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Special Types of Ternary Semigroups

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Abstract: *The main goal of this paper is to initiate the notions of U-ternarysemigroup and V-ternary semigroup in the class of orbitary ternarysemigroups. We study prime ideals and maximal ideals in a U-ternarysemigroup and characterize V-ternary semigroup. It is proved that if T is a globally idempotent ternarysemigroups with maximal ideal, then either T is a V-ternarysemigroup or T has a unique maximal ideal which is prime. Finally we proved that a ternarysemigroup T is a V-ternarysemigroup if and only if T has atleast one proper prime ideal and if* $\{p_\alpha\}$ *is the family of all proper prime ideals, then* $< x > = T$ *for* $x \in T \setminus U p_\alpha$ *or T is a simple ternarysemigroup.*

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Mathematics Subject Classification: 06F05, 20N99.

1. Introduction

The concept of a semigroup is so simple and natural that it is hard to say when it first appeared. The algebraic structure of semigroups was widely studied by Clifford [2] etc. The concept of ternary algebraic system was instigate by Lehmer [3] in 1932, but in advance such formation was investigated by Kasner [4] who gave the ideal of n-ary algebras. The concept of ternary semigroup was familiar to Banach. Who is assigned with example of a ternarysemigroup which cannot reduced to a semigroup. Glimer [6] studied about U-rings and this notion was introduced by Satyanarayana [5] in commutative semigroups. Anjaneyulu and Ramakotaiah [1] introduced the notions of U-semigroups and V-semigroups in the class of orbitary semigroups. In [8] Sarala and Anjaneyulu studied the notions of ideals in ternarysemigroups. In this paper we initiate the notion of a U-ternary semigroup, V-ternary semigroups and distinguish the U and V-ternarysemigroups.

2. Preliminary Notes

Definition 2.1: Let $T \neq \emptyset$. Then T is called a ternarysemigroup if being existence a mapping from $T \times T \times T$ to T which maps $(pqr) \to [pqr]$ satisfying the condition

:$[(pqr)st] = [p(qrs)t] = [pq(rst)]$ for all $p, q, r, s, t \in T$.

Definition 2.2: An idempotent component $e \in T$ is said to be left (or lateral or right) identity of the if $eaa = a$(or $aea = a$ or $aae = a$) for all $a \in T$.

Left (or lateral or right) identity may not be unique. But if *e* is an identity (i.e. e plays the role of left lateral and right identity simultaneously) then *e* is unique.

3. Main Results

Definition 3.1: For any ideal P in a ternarysemigroup T, if the intersection of all primeideals accommodate P is known the prime radical of the ideal P and is represented by rad P or $\sqrt{P}$.

Definition 3.2: A ternarysemigroup T is called as a **U-ternarysemigroup**, provided for any ideal A in T specified $\sqrt{A}$=T implies A=T.

The following table1show an example of a U-ternarysemigroup.

Table. 1.

.	p	q	r	s
p	p	p	p	p
q	p	p	p	q
r	p	p	p	p
s	p	p	r	s

Theorem 3.3: A ternarysemigroup T is an U-ternarysemigroup if either T has a left identity or T is originate by an idempotent.

Proof: Assume T has a left identity e. If A be any proper ideal such that $\sqrt{A}$=T. Since $\sqrt{A} \subseteq$\{$x \in$ T: $x^n \in$A for an odd natural numbers n\} = T. So there is a natural number n specified $e^n \in$ A and hence $e \in$ A. Thus T = eTT $\subseteq$ A, a contradiction. Then T is a U-ternary semigroup.

Suppose T is generated by an idempotent e. As above we can prove that for an ideal A in T, if $\sqrt{A}$ = T, Then $e \in$ A and hence A=T. So T is a U-Ternarysemigroup.

Theorem 3.4: A ternarysemigroup T is a U-ternarysemigroup if either T has a right identity or T is originate by an idempotent.

Proof: Assume T has a right identity e. If A be any proper ideal such that $\sqrt{A}$ = T. Since $\sqrt{A} \subseteq$ \{$x \in$ T: $x^n \in$ A for an odd natural numbers n\} = T . So there exist a natural number n such that $e^n \in$ A and hence $e \in$ A. Thus T=TTe $\subseteq$ A, a contradiction. Therefore T is a U-ternarysemigroup.

Suppose T is generated by an idempotent e. As above we can prove that for an ideal A in T, if $\sqrt{A}$=T then $e \in$ A and hence A=T. So T is a U-ternarysemigroup.

Theorem 3.5: A ternarysemigroup T is a U-ternarysemigroup then either T has a lateral identity or T is generated by an idempotent.

Proof: The proof is same as to Theorem 3.4.

Theorem 3.6: A ternarysemigroup T is a U-ternarysemigroup then either T has an identity or T is generated by an idempotent.

Proof: By theorem 3.3, 3.4and 3.5, T is a U-ternarysemigroup.

From the above example of U-ternarysemigroup, we remark that there are U-ternarysemigroups neither containing left (right, lateral) identity nor generated by an idempotent.

Definition 3.7: If A is an ideal of a ternarysemigroup T then A is known **Proper Ideal** if A $\neq$ T.

Definition 3.8: If A is an ideal of a ternarysemigroup T then A is known a **Prime Ideal** provided PQR $\subseteq A$: P, Q and R are ideals of T, then either P $\subseteq$ A or Q $\subseteq A$ or R $\subseteq A$.

Theorem 3.9: A ternarysemigroup T is a U-ternary semigroup $\leftrightarrow$ every proper ideal is accommodate in a proper prime ideal.

Proof: Assume T is a U-ternarysemigroup. Let A be a actual ideal in T. If A is not accommodate in any proper prime ideal, then $\sqrt{A}$ = T. Since T is a U-ternarysemigroup, we have A is equal to T, a contradiction. So every proper ideal is accommodate in a actual prime ideal. Conversely if every actual ideal is accommodate in a actual prime ideal, then clearly T is a U-ternary semigroup.

Theorem 3.10: Let T be a U-ternarysemigroup. If $\{p_\alpha\}$ is the prime ideals in T and if P is a greatest element in this collection, then P is a greatest ideal in T.

Proof: Assume T be a U-ternarysemigroup. If P is not a maximal ideal in T, then there is a actual ideal A in T accommodate P properly. Since P is a maximal element in the set of all proper prime ideals in T, we have A is not accommodate in any actual prime ideal. So $\sqrt{A}$ = T. Since T is a U-ternarysemigroup, A = T, a negation. Hence P is a maximal ideal in T.

Definition 3.11: If a ternarysemigroup T is called a Dimension n or n-Dimensional if there exist a strictly ascending chain $p_o \subset p_1 \subset p_2 \subset .....p_n$ of prime (proper) ideals in T, but no such a chain of n+2 proper prime ideals exist in T where n is an odd natural number.

Theorem 3.12: If A is a proper ideal in the finite dimensional U-ternary semigroup T, then A is contained in a maximal ideal.

Proof: By theorem 3.9, A is accommodate in a proper prime ideal p_o . If p_o is not a largest ideal, then by theorem 3.10, a proper prime ideal p_1 such that $p_0 \subset p_1$. If $p_1$is largest we are through otherwise p_1 is properly accommodate in a proper prime ideal p_2 in T the processes of choosing p_i's must cease in a some number of steps because of the finite dimensionality of T. Hence A is accommodate in a largest ideal.

In a commutative ring it is proved that every finite dimensional U-ring is a union of maximal ideals [6]. But in ternarysemigroups this is not true, as in the ternarysemigroup T in above example is a finite dimensional U-ternarysemigroup with the unique maximal ideal {*a*, *b*, *c*}.

Definition 3.13: A ternarysemigroup T is called a **V-ternarysemigroup** if for any element $a \in$ T, $\sqrt{< a >}$ = T implies < *a* > = T.

Every U-ternarysemigroup is a V-ternarysemigroup. However V-ternarysemigroup is not compulsory a U-ternarysemigroup.

Assume T be the ternarysemigroup of all odd natural numbers greater than 1, under usual multiplication. The ideal A = {3, 5 ...} is not accommodate in any proper prime ideal and hence by theorem 3.9, T is not a U-ternarysemigroup. Clearly every principal ideal is accommodate in a proper prime ideal. So T is a V-ternarysemigroup.

Definition 3.14: If A is an ideal of a ternarysemigroup T then A is known a **globally idempotent ideal** if A^3 = A.

Definition 3.15: A ternarysemigroup T is called a **globally idempotent ternarysemigroup** provided T^3 = T.

Theorem 3.16: If T is a globally idempotent ternarysemigroup with maximal ideals, then either T is a V-ternarysemigroup or T has a single largest ideal which is prime.

Proof: Let S = $\{a \in T: \sqrt{< a >} \neq T\}$. If S = $\emptyset$, then for every $a \in$T, $\sqrt{< a >}$ = T and so T has no proper prime ideals. But maximal ideals are prime [7]. Hence this case is inadmissible. Clearly S is an ideal in T. If S ≠ T, then S is the unique largest ideal. For, let M be any maximal ideal. Since T = T^3, M is a prime ideal and so $\sqrt{M}$ = M. Now if $a \in M \setminus S$, then T = $\sqrt{< a >} \subseteq \sqrt{M}$ = M. Thus M $\subseteq T$ and so M = T. Then only other possibility is S = T, in which case T is a V-ternary semigroup.

It is clearly a ternarysemigroup T is globally idempotent double implies maximal ideals in T are prime. So if a ternarysemigroup T contains unique maximal ideal which is prime, then T is globally idempotent. But from the above example of V-ternarysemigroup, we remark that there are V-ternarysemigroups containing maximal ideals which are not globally idempotent.

Definition 3.17: A ternarysemigroup T is called a **simple ternarysemigroup** if T has no proper ideals.

Theorem 3.18: If a ternarysemigroup T is a V-ternarysemigroup double implies T has atleast one actual prime ideal and if $\{p_\alpha\}$ is the family of all actual prime ideals, the <*x*>=T for *x*∈T \U p_α or T is a simple ternarysemigroup.

Proof: Let T ne a V-ternarysemigroup. If T has no actual prime ideals, then $\sqrt{< a >}$ = T for every a ∈ T. This implies < a > = T and hence T is a simple ternarysemigroup. So assume T has proper prime ideals. Then for any a ∈ T \U p_α, $\sqrt{< a >}$ = T, since a does not belong to any actual prime ideal. Thus < a >=T. Conversely assume if a is any element in T such that < a > not equal to T. If a ∈ T\U$p_\alpha$, then,< a > = T. So a ∈ U p_α and hence $\sqrt{< a >} \neq$ T. Therefore T is a V-ternarysemigroup.

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