Combustion Fronts in Porous Media: Modeling and Analytical Simulation

R. O. Olayiwola

Department of Mathematics,
Federal University of Technology,
Minna, Nigeria
olayiwolarasaq@yahoo.co.uk

A. A. Mohammed

Department of Mathematics,
Federal University of Technology,
Minna, Nigeria
adamualhaj@yahoo.com

Falaye, A. A.

Department of Mathematics,
Federal University of Technology,
Minna, Nigeria
falaye.adeyinka@futminna.edu.ng

Adetutu, O. M.

Department of Statistics,
Federal University of Technology,
Minna, Nigeria
ola.adetutu@futminna.edu.ng

Abstract: *In this paper, a mathematical model for forward propagation of a combustion front with Arrhenius kinetics through a porous medium is presented. The reaction involves oxygen and a solid fuel. We assume that this solid fuel depends on the space variable. Under an incompressible assumption, we obtain a simple model whose variables are temperature, oxygen mass fraction, fuel concentration, seepage velocity and pressure. We also assume that the amount of gas produced by the reaction is equal to the amount of gas consumed by it. By actual solution, we prove the existence and uniqueness of solution of the model. We examine the properties of solution under certain conditions. Using large activation energy asymptotics, the analytical solution of the model was provided via Homotopy perturbation method (HPM) and the results are presented graphically. It is discovered that the influence of Frank-Kamenetskii number on the heat transfer and species consumption is quite significant..*

Keywords: *Combustion front, Solid fuel, Porous medium, Homotopy Perturbation Method (HPM).*

1. INTRODUCTION

Combustion fronts in porous media have been studied by many authors during the last few decades. In particular, for combustion processes in oil recovery, models and results of numerical simulations have been presented. One of the first models of combustion in a petroleum reservoir was formulated by Gottfried [1]. The model consists of a system of six partial differential equations describing the flow of oil, water, and gas through the porous medium, together with a chemical reaction between oxygen and oil. Crookston and Culham [2] presented a general model for thermal recovery processes, as well as associated numerical procedures. In addition to the aspects of combustion processes modelled by Gottfried, they included such aspects as coke formation and oxidation.

These models are nonlinear reaction–diffusion–convection systems derived from the principle of conservation. In vector form in one space dimension, they have the form

$$H(U)_t + F(U)_x = (B(U)U_x)_x + G(U), \quad 0 < x < l, \ t > 0, \tag{1}$$

where *l* is the length of the porous medium and *U* is the vector of unknown quantities, such as temperature and densities. The first and second terms in (1) represent, respectively, accumulation and transport by convection of these quantities; the function *G* represents source terms due to chemical reactions and heat loss; and the term $(B(U)U_x)_x$ represents diffusion of heat, mass, etc.

The combustion process is described by the solution of the system (1), with suitable initial and boundary conditions.
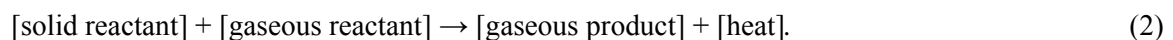
In the more recent literature, several authors have studied the oxidation of crude oil with air injected in porous media. These include Ayeni [3] who studied thermal runaway phenomena while investigating the reaction of oxygen and hydrogen. He provided useful theorems on such flows. Olayiwola and Ayeni [4] studied the continuity, momentum and coupled nonlinear energy and species convection-diffusion equations describing the in-situ combustion process in porous media. The numerical solution was provided using large activation energy asymptotics and shooting method.

Some recent work identifies combustion fronts with traveling waves, and proves their existence using geometric methods. Da Mota et al. [5, 6] used this approach to study combustion fronts in a two-phase (oil and oxygen) model. Combustion fronts were identified with traveling waves connecting an unburned state ahead of the front to a burned state behind it.

In this work, we study a model for combustion of oxygen and a solid fuel such as coke in a porous medium. As in [4], we assume the fuel depends on the space variable x. We also assume that the amount of gas produced by the reaction is equal to the amount of gas consumed by it. We shift and rescale the temperature so that 0 corresponds to the initial temperature of the porous medium T_0, which are also take to be the ignition temperature. Physically relevant solutions must have $0 \leq x \leq l$. However, in order to have a clear picture of solution in this region, we consider all solutions in the upper half plane $x \geq 0$. We prove the existence and uniqueness of solution. We also examine the properties of solution. To simulate the flow analytically, we assume that the incoming mixture is at the burner temperature.

2. MODEL FORMULATION

We consider a horizontal one-dimensional porous medium with an initially available concentration of a solid fuel such as coke. The space variable is x, $0 < x < \infty$. The chemical reaction in the medium takes the simple form

$$[\text{solid reactant}] + [\text{gaseous reactant}] \rightarrow [\text{gaseous product}] + [\text{heat}]. \tag{2}$$

To formulate balance equations, we assume that fuel depends on space variable x. We also assume that the fluids are incompressible. Thus ρ_g and ρ_f are constants. The state variables depending on (x) are temperature T, oxygen mass fraction in the gas phase C_{ox}, fuel concentration C_f, seepage velocity v and pressure p.

The following steady state equations are assumed to hold in the porous medium:

Balance of energy

$$v\frac{dT}{dx} = \frac{\lambda}{\rho_g c_{pg}}\left(\frac{d^2T}{dx^2}\right) + \frac{Q\omega}{\rho_g c_{pg}} \tag{3}$$

Balance of oxygen mass

$$v\frac{dC_{ox}}{dx} = D_{ox}\left(\frac{d^2C_{ox}}{dx^2}\right) - \frac{s_{ox}\omega}{\rho_g} \tag{4}$$

Balance of fuel mass

$$v\frac{dC_f}{dx} = D_f\left(\frac{d^2C_f}{dx^2}\right) - \frac{(1-s_{sf})\omega}{\rho_f} \tag{5}$$

Balance of total gas mass

$$\rho_g\frac{dv}{dx} = s_g\omega \tag{6}$$

Darcy's law

$$v = -k \frac{\partial p}{\partial x}$$ (7)

The rate of coke consumption in the chemical reaction, in medium, is assumed to be given by a version of Arrhenius's law:

$$\omega = \begin{cases} A(C_{ox} p)^a C_f e^{-\frac{E}{RT}}, & for \ T > 0 \\ 0, & for \ T \le 0 \end{cases},$$ (8)

where A is the Arrhenius constant, E the activation energy, a the order of the gaseous reaction rate, and R is the gas constant, ρ_g is the gas density, ρ_f is the fuel density, λ is thermal conductivity, c_{pg} is heat capacity of gas at constant pressure, ω is rate of coke consumption in the chemical reaction, Q is heat of reaction, s_{ox} is the mass–weighted stoichiometric coefficients for oxygen, s_{sf} is the mass–weighted stoichiometric coefficients for solid fuel, s_g is the mass–weighted stoichiometric coefficients for gas, k is the flow resistance, which is directly proportional to rock permeability and inversely proportional to gas viscosity.

The quantity s_g may be positive, negative, or zero, depending on whether the amount of gas produced by the reaction is more than, less than, or equal to the amount of gas consumed by it. Here, we will assume $s_g = 0$, $\rho_g = \rho_f = \rho$.

From the balance of total gas mass (6), we obtain

$$v = constant$$ (9)

We let

$$v = -u_0, \quad u_0 > 0$$

From the Darcy's law, we obtain

$$p = \frac{u_0}{k} x + p_0,$$ (10)

where p_0 is the initial total gas pressure.

From now on, we shall assume that the pressure is constant. Although these assumptions could be relaxed in the future, they considerably simplify the equations. With these simplifications, Equations (3) – (8) reduces to the following system modeling temperature, oxygen mass fraction and fuel concentration.

$$u_0 \frac{dT}{dx} + \frac{\lambda}{\rho c_{pg}} \left(\frac{d^2 T}{dx^2} \right) + \frac{Q\omega}{\rho c_{pg}} = 0$$ (11)

$$u_0 \frac{dC_{ox}}{dx} = D_{ox} \left(\frac{d^2 C_{ox}}{dx^2} \right) - \frac{s_{ox}\omega}{\rho} = 0$$ (12)

$$u_0 \frac{dC_f}{dx} = D_f \left(\frac{d^2 C_f}{dx^2} \right) - \frac{(1 - s_{sf})\omega}{\rho} = 0$$ (13)

The boundary conditions were formulated as follows:

Boundary conditions:

$$\left. T \right|_{x=0} = T_b, \qquad \left. C_{ox} \right|_{x=0} = C_0, \qquad \left. C_f \right|_{x=0} = C_{f0} \right\}, \tag{14}$$

$$\lim_{\eta \to \infty} T(x) = T_0, \ \lim_{\eta \to \infty} C_{ox}(x) = \lim_{\eta \to \infty} C_f(x) = 0 \right\}$$

where the subscript b means burner.

3. METHOD OF SOLUTION

3.1 Existence and Uniqueness of Solution

Theorem 1: Let $D_f = D_{ox} = \dfrac{\lambda}{\rho c_{pg}} = \lambda_1$, $s_{ox} = 1 - s_{sf}$. Then there exists a unique solution of problem (11) - (13) satisfy (14).

Proof: Let $D_f = D_{ox} = \dfrac{\lambda}{\rho c_{pg}} = \lambda_1$, $s_{ox} = 1 - s_{sf}$ and $\phi = \left(s_{ox} T + \dfrac{Q}{2c_{pg}} \left(C_{ox} + C_f \right) \right)$.

Then (11) - (14) become

$$u_0 \frac{d\phi}{dx} + \lambda_1 \frac{d^2\phi}{dx^2} = 0 \tag{15}$$

$$\phi(0) = s_{ox} T_b + \frac{Q}{2c_{pg}} \left(C_0 + C_{f0} \right), \quad \phi(x) \to s_{ox} T_0 \ as \ x \to \infty \tag{16}$$

We obtain the solution of problem (15) as

$$\phi(x) = \left(s_{ox}(T_b - T_0) + \frac{Q}{2c_{pg}} \left(C_0 + C_{f0} \right) \right) e^{-\frac{u_0}{\lambda_1}x} + s_{ox} T_o \tag{17}$$

Then, we obtain

$$T(x) = \left((T_b - T_0) + \frac{Q}{2s_{ox}c_{pg}} \left(C_0 + C_{f0} \right) \right) e^{-\frac{u_0}{\lambda_1}x} + T_o - \frac{Q}{2s_{ox}c_{pg}} \left(C_{ox}(x) + C_f(x) \right) \tag{18}$$

$$C_{ox}(x) = \left(\frac{2s_{ox}c_{pg}}{Q} (T_b - T_0) + \left(C_0 + C_{f0} \right) \right) e^{-\frac{u_0}{\lambda_1}x} + \frac{2s_{ox}c_{pg}}{Q} T_o - \left(\frac{2s_{ox}c_{pg}}{Q} T(x) + C_f(x) \right) \tag{19}$$

$$C_f(x) = \left(\frac{2s_{ox}c_{pg}}{Q} (T_b - T_0) + \left(C_0 + C_{f0} \right) \right) e^{-\frac{u_0}{\lambda_1}x} + \frac{2s_{ox}c_{pg}}{Q} T_o - \left(\frac{2s_{ox}c_{pg}}{Q} T(x) + C_{ox}(x) \right) \tag{20}$$

Hence, there exists a unique solution of problem (11) - (14). This completes the proof.

3.2 Non-dimensionalisation

We scale the length by using $x^* = \dfrac{\alpha}{v^i}$, where v^i is the injection velocity and α the effective thermal diffusivity. We introduce dimensionless variables for space,

$$x' = \frac{x}{x^*} \tag{21}$$

We also introduce dimensionless variables for temperature, fuel concentration, oxygen mass fraction, velocity, pressure and density;

$$\theta = \frac{E}{RT_0^2}(T - T_0), \quad p' = \frac{p}{p^*}, \quad \rho' = \frac{\rho}{\rho^*}, \quad C_{ox}' = \frac{C_{ox}}{C_{ox}^0}, \quad C_f' = \frac{C_f}{C_f^0}, \quad u_0 = \frac{\lambda_1}{x^*} \tag{22}$$

where T_0 is the initial temperature of the porous medium and C_f^0, C_{ox}^0 are the initial fuel concentration and initial oxygen mass fraction, respectively; x^*, p^*, ρ^* are reference values for space, pressure and density, respectively.

Using (19) and (20), with these dimensionless variables, and after dropping the prime, the system (11) – (14) become

$$\frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} + \delta\left(\gamma e^{-x} - \upsilon\theta - C_f^0 C_f\right)^a \left(\gamma e^{-x} - \upsilon\theta - C_{ox}^0 C_{ox}\right) e^{\frac{\theta}{1+\epsilon\theta}} = 0 \tag{23}$$

$$\frac{d^2 C_{ox}}{dx^2} + \frac{d C_{ox}}{dx} - \beta\left(\gamma e^{-x} - \upsilon\theta - C_f^0 C_f\right)^a \left(\gamma e^{-x} - \upsilon\theta - C_{ox}^0 C_{ox}\right) e^{\frac{\theta}{1+\epsilon\theta}} = 0 \tag{24}$$

$$\frac{d^2 C_f}{dx^2} + \frac{d C_f}{dx} - \sigma\left(\gamma e^{-x} - \upsilon\theta - C_f^0 C_f\right)^a \left(\gamma e^{-x} - \upsilon\theta - C_{ox}^0 C_{ox}\right) e^{\frac{\theta}{1+\epsilon\theta}} = 0 \tag{25}$$

$$\left.\begin{array}{ll} \theta(0) = \theta_b, & \theta(x) \to 0 \ as \ x \to \infty \\[2mm] C_{ox}(0) = C_0, & C_{ox}(x) \to 0 \ as \ x \to \infty \\[2mm] C_f(0) = C_{f0}, & C_f(x) \to 0 \ as \ x \to \infty \end{array}\right\} \tag{26}$$

where

$$\epsilon = \frac{RT_0}{E}, \qquad \upsilon = \frac{2 s_{ox} c_{pg} \in T_0}{Q}, \qquad \gamma = \frac{2 s_{ox} c_{pg}}{Q}(T_b - T_0) + (C_0 + C_{f0}),$$

$$\beta = \frac{\alpha^2 s_{ox} A\left(C_{ox}^0 p^* p'\right)^a C_f^0 C_{ox}'^a C_f' e^{-\frac{E}{RT_0}}}{\rho^* \rho' C_{ox}^0 v'^2 \lambda_1}, \qquad \sigma = \frac{\alpha^2 (1 - s_{sf}) A\left(C_{ox}^0 p^* p'\right)^a C_f^0 C_{ox}'^a C_f' e^{-\frac{E}{RT_0}}}{\rho^* \rho' C_f^0 v'^2 \lambda_1},$$

$$\delta = \frac{\alpha^2 Q A\left(C_{ox}^0 p^* p'\right)^a C_f^0 C_{ox}'^a C_f' e^{-\frac{E}{RT_0}}}{\rho^* \rho' c_{pg} \in T_0 v'^2 \lambda_1} : \text{Frank-Kamenetskii number}$$

3.3 Properties of Solution

In this section, we consider equation (23) when $\in \to 0$, $\theta_b = 0$ and transform the equation from infinite domain to finite domain, using the transformation $y = e^{-x}$.

We obtain

$$\frac{d^2\theta}{dy^2} + \frac{\delta}{y^2}\left(\gamma y - \upsilon\theta - C_f^0 C_f\right)^a \left(\gamma y - \upsilon\theta - C_{ox}^0 C_{ox}\right) e^{\theta} = 0 \tag{27}$$

$$\theta(0) = 0, \qquad \theta(1) = 0$$

Theorem 2: Let $\upsilon = C_{ox}^0 = C_f^0 = 0$ and $a = 1$ in (27). Then $\theta(y)$ is symmetric about $y = \dfrac{1}{2}$.

Proof: Let $\upsilon = C_{ox}^0 = C_f^0 = 0$ and $a = 1$ in (27). We obtain

$$\frac{d^2\theta(y)}{dy^2} + \delta_1 e^{\theta(y)} = 0$$

$$\theta(0) = 0, \qquad \theta(1) = 0,$$

Let $z = 2y - 1$

Then

$$\frac{d^2}{dy^2} = 4\frac{d^2}{dz^2}$$

So the problem becomes

$$\frac{d^2\theta(z)}{dz^2} + \frac{\delta_1}{4} e^{\theta(z)} = 0$$

$$\theta(-1) = \theta(1) = 0$$

It suffices to show that $\theta(-z) = \theta(z)$.

Replace z by $-z$. We obtain

$$\frac{d^2\theta(-z)}{d(-z)^2} + \frac{\delta_1}{4} e^{\theta(-z)} = 0$$

Hence θ is symmetric about $z = 0$ i.e. θ is symmetric about $y = \dfrac{1}{2}$. This completes the proof.

Theorem 3: Let $\upsilon = C_{ox}^0 = C_f^0 = 0$ and $a = 1$ in (27). Then $\theta'\left(\dfrac{1}{2}\right) = 0$.

Proof: Let $\upsilon = C_{ox}^0 = C_f^0 = 0$ and $a = 1$ in (27). We obtain

$$\frac{d^2\theta(y)}{dy^2} + \delta_1 e^{\theta(y)} = 0$$

$$\theta(0) = 0, \qquad \theta(1) = 0,$$

Since $\theta(y)$ is symmetric about $y = \dfrac{1}{2}$. Then $\theta'\left(\dfrac{1}{2}\right) = 0$. This completes the proof.

Theorem 4: Let $\upsilon = C_{ox}^0 = C_f^0 = 0$ and $a = 1$ in (27). Then $\theta'(y) > 0$ for $y \in \left(0, \dfrac{1}{2}\right)$.

Proof: Let $\upsilon = C_{ox}^0 = C_f^0 = 0$ and $a = 1$ in (27). We obtain

$$\frac{d^2\theta(y)}{dy^2} = -\delta_1 e^{\theta(y)}$$

$$\theta(0) = 0, \qquad \theta(1) = 0,$$

Using Ayeni [3], we obtain

$$\theta(y) = \delta_1 \int_0^{\frac{1}{2}} k(y,t) e^{\theta(t)} dt,$$

where

$$k(y,t) = \begin{cases} y, & 0 \le y \le t \\ \\ t, & t \le y \le \dfrac{1}{2} \end{cases}$$

So

$$\theta'(y) = \delta_1 \left[y\, e^{\theta(y)} + \int_y^{\frac{1}{2}} e^{\theta(t)} dt - y\, e^{\theta(y)} \right]$$

$$= \delta_1 \int_y^{\frac{1}{2}} e^{\theta(t)} dt$$

Hence, $\theta(y)$ is strictly monotonically increasing for $y \in \left(0, \dfrac{1}{2}\right)$. This completes the proof.

3.4 Solution by Homotopy Perturbation Method

Homotopy perturbation method (HPM) was first proposed by He and was successfully applied to various engineering problems [7].

Here, we consider equations (23) – (26) when $\in\,\to 0$, $C_f^0 = C_{ox}^0 = 0$, $a = 1$ and using the transformation $y = e^{-x}$.

We obtain

$$\frac{d^2\theta}{dy^2} + \frac{\delta}{y^2}(\gamma y - \upsilon\theta)^2\, e^\theta = 0 \tag{28}$$

$$\theta(0) = 0, \qquad \theta(1) = \theta_b$$

$$\frac{d^2 C_{ox}}{dy^2} - \frac{\beta}{y^2}(\gamma y - \upsilon\theta)^2\, e^\theta = 0 \tag{29}$$

$$C_{ox}(0) = 0, \qquad C_{ox}(1) = C_0$$

$$\frac{d^2 C_f}{dy^2} - \frac{\sigma}{y^2}(\gamma y - \upsilon\theta)^2\, e^\theta = 0 \tag{30}$$

$$C_f(0) = 0, \qquad C_f(1) = C_{f0}$$

Using the approximation (see Ayeni [8]):

$$\exp(\theta) \approx 1 + (e - 2)\theta + \theta^2 \tag{31}$$

We apply Homotopy-perturbation to equations (28) – (30), where details can be found in [7] and obtain the following approximated solutions:

$$\theta(y) = \theta_b y + \delta\left(\left(\frac{1}{2}A + \frac{1}{6}B + \frac{1}{12}C\right) - \left(\frac{1}{2}Ay^2 + \frac{1}{6}By^3 + \frac{1}{12}Cy^4\right)\right) +$$

$$\left(\begin{array}{l} \left(\begin{array}{l} -A_1 D - \dfrac{1}{12}A_1 A - \dfrac{1}{72}A_1 B - \dfrac{1}{240}A_1 C + \dfrac{1}{2}B_1 D - \dfrac{1}{24}B_1 A - \\ \dfrac{1}{120}B_1 B - \dfrac{1}{360}B_1 C + \dfrac{1}{6}C_1 D - \dfrac{1}{40}C_1 A - \dfrac{1}{180}C_1 B - \dfrac{1}{504}C_1 C \end{array}\right) - \\ \delta^2\left(\begin{array}{l} A_1 D(\ln y - 1)y - \dfrac{1}{12}A_1 Ay^3 - \dfrac{1}{72}A_1 By^4 - \dfrac{1}{240}A_1 Cy^5 + \dfrac{1}{2}B_1 Dy^2 - \\ \dfrac{1}{24}B_1 Ay^4 - \dfrac{1}{120}B_1 By^5 - \dfrac{1}{360}B_1 Cy^6 + \dfrac{1}{6}C_1 Dy^3 - \dfrac{1}{40}C_1 Ay^5 - \\ \dfrac{1}{180}C_1 By^6 - \dfrac{1}{504}C_1 Cy^7 \end{array}\right) \end{array}\right) \quad (32)$$

$$C_{ox}(y) = C_0 y + \beta\left(\left(\frac{1}{2}Ay^2 + \frac{1}{6}By^3 + \frac{1}{12}Cy^4\right) - \left(\frac{1}{2}A + \frac{1}{6}B + \frac{1}{12}C\right)\right)$$

$$+ \beta\delta\left(\begin{array}{l} \left(\begin{array}{l} A_1 D(\ln y - 1)y - \dfrac{1}{12}A_1 Ay^3 - \dfrac{1}{72}A_1 By^4 - \dfrac{1}{240}A_1 Cy^5 + \dfrac{1}{2}B_1 Dy^2 - \\ \dfrac{1}{24}B_1 Ay^4 - \dfrac{1}{120}B_1 By^5 - \dfrac{1}{360}B_1 Cy^6 + \dfrac{1}{6}C_1 Dy^3 - \dfrac{1}{40}C_1 Ay^5 - \\ \dfrac{1}{180}C_1 By^6 - \dfrac{1}{504}C_1 Cy^7 \end{array}\right) \\ -\left(\begin{array}{l} -A_1 D - \dfrac{1}{12}A_1 A - \dfrac{1}{72}A_1 B - \dfrac{1}{240}A_1 C + \dfrac{1}{2}B_1 D - \dfrac{1}{24}B_1 A - \\ \dfrac{1}{120}B_1 B - \dfrac{1}{360}B_1 C + \dfrac{1}{6}C_1 D - \dfrac{1}{40}C_1 A - \dfrac{1}{180}C_1 B - \dfrac{1}{504}C_1 C \end{array}\right) \end{array}\right) \quad (33)$$

$$C_f(y) = C_{f0} y + \sigma\left(\left(\frac{1}{2}Ay^2 + \frac{1}{6}By^3 + \frac{1}{12}Cy^4\right) - \left(\frac{1}{2}A + \frac{1}{6}B + \frac{1}{12}C\right)\right) +$$

$$\sigma\delta\left(\begin{array}{l} \left(\begin{array}{l} A_1 D(\ln y - 1)y - \dfrac{1}{12}A_1 Ay^3 - \dfrac{1}{72}A_1 By^4 - \dfrac{1}{240}A_1 Cy^5 + \dfrac{1}{2}B_1 Dy^2 - \\ \dfrac{1}{24}B_1 Ay^4 - \dfrac{1}{120}B_1 By^5 - \dfrac{1}{360}B_1 Cy^6 + \dfrac{1}{6}C_1 Dy^3 - \dfrac{1}{40}C_1 Ay^5 - \\ \dfrac{1}{180}C_1 By^6 - \dfrac{1}{504}C_1 Cy^7 \end{array}\right) \\ -\left(\begin{array}{l} -A_1 D - \dfrac{1}{12}A_1 A - \dfrac{1}{72}A_1 B - \dfrac{1}{240}A_1 C + \dfrac{1}{2}B_1 D - \dfrac{1}{24}B_1 A - \\ \dfrac{1}{120}B_1 B - \dfrac{1}{360}B_1 C + \dfrac{1}{6}C_1 D - \dfrac{1}{40}C_1 A - \dfrac{1}{180}C_1 B - \dfrac{1}{504}C_1 C \end{array}\right) \end{array}\right) \quad (34)$$

Then $\theta(y)$, $C_{ox}(y)$ and $C_f(y)$ can be expressed as function of x using previous transformation, $y = e^{-x}$. The computations were done using computer symbolic algebraic package MAPLE.

where

$$q = e - 2, \quad A = (\gamma - \upsilon\theta_b)^2, \quad B = q\theta_b(\gamma - \upsilon\theta_b)^2, \quad C = \theta_b^2(\gamma - \upsilon\theta_b)^2,$$

$$D = \left(\frac{1}{2}A + \frac{1}{6}B + \frac{1}{12}C\right), \quad A_1 = -2\upsilon(\gamma - \upsilon\theta_b), \quad B_1 = q(\gamma^2 - 4\upsilon\gamma\theta_b + 3\upsilon^2\theta_b^2),$$

$$C_1 = 2\theta_b(\gamma^2 - 3\upsilon\gamma\theta_b + 2\upsilon^2\theta_b^2)$$

4. RESULTS AND DISCUSSION

Here the existence and uniqueness of solution of the Problem is proved by actual solutions. Also, under certain condition, we have examined the properties of solution. Analytical solutions of equations (28) - (30) are achieved via Homotopy perturbation method and computed for the values of $\gamma = 1,\ \beta = 0.1,\ \sigma = 0.3,\ \upsilon = 1,\ e = 2.718$. The species concentration and temperature values are depicted graphically in Figs. 1 - 6.

From Figs. 1 and 2, we can conclude that with the increase of Frank-Kamenetskii number (δ), temperature increases.
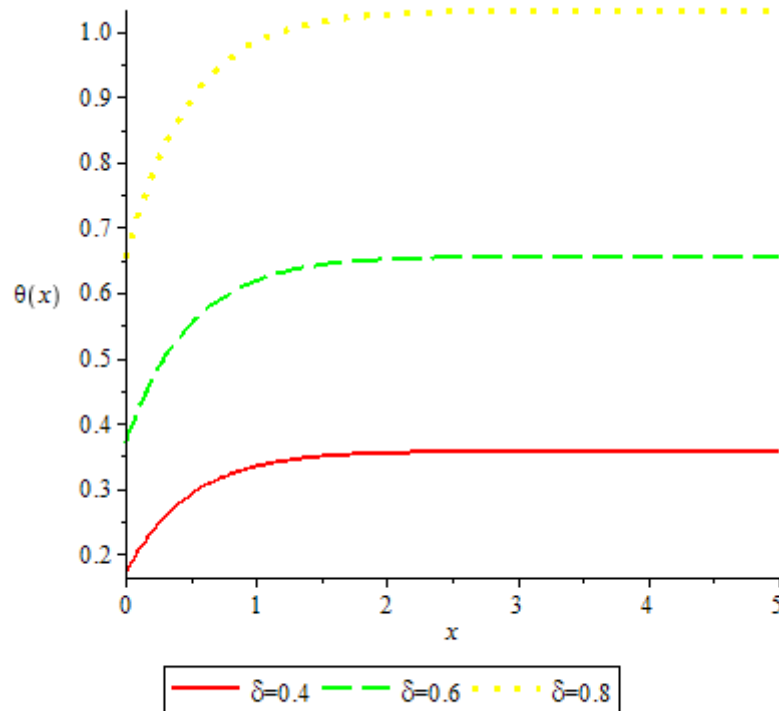
Fig. 1: Temperature variation versus distance x for various values of δ at
γ = 1, β = 0.1, σ = 0.3, υ = 1, ε = 0.01, e = 2.718, θ_b = 0.01

Fig. 2: Temperature variation versus distance x for various values of δ at
γ = 1, β = 0.1, σ = 0.3, υ = 1, ε = 0.01, e = 2.718, θ_b = 0.01

From Figs. 3 and 4, we can conclude that with the increase of Frank-Kamenetskii number (δ), oxygen mass fraction decreases.

Fig. 3: Oxygen mass fraction variation versus distance x for various values of δ at $\gamma = 1$, $\beta = 0.1$, $\sigma = 0.3$, $\upsilon = 1$, $e = 2.718$, $\theta_b = 0.01$, $C_0 = 0.1$
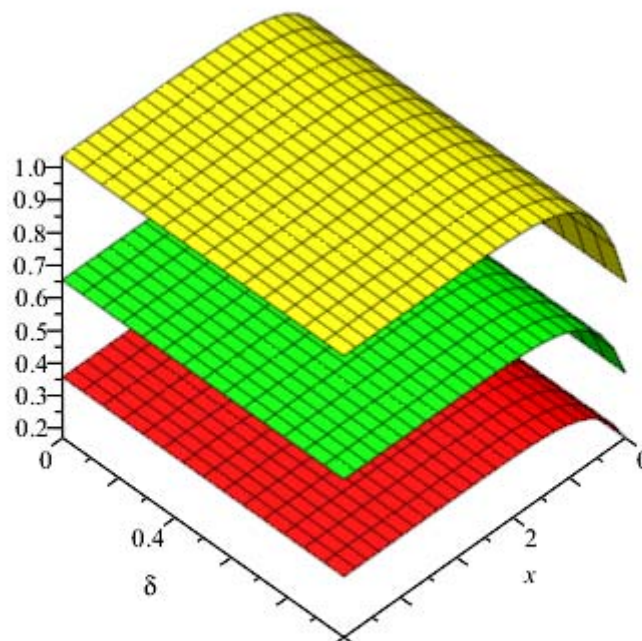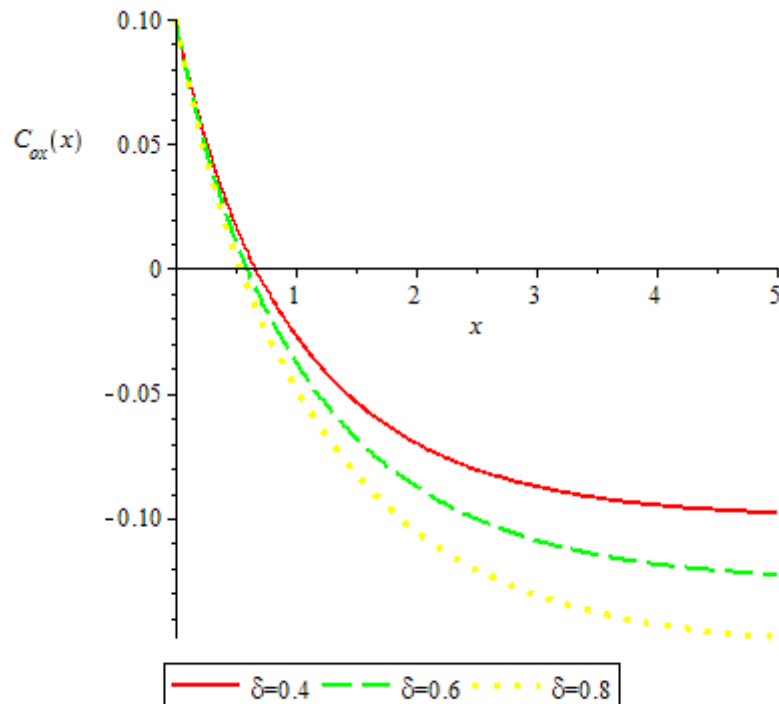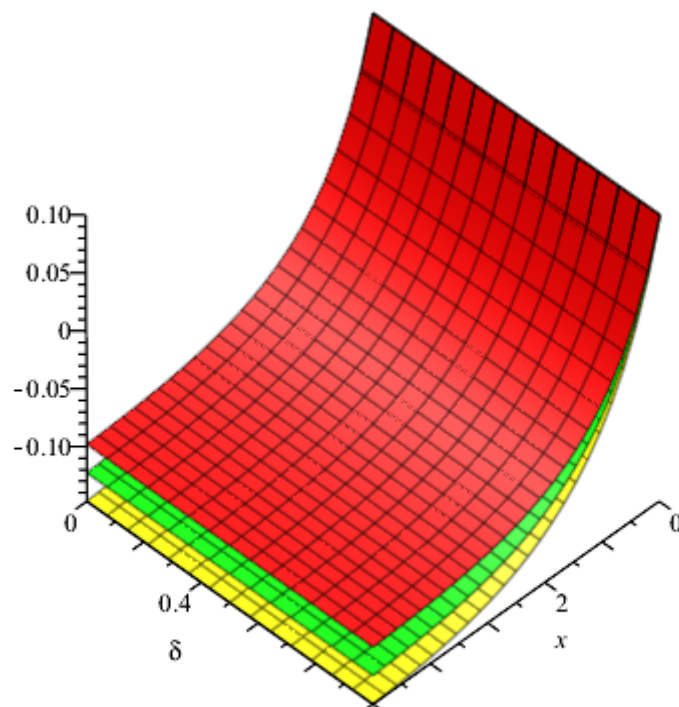

Fig. 4: Oxygen mass fraction variation versus distance x for various values of δ at $\gamma = 1$, $\beta = 0.1$, $\sigma = 0.3$, $\upsilon = 1$, $e = 2.718$, $\theta_b = 0.01$, $C_0 = 0.1$

From Figs. 5 and 6, we can conclude that with the increase of Frank-Kamenetskii number (δ), fuel mass fraction decreases.
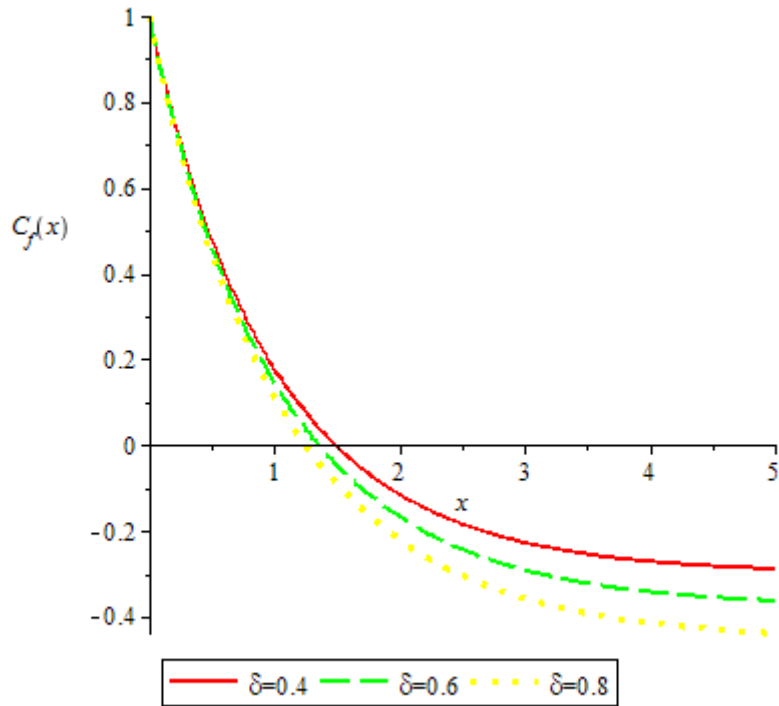

Fig. 5: Fuel mass fraction variation versus distance x for various values of δ at $\gamma = 1$, $\beta = 0.1$, $\sigma = 0.3$, $\upsilon = 1$, $e = 2.718$, $\theta_b = 0.01$, $C_{f0} = 1$
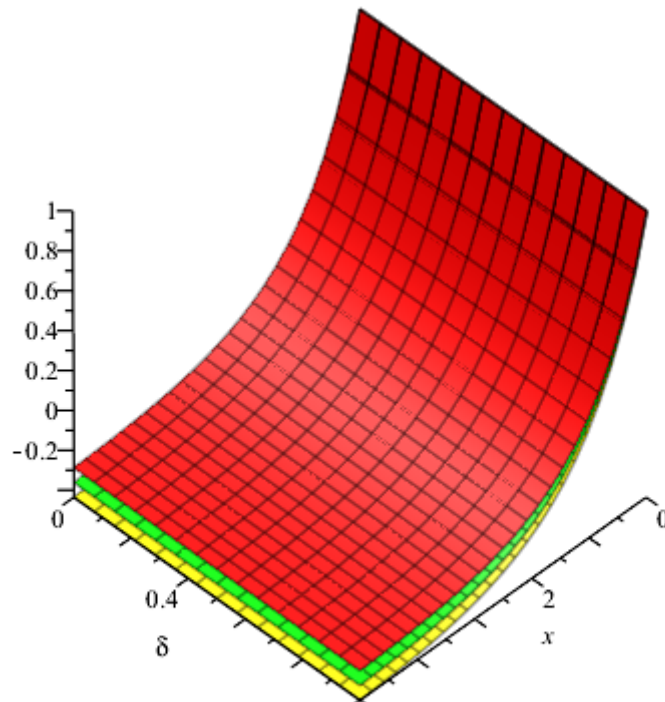

Fig. 6: Fuel mass fraction variation versus distance x for various values of δ at $\gamma = 1$, $\beta = 0.1$, $\sigma = 0.3$, $\upsilon = 1$, $e = 2.718$, $\theta_b = 0.01$, $C_{f0} = 1$

5. CONCLUSION

From the studies made on this paper we conclude that Frank-Kamenetskii number enhance the temperature and decrease the species concentration. These occur as a result of increase in heat of reaction. When the heat of reaction is high, the rate of conversion of solid fuel into light oils, water and gas is high and consequently, the recovery rate is boosted. This is of great economic importance.

The results may be used as a preliminary predictive tool to study mathematically the forward propagation of a combustion front through a porous medium. The work may be extended to more complex cases such as when the amount of gas produced by the reaction is more than or less than the amount of gas consumed by it and therefore, recommended for further research.

REFERENCES

[1] Gottfried B. S., A mathematical model of thermal oil recovery in linear systems, Soc. Pet. Eng. J. 196 - 210 (1965).

[2] Crookston R. B. and Culham W. E., A numerical simulation model for thermal recovery processes, Soc. Pet. Eng. J. 9, 37 - 58 (1979).

[3] Ayeni R. O., Thermal Runaway, Ph.D. Thesis Cornell University USA. Nigeria. Pp. 25 (1978)

[4] Olayiwola R. O. and Ayeni R. O., In-situ combustion simulation for heavy oil reservoirs, WSEAS Transaction on Mathematics. 10(9), 301 - 309 (2011).

[5] Da Mota J. C., Dantas W. and Marchesin D., Traveling waves for combustion in porous media, Int. Ser. Num. Math. Birkhauser. 129, 177 - 187 (1999).

[6] Da Mota J. C., Dantas W. and Marchesin D., Combustion fronts in porous media, SIAM J. Appl. Math. 62, 2175 - 2198 (2002).

[7] He J. H., Some asymptotic methods for strongly nonlinear equations, Int. J. Modern Phys. B. 20(10), 1141 - 1199 (2006).

[8] Ayeni R. O., On the explosion of chain-thermal reactions, J. Austral. Math. Soc. (Series B). 24, 194 - 202 (1982).

AUTHORS' BIOGRAPHY

Dr. R. O. Olayiwola: He is a Senior Lecturer in the Department of Mathematics, Federal University of Technology, Minna, Nigeria. He had his Ph.D in Applied Mathematics from Ladoke Akintola University of Technology, Ogbomoso, Nigeria. He is a member of Nigeria Mathematical Society (NMS) and Nigerian Association of Mathematical Physics (NAMP). His area of Specialization is in Fluid Dynamics and Combustion Theory.

Dr. A. A. Mohammed: He is a Senior Lecturer in the Department of Mathematics, Federal University of Technology, Minna, Nigeria. He had his Ph.D in Applied Mathematics from Federal University of Technology, Minna, Nigeria. He is a member of Nigeria Mathematical Society (NMS) and Nigerian Association of Mathematical Physics (NAMP). His area of Specialization is in Dynamics.

Mr. Falaye A. A.: He is a Lecturer in the Department of Mathematics, Federal University of Technology, Minna, Nigeria. He holds B. Tech (Mathematics/ Computer Science) and M.Sc. (Computer Science). He is a member of Nigeria Mathematical Society (NMS) and Nigeria Computer Society (NCS). His area of interest are risk and security in Network System, mobile and internet banking and fluid mechanics.

Mr. Adetutu O. M: He is an Assistant Lecturer in the Department of Statistics, Federal University of Technology, Minna, Nigeria. He had his M.Sc in Statistics from University of Ilorin, Ilorin, Nigeria. He is a member of Nigeria Statistical Association (NSA). His area of Specialization is in Micro-array Analysis and its Applications.