Legendre and Chebyshev multi wavelet method, 0≤x,t≤1*

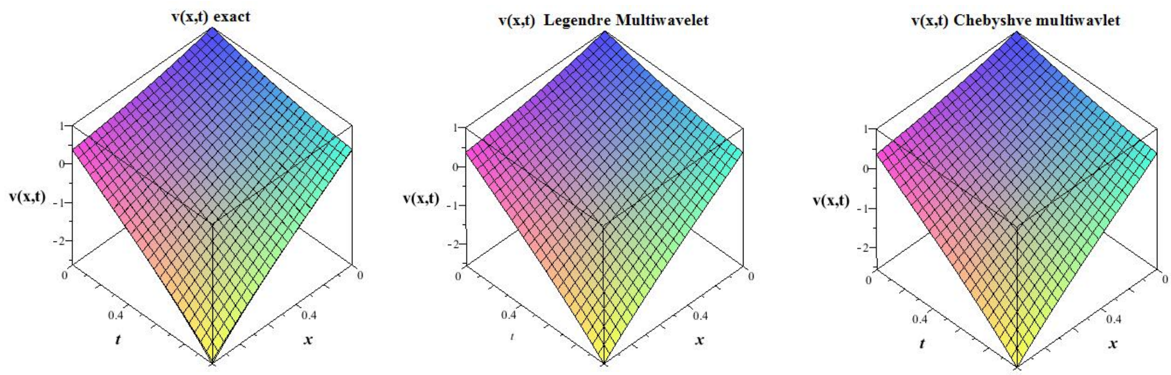

Fig.1b *Exact and approximate solution of v(x, t) by Legendre and Chebyshev multi wavelet method, 0≤x,t≤1*

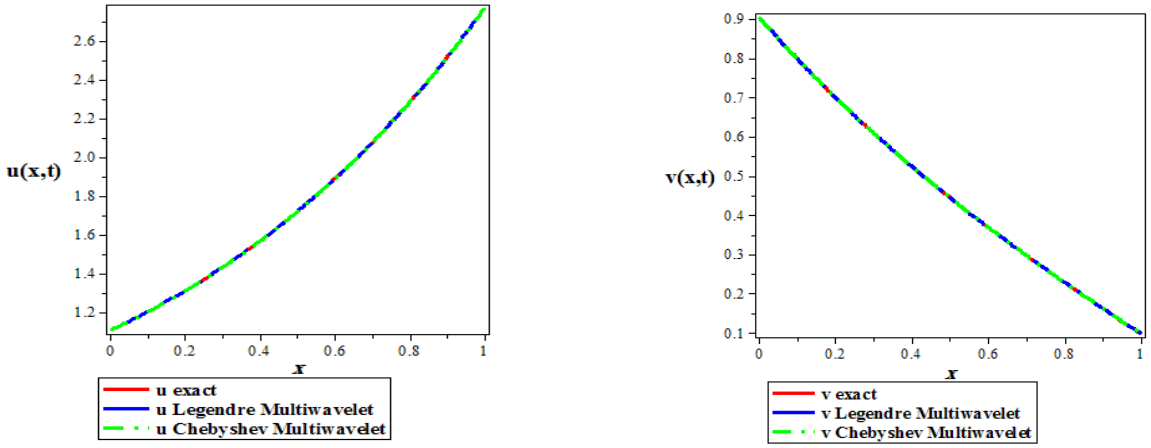

Fig. 1c. *Exact, Legendre Chebyshev multiwavelet solution of u(x, t) and v(x t) for 0 ≤ x ≤ 1, t=0.1*

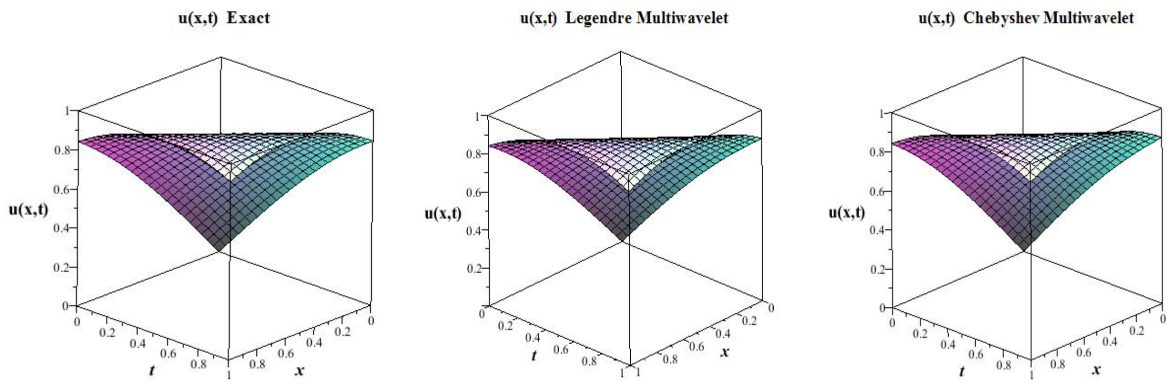

Fig. 2a *Exact and approximate solution of u(x, t) by Legendre and Chebyshev multi wavelet method, 0≤x, t≤1*

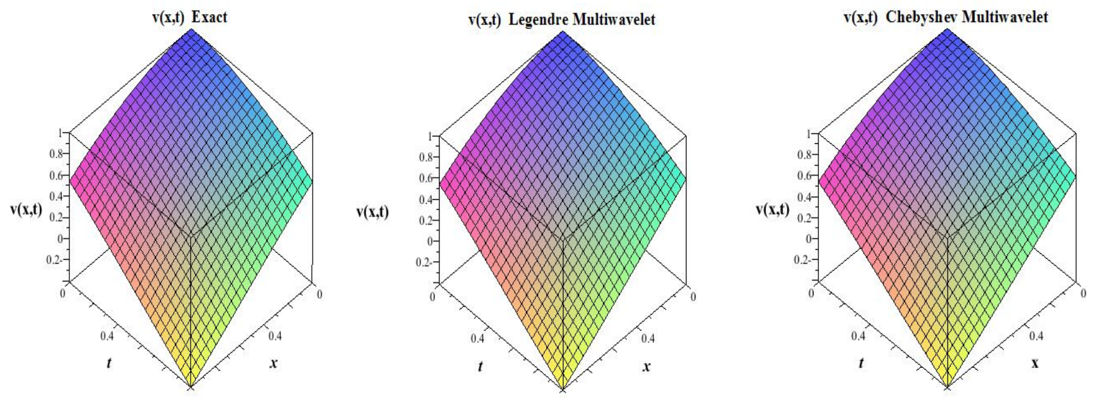

Fig. 2b *Exact and approximate solution of v(x, t) by Legendre and Chebyshev multi wavelet method, 0≤x, t≤1*

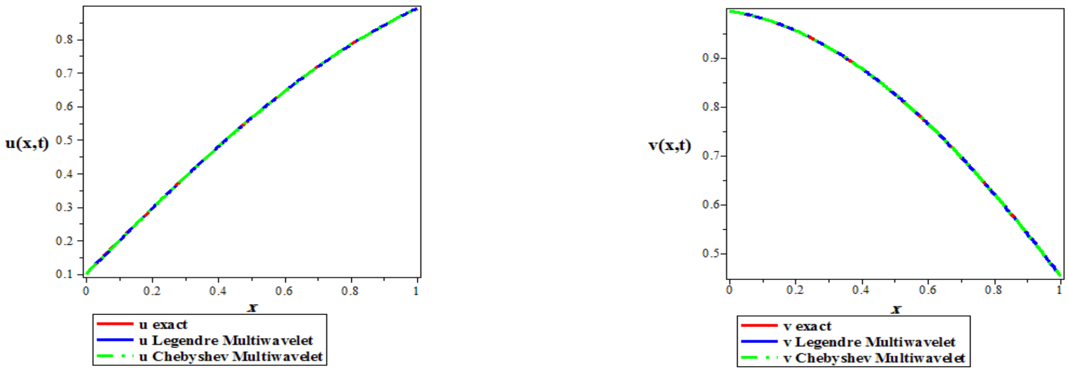

Fig. 2c. *Exact, Legendre Chebyshev multiwavelet solution of u(x, t) and v(x t) for 0 ≤ x ≤ 1, t=0.1*

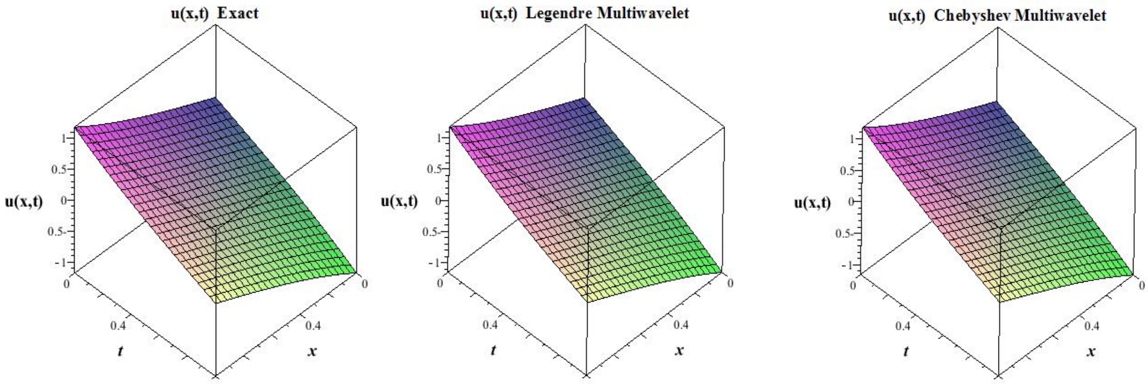

Fig. 3a *Exact and approximate solution of u(x, t) by Legendre and Chebyshev multi wavelet method, 0≤x, t≤1*

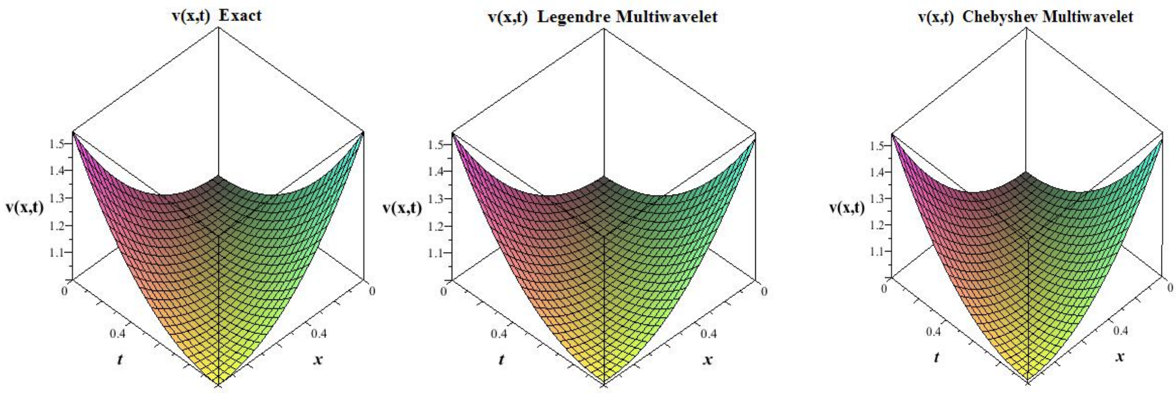

Fig. 3b *Exact and approximate solution of v(x, t) by Legendre and Chebyshev multi wavelet method, 0≤x, t≤1*

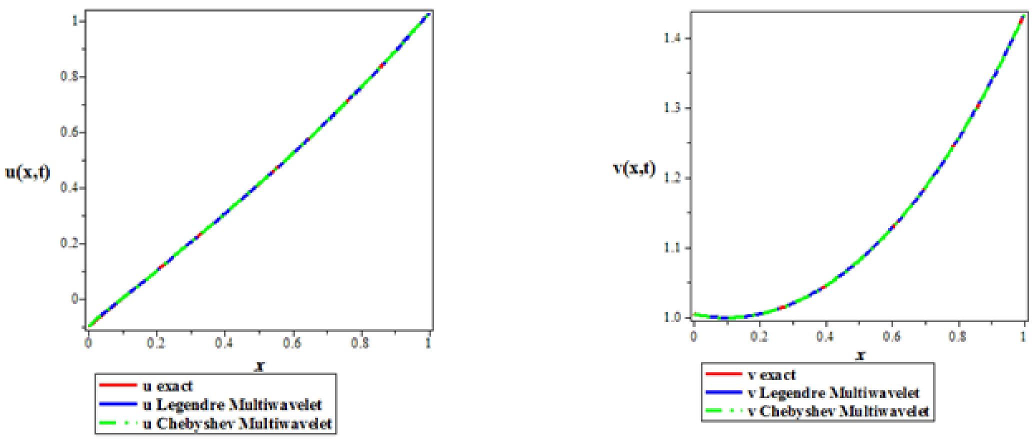

Fig. 3c. *Exact, Legendre Chebyshev multiwavelet solution of u(x, t) and v(x t) for 0 ≤ x ≤ 1, t=0.1*

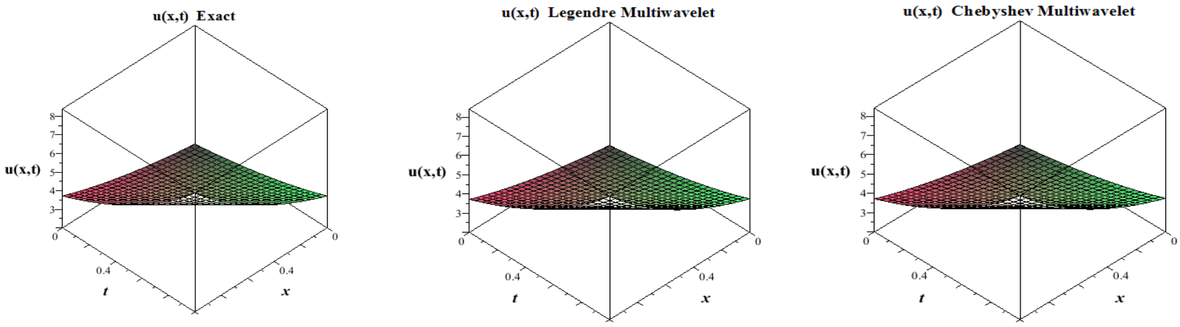

Fig. 4a *Exact and approximate solution of u(x, t) by Legendre and Chebyshev multi wavelet method, 0≤x, t≤1*

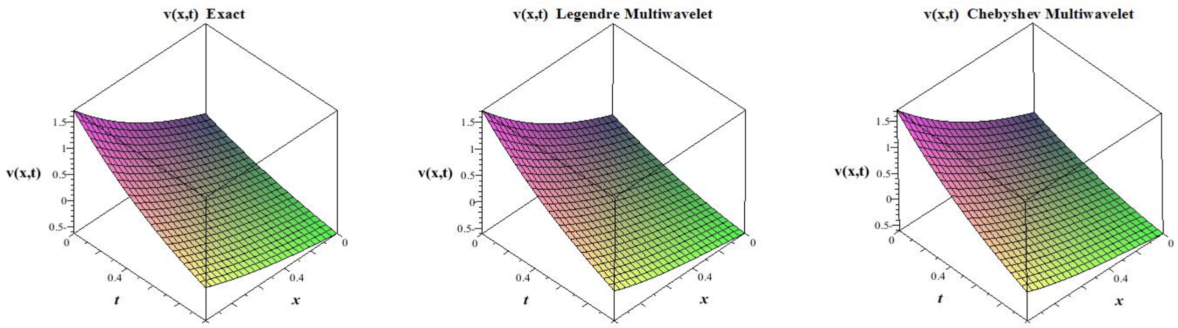

Fig. 4b *Exact and approximate solution of v(x, t) by Legendre and Chebyshev multi wavelet method, 0≤x,t≤1*

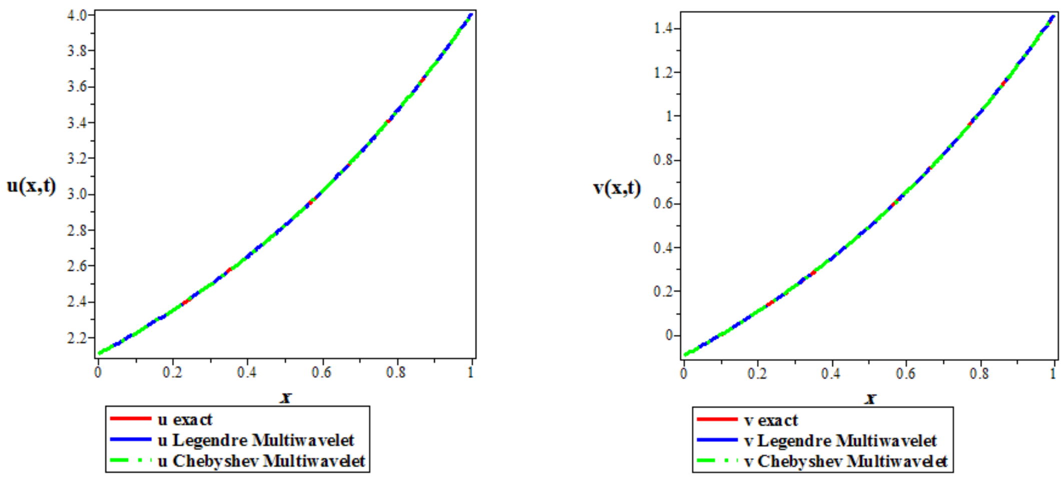

Fig. 4c. *Exact, Legendre Chebyshev multiwavelet solution of u(x, t) and v(x t) for 0 ≤ x ≤ 1, t=0.1*

Table 1. *The error of u(x,t) and v(x,t) of example 1*

x , t=0.1	$\|u_{ex} - u_{Legendre}\|$	$\|v_{ex} - v_{Legendre}\|$	$\|u_{ex} - u_{Chebyshev}\|$	$\|v_{ex} - v_{Chebyshev}\|$
0	4.3×10^{-5}	7.9×10^{-5}	4.3×10^{-5}	7.9×10^{-5}
0.1	3.3×10^{-5}	1.7×10^{-5}	3.3×10^{-5}	1.7×10^{-5}
0.2	8.3×10^{-6}	2.9×10^{-5}	8.3×10^{-6}	2.9×10^{-5}
0.3	1.3×10^{-5}	4.9×10^{-6}	1.3×10^{-5}	4.9×10^{-6}
0.4	2.1×10^{-5}	2.2×10^{-5}	2.1×10^{-5}	2.2×10^{-5}
0.5	1.2×10^{-5}	3.1×10^{-5}	1.2×10^{-5}	3.1×10^{-5}
0.6	5.6×10^{-6}	1.6×10^{-5}	5.6×10^{-6}	1.6×10^{-5}
0.7	2.2×10^{-5}	1.5×10^{-5}	2.2×10^{-5}	1.5×10^{-5}
0.8	1.8×10^{-5}	3.4×10^{-5}	1.8×10^{-5}	3.4×10^{-5}
0.9	3.1×10^{-5}	5.0×10^{-6}	3.1×10^{-5}	4.9×10^{-6}
1.0	1.5×10^{-4}	1.7×10^{-4}	4.3×10^{-5}	7.9×10^{-5}

Table 2. *The error of u(x,t) and v(x,t) of example 2*

x , t=0.1	$\lvert u_{ex} - u_{Legendre}\rvert$	$\lvert v_{ex} - v_{Legendre}\rvert$	$\lvert u_{ex} - u_{Chebyshev}\rvert$	$\lvert v_{ex} - v_{Chebyshev}\rvert$
0	1.6×10^{-4}	2.1×10^{-4}	1.6×10^{-4}	2.1×10^{-4}
0.1	6.1×10^{-5}	1.2×10^{-4}	6.2×10^{-5}	1.2×10^{-4}
0.2	3.2×10^{-5}	8.7×10^{-5}	3.2×10^{-5}	8.7×10^{-5}
0.3	3.8×10^{-5}	7.1×10^{-5}	3.8×10^{-5}	7.1×10^{-5}
0.4	5.3×10^{-5}	6.6×10^{-5}	5.4×10^{-5}	6.6×10^{-5}
0.5	6.2×10^{-5}	6.1×10^{-5}	6.2×10^{-5}	6.1×10^{-5}
0.6	5.5×10^{-5}	4.9×10^{-5}	5.6×10^{-5}	4.9×10^{-5}
0.7	3.6×10^{-5}	3.1×10^{-5}	3.6×10^{-5}	3.1×10^{-5}
0.8	1.1×10^{-5}	1.1×10^{-5}	1.1×10^{-5}	1.1×10^{-5}
0.9	4.4×10^{-7}	3.3×10^{-7}	3.1×10^{-7}	5.1×10^{-7}
1.0	2.1×10^{-4}	1.7×10^{-4}	1.6×10^{-4}	2.1×10^{-4}

Table 3. *The error of u(x,t) and v(x,t) of example 3*

x , t=0.1	$\lvert u_{ex} - u_{Legendre}\rvert$	$\lvert v_{ex} - v_{Legendre}\rvert$	$\lvert u_{ex} - u_{Chebyshev}\rvert$	$\lvert v_{ex} - v_{Chebyshev}\rvert$
0	5.9×10^{-5}	3.2×10^{-5}	5.9×10^{-5}	3.2×10^{-5}
0.1	7.3×10^{-6}	1.7×10^{-5}	7.3×10^{-6}	1.7×10^{-5}
0.2	1.3×10^{-5}	7.4×10^{-7}	1.3×10^{-5}	7.4×10^{-7}
0.3	5.3×10^{-6}	1.5×10^{-5}	5.1×10^{-6}	1.5×10^{-5}
0.4	2.5×10^{-5}	2.2×10^{-5}	2.5×10^{-5}	2.2×10^{-5}
0.5	3.1×10^{-5}	1.9×10^{-5}	3.1×10^{-5}	1.9×10^{-5}
0.6	2.1×10^{-5}	9.4×10^{-6}	2.1×10^{-5}	9.5×10^{-6}
0.7	4.5×10^{-7}	1.1×10^{-6}	5.0×10^{-7}	1.3×10^{-6}
0.8	1.1×10^{-5}	1.6×10^{-6}	1.2×10^{-5}	1.8×10^{-6}
0.9	1.6×10^{-5}	2.9×10^{-5}	1.6×10^{-5}	2.9×10^{-5}
1.0	2.1×10^{-4}	1.1×10^{-4}	5.9×10^{-5}	3.2×10^{-5}

Table 4. *The error of u(x,t) and v(x,t) of example 4*

x , t=0.1	$\lvert u_{ex} - u_{Legendre}\rvert$	$\lvert v_{ex} - v_{Legendre}\rvert$	$\lvert u_{ex} - u_{Chebyshev}\rvert$	$\lvert v_{ex} - v_{Chebyshev}\rvert$
0	3.6×10^{-5}	6.0×10^{-5}	3.9×10^{-5}	6.0×10^{-5}
0.1	1.1×10^{-4}	3.1×10^{-5}	1.1×10^{-4}	3.0×10^{-5}
0.2	1.3×10^{-4}	5.5×10^{-5}	1.3×10^{-4}	5.5×10^{-5}
0.3	8.1×10^{-5}	9.5×10^{-5}	8.1×10^{-5}	9.6×10^{-5}
0.4	1.1×10^{-5}	1.2×10^{-4}	1.1×10^{-5}	1.2×10^{-4}
0.5	4.3×10^{-5}	1.2×10^{-2}	4.2×10^{-5}	1.2×10^{-4}
0.6	6.4×10^{-5}	9.3×10^{-5}	6.4×10^{-5}	9.3×10^{-5}
0.7	5.2×10^{-5}	4.9×10^{-5}	5.2×10^{-5}	4.8×10^{-5}
0.8	2.2×10^{-5}	2.3×10^{-5}	2.2×10^{-5}	2.2×10^{-5}
0.9	1.6×10^{-5}	6.4×10^{-5}	1.6×10^{-5}	6.4×10^{-5}
1.0	9.5×10^{-5}	2.4×10^{-4}	3.9×10^{-5}	6.0×10^{-5}

5. CONCLUSION

In the current work the Legendrmultiwavelet and Chebyshev multiwavelet have been applied for solving linear second order system of PDE's by reducing the linear first order system of PDE'sinto system of algebraic equations and with solving this system we obtained approximate solution of the problem. In addition, an illustrative example have been included to demonstrate the validity and applicability of the methods.

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