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# On the Cubic Equation with Four Unknowns $x^3 + y^3 = 31(k^2 + 3s^2)zw^2$

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**Abstract:** *The homogeneous cubic equation with four unknowns represented by the Diophantine equation* $x^3 + y^3 = 31(k^2 + 3s^2)zw^2$ *is analyzed for its patterns of non – zero integral solutions. A few interesting properties between the solutions and special numbers are presented.*

**Keywords:** *Cubic equation with four unknowns, Integral solutions.*
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## 1. INTRODUCTION

The Diophantine equations offer an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-15] for cubic equations with four unknowns. This communication concerns with yet another interesting equation $x^3 + y^3 = 31(k^2 + 3s^2)zw^2$ representing the homogeneous cubic equation with four unknowns for determining its infinitely many non-zero integral points. Also a few interesting properties are presented.

## 2. NOTATIONS USED

- $t_{m,n}$ - Polygonal number of rank $n$ with size $m$.
- $P_n^m$ - Pyramidal number of rank $n$ with size $m$.
- $gn_a$ - Gnomonic number of rank $a$
- $so_n$ - Stella octangular number of rank $n$
- $pr_n$ - Pronic number of rank $n$
- $CP_{m,n}$ - Centered pyramidal number of rank $n$ with size $m$.

### 2.1 Method of Analysis

The cubic diophantine equation with four unknowns to be solved for getting non-zero integral solutions is

$$x^3 + y^3 = 31(k^2 + 3s^2)zw^2 \tag{1}$$

Introduction of the transformation

$$x = u + v,\ y = u - v \text{ and } z = 2uv \tag{2}$$

in (1) leads to $u^2+3v^2=31(k^2+3s^2)zw^2$ (3)

Now, we solve (3) through different methods and thus obtain different patterns of solutions to (1).

*2.1.1 Pattern -I*

Assume $w=w(a,b)=a^2+3b^2$ (4)

where a and b are non zero distinct integers

Write 31 as $31=(2+i3\sqrt{3})(2-i3\sqrt{3})$ (5)

Using (4) & (5) in (3) and applying the method of factorization, define

$$u+i\sqrt{3}v=(2+i3\sqrt{3})(k+i\sqrt{3}s)(a+i\sqrt{3}b)^2$$

Equating the real and imaginary parts, we have

$$u=k(2a^2-6b^2-18ab)+s(-9a^2+27b^2-12ab)$$
$$v=k(3a^2-9b^2+4ab)+s(2a^2-6b^2-18ab)$$

Hence in view of (2), the values of x, y, z are given by

$$x=x(k,s,a,b)=k(5a^2-15b^2-14ab)+s(-7a^2+21b^2-30ab)$$
$$y=y(k,s,a,b)=k(-a^2+3b^2-22ab)+s(-11a^2+33b^2+6ab) \quad (6)$$
$$z=z(k,s,a,b)=k(4a^2-12b^2-36ab)+s(-18a^2+54b^2-24ab)$$

Thus (4) and (6) represent the non zero integral solutions to (1).

A few interesting properties observed are as follows:

1. $x(1,1,a,1)+5y(1,1,a,1)+t_{126,a}\equiv 1 \pmod{185}$
2. $x(k,s,a,1)-kt_{12,a}+st_{16,a}+15k-21s\equiv 0 \pmod{2a}$
3. $x(k,k,t_{3,a},t_{3,a+2})+5y(k,k,t_{3,a},t_{3,a+2})=-62k(2t_{3,a}-3\Pr_{a+2}+2Pt_a)$
4. $[x(k,s,a,b)+y(k,s,a,b)]^2=z^2(k,s,a,b)$

*2.1.2 Pattern –II*

Rewrite (3) as $u^2+3v^2=31(k^2+3s^2)zw^2*1$ (7)

Write 1 as $1=\frac{1}{4}(1+i\sqrt{3})(1-i\sqrt{3})$ (8)

Following the procedure similar to pattern-I, the corresponding non-zero distinct integral solutions of (1) are found to be

$$x=x(k,s,a,b)=k(-a^2+3b^2-22ab)+s(-11a^2+33b^2+6ab)$$
$$y=y(k,s,a,b)=k(-6a^2+18b^2-8ab)+s(-4a^2+12b^2+36ab)$$
$$z=z(k,s,a,b)=k(-7a^2+21b^2-30ab)+s(-15a^2+45b^2+42ab)$$

along with (4).

**Properties:**

1. $x(-1,1,3,b)+z(-1,1,3,b)-ct_{54,b}\equiv -164 \pmod{246}$
2. $7x(1,s,a,a-1)-z(1,s,a,a-1)+20(S_a-1)+8t_{3,a}\equiv 0 \pmod{62}$

3. $x(-11,1,a,a+1)-2Ct_{248,a}+2=0$
4. $31\{6x(1,1,a(a+1),a+2)-y(1,1,a(a+1),a+2)+744P_a^3+62(\Pr_a)^2\}$ is a Nasty number.

*2.1.3 Pattern –III*

Instead of (5), write 31 as $31=\frac{1}{4}(7+i5\sqrt{3})(7-i5\sqrt{3})$

Following the procedure similar to pattern-I, and performing a few calculations, the corresponding non-zero distinct integral solutions of (1) are given by

$$x=x(k,s,a,b)=k(6a^2-18b^2-8ab)+s(-4a^2+12b^2-36ab)$$
$$y=y(k,s,a,b)=k(a^2-3b^2-22ab)+s(-11a^2+33b^2-6ab)$$
$$z=z(k,s,a,b)=k(7a^2-21b^2-30ab)+s(-15a^2+45b^2-42ab)$$

along with (4).

**Properties:**

1. $x(5,3,2,b(b+1))+54(\Pr_b)^2+2ct_{296,b}\equiv 0(\text{mod}74)$
2. $z(1,1,a,a(a+1))-96(t_{3,a})^2+144P_a^5+t_{18,a}\equiv a(\text{mod}7)$
3. $7y(k,s,a,2a^2-1)-z(k,s,a,2a^2-1)+124kSO_a\equiv 0(\text{mod }62)$
4. $93\{x(k,1,(a+1),a)-6y(k,1,(a+1),a)-124k\Pr_a+186t_{4,a}\}$ is a Nasty number.

*2.1.4 Pattern –IV*

Instead of (8), write 1 as $1=\frac{1}{49}(1+i4\sqrt{3})(1-i4\sqrt{3})$

Following the procedure similar to pattern-III, and performing a few calculations, the corresponding non-zero distinct integral solutions of (1) are

$$x=x(k,s,a,b)=k(-70a^2+210b^2-1064ab)+s(-532a^2+1596b^2+420ab)$$
$$y=y(k,s,a,b)=k(-301a^2+903b^2-322ab)+s(-161a^2+483b^2+180ab)$$
$$z=z(k,s,a,b)=k(-371a^2+1113b^2-1386ab)+s(-693a^2+2079b^2+2226ab)$$

**Properties:**

1. $x(-1,1,a,a(a+1))-1386(\Pr_a)^2-2968P_a^5-t_{36,a}\equiv a(\text{mod}16)$
2. $x(k,s,a,b)+y(k,s,a,b)-z(k,s,a,b)=0$
3. $[x(k,s,a,b)+y(k,s,a,b)]^2-z^2(k,s,a,b)=0$
4. $12(x^2(k,s,a,b)+y^2(k,s,a,b))-6z^2(k,s,a,b)$ is a Nasty number.

## 3. CONCLUSION

To conclude, one may search for other patterns of solutions and their corresponding properties.

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