International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 2, Issue 1, January - 2014, PP 83-93
ISSN 2347-307X (Print) & ISSN 2347-3142 (Online)
www.arcjournals.org

Geometric Progression in Operations Research (PERT) – A Special Case Study

Dr. Kanduri Venkata Lakshmi Narasimhacharyulu
Associate Professor
Department of Mathematics,
Bapatla Engineering College,
Bapatla-522101,India.
kvlna@yahoo.com

I.Pothuraju
Assistant Professor
Department of Mathematics
Bapatla Engineering College,
Bapatla, Andhra Pradesh, India.
im_praju@yahoo.com

Abstract: *The present paper looks into the influence of Geometric Progression (G.P) which will whether endorse the network or not in a special case. The huge network is illustrated in a systematic way with 46 nodes and 60 activities. G.P is employed on a most likely time estimate among the three time estimates namely optimistic, most likely and pessimistic. The investigation has been done on the considered network. Some remarkable results are found. All float values are also computed. Critical path is identified and project analysis has been carried out. Periodical analysis is also established with standard normal distribution curves which are illustrated wherever necessary.*

Keywords: *Network, Time estimates, Float, Critical path, Normal distribution.*

AMS Classification: ***90-08, 90B10, 90C90***

1. Introduction

Project networks are best representations of reality. The networks have some difficulty to control the realistic influences. In such case, no advantage will be occurred in their execution. In general networks used to predict and explain phenomena with a high degree of accuracy. Very large number of variables may be commanded to anticipate a phenomenon with consummate accuracy. The best way is to find the correct variable and right relationship between them.

The normal distribution curve and it's properties have been employed for the project with large size network. It necessitates all activities of the project. The three time estimates estimate an individual activity as optimistic time , pessimistic time and most probable time. These three time estimates indicate the measure of uncertainty likely to be encountered in performing an individual activity. The time estimates are mainly based on human observation, experience and knowledge of the estimator about the performance of the particular activity.

Levin and Kirkpatrick [3] explained about planning and control of a project with the aid of PERT and CPM in 1966.Wiest and Levy [4] looked into new ideas on Networks with management guide to PERT/CPM for research scholars in the area of operations research. PERT algorithm was established with various models by Billy E.Gillett [5] .He attempted different models of operations Research in 1979. S.D Sharma [6] discussed the applications of PERT&CPM techniques. K.V.L.N.Acharyulu et.al [1] analyzed some curious cases of PERT and Game theory problems.

The authors examined in this paper to classify whether Geometric progression in a peculiar case will assert a network or not. The Geometric progression is employed on most likely time estimate among the three time estimations. Project analysis is also accomplished. Total Float, Free float and Independent Float are calculated in the part of confirming the critical path. The standard normal distribution curves are illustrated with the aid of Mat lab wherever essential.

2. Construction of Network

A network is constructed with 46 nodes and 60 activities in a systematic way for investigating the influence of **Geometric Progression**. G.P is applied on most likely time estimate (m) in case (I) among the three estimates. Network does not accept any error and dummy activity.

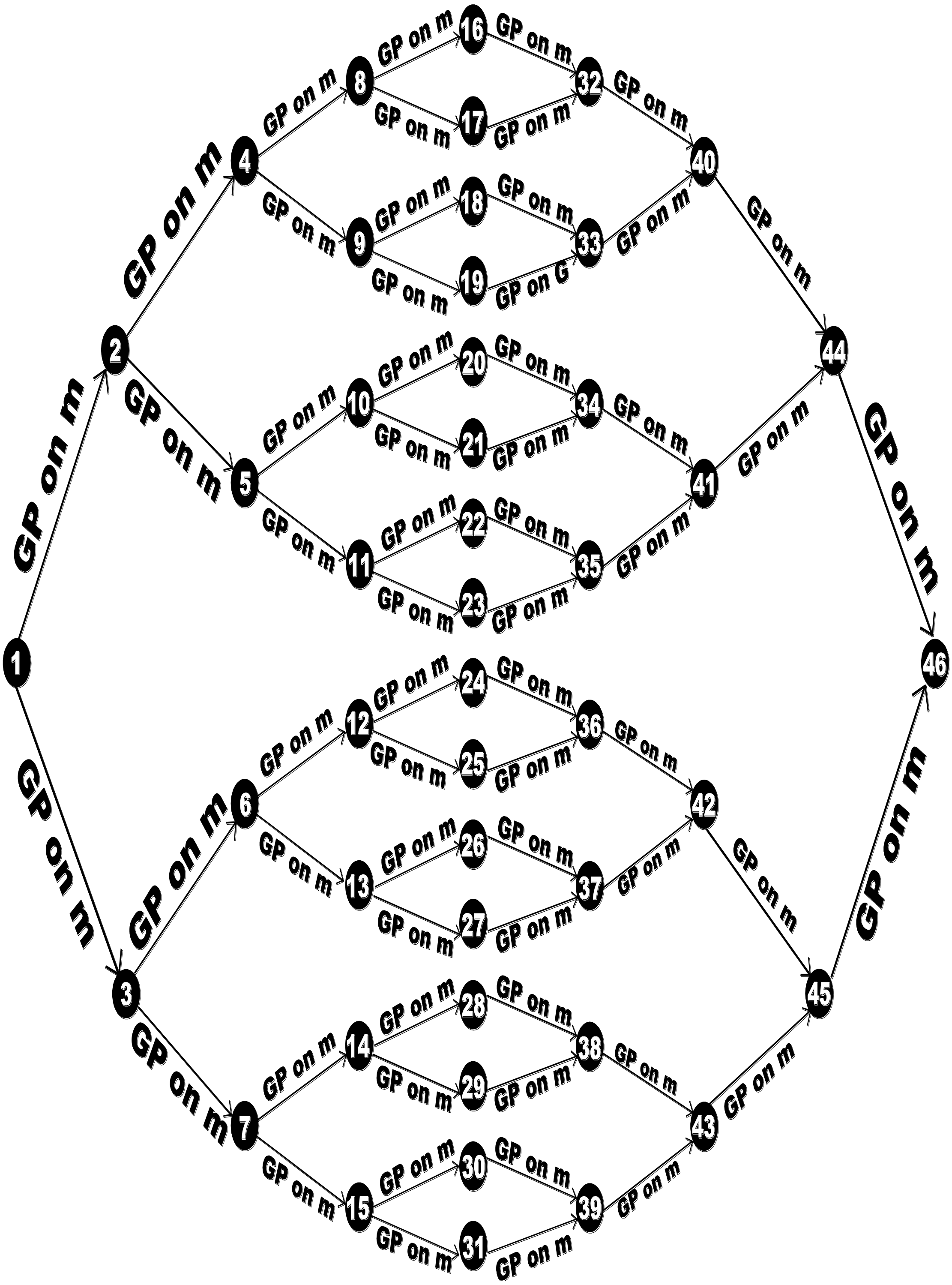

Fig.1. *Drawn Network having 46 nodes with 60 activities*

3. Preliminaries and Notations

(i).**TE**= Earliest excepted completion time of event (TE)

Def: For the fixed value of j=TE(j)=Max[TE(i)+ET(i,j)] which ranges over all activities from i-j.

(ii).**TL**= Latest allowable event completion time (TL)

Def: For the fixed value of i=TL(i)=Min[TL(j)+ET(i,j)] which ranges over all activities from i-j.

(iii).**ET**= Excepted completion time of activity (I,J)

(iv). **a** = Optimistic time estimate

(v). **m** = Most likely time estimate

(vi). **b** = Pessimistic time estimate

(vii). **ES** = Earliest start of an activity

(viii). **EF** = Earliest finish of an activity

(ix). **LS** = Latest start of an activity

(x). **LF** = Latest finish of an activity

(xi). **TF** = Total Float

Def: TF of activity i-j = LF_{i-j}-EF_{i-j} (or) LS_{i-j}-ES_{i-j}

(xii).**FF** = Free Float

Def: FF of activity i-j = TF - (TL-TE) of node j

(xiii).**IF** = Independent Float

Def: IF of activity i-j = FF - (TL-TE) of node i

(xiv)**.SE=**Slack event time

(xv).**CPI**=Critical Path Indicator

(xvi).**SCT**= Scheduled Time

(xvii). σ = Standard deviation of project length

4. Material and Methods

Step 1: Draw the project network completion time

Step 2: Compute the excepted duration of each activity by using the formula $ET = \frac{a+4m+b}{6}$.

From the time estimates a,m and p. Also calculate the excepted variance. σ^2 of each activity

Step 3: Calculate TE, TL

Step 4: Find Total Float, Free Float and Independent Float

Step 5: Find the critical path and identify the critical activities

Step 6: Compute project length which is a square root to sum of variance of all the critical activities.

Step 7: From the standard normal variable $z = \frac{SCT - ET}{\sigma}$, Where SCT is scheduled Completion time of event, σ =standard deviation of project length. Using the standard normal curve, we can estimate the probability of completing project within specified time.

5. Results

By using CPM and PERT algorithms on the Network, the critical path is identified from the scientific computations of the following tables from Table-1 to Table-6.The tables consists of all activities, Time estimates, ET, Varience.ES, EF, LS, LF and all Float values. The Critical path indicator provides at each critical Activity in each table.

Table-1. *Scientific Computations of First two Levels*

Activity	Time Estimates			ET	σ^2	Earliest[E]		Latest[L]		TF	FF	IF	CPI
	a	m	b			ES	EF	LS	LF				
1--2	1	1.414	2	1.442	0.027	0	1.442	60.036	61.478	60.036	0	0	
1--3	3	3.464	4	3.473	0.027	0	3.473	0	3.473	0	0	0	*
2--4	5	5.477	6	5.484	0.027	1.442	6.926	89.484	94.968	88.042	0	-60.036	
2--5	7	7.483	8	7.488	0.027	1.442	8.93	61.478	68.966	60.036	0	-60.036	
3--6	9	9.486	10	9.49	0.027	3.473	12.963	31.475	40.965	28.002	0	0	

Table-2. *Scientific Computations of Third Level*

Activity	Time Estimates			ET	σ^2	Earliest[E]		Latest[L]		TF	FF	IF
	a	m	b			ES	EF	LS	LF			
4--8	13	13.49	14	13.493	0.027	6.926	20.419	106.969	120.462	100.043	0	-88.042
4--9	15	15.491	16	15.494	0.027	6.926	22.42	94.968	110.462	88.042	0	-88.042
5--10	17	17.492	18	17.494	0.027	8.93	26.424	80.968	98.462	72.038	0	-60.036
5--11	19	19.493	20	19.495	0.027	8.93	28.425	68.966	88.461	60.036	0	-60.036
6--12	21	21.494	22	21.496	0.027	12.963	34.459	53.265	74.461	40.002	0	-28.002
6--13	23	23.494	24	23.496	0.027	12.963	36.459	40.965	64.461	28.002	0	-28.002
7--14	25	25.495	26	25.496	0.027	14.965	40.461	26.965	52.461	12	0	0
7--15	27	27.495	28	27.496	0.027	14.965	42.461	14.965	42.461	0	0	0

Table-3. *Scientific Computations of Fourth Level*

Activity	Time Estimates			ET	σ^2	Earliest[E]		Latest[L]		TF	FF	IF
	a	m	b			ES	EF	LS	LF			
8--16	29	29.495	30	29.496	0.027	20.419	49.915	124.463	153.959	104.044	0	-100.043
8--17	31	31.496	32	31.497	0.027	20.419	51.916	120.462	151.959	100.043	0	-100.043

9--18	33	33.496	34	33.497	0.027	22.42	55.917	114.462	147.959	92.042	0	-88.042
9--19	35	35.496	36	35.497	0.027	22.42	57.917	110.462	145.959	88.042	0	-88.042
10--20	37	37.496	38	37.497	0.027	26.424	63.921	102.462	139.959	76.038	0	-72.038
10--21	39	39.496	40	39.497	0.027	26.424	65.921	98.462	137.959	72.038	0	-72.038
11--22	41	41.496	42	41.497	0.027	28.425	69.922	92.462	133.959	64.037	0	-60.036
11--23	43	43.497	44	43.498	0.027	28.425	71.923	88.461	131.959	60.036	0	-60.036
12--24	45	45.497	46	45.498	0.027	34.459	79.957	78.461	123.959	44.002	0	-40.002
12--25	47	47.497	48	47.498	0.027	34.459	81.957	74.459	121.957	40	0	-40.002
13--26	49	49.497	50	49.498	0.027	36.459	85.957	68.459	117.957	32	0	-28.002
13--27	51	51.497	52	51.498	0.027	36.459	87.957	64.461	115.959	28.002	0	-28.002
14--28	53	53.497	54	53.498	0.027	40.461	93.959	56.461	109.959	16	0	-12
14--29	55	55.497	56	55.498	0.027	40.461	95.959	52.461	107.959	12	0	-12
15--30	57	57.497	58	57.498	0.027	42.461	99.959	46.461	103.959	4	0	0
15--31	59	59.497	60	59.498	0.027	42.461	101.959	42.461	101.959	0	0	0

Table-4. *Scientific Computations of Fifth Level*

Activity	Time Estimates			ET	σ^2	Earliest[E]		Latest[L]		TF	FF	IF
	a	m	b			ES	EF	LS	LF			
16-32	61	61.497	62	61.498	0.027	49.915	111.413	153.959	215.457	104.044	4.001	-100.043
17-32	63	63.498	64	63.498	0.027	51.916	115.414	151.959	215.457	100.043	0	-100.043
18-33	65	65.498	66	65.498	0.027	55.917	121.415	147.959	213.457	92.042	4	-88.042
19-33	67	67.498	68	67.498	0.027	57.917	125.415	145.959	213.457	88.042	0	-88.042
20-34	69	69.498	70	69.498	0.027	63.921	133.419	139.959	209.457	76.038	4	-72.038
21-34	7	71.49	7	71.49	0.02	65.92	137.4	137.9	209.4	72.03	0	-

	1	8	2	8	7	1	19	59	57	8		72.038
22-35	73	73.498	74	73.498	0.027	69.922	143.42	133.959	207.457	64.037	4.001	-60.036
23-35	75	75.498	76	75.498	0.027	71.923	147.421	131.959	207.457	60.036	0	-60.036
24-36	77	77.498	78	77.498	0.027	79.957	157.455	123.959	201.457	44.002	4	-40.002
25-36	79	79.498	80	79.498	0.027	81.957	161.455	121.959	201.457	40.002	0	-40.002
26-37	81	81.498	82	81.498	0.027	85.957	167.455	117.959	199.457	32.002	4	-28.002
27-37	83	83.498	84	83.498	0.027	87.957	171.455	115.959	199.457	28.002	0	-28.002
28-38	85	85.498	86	85.498	0.027	93.951	179.449	109.959	195.457	16.008	4.008	-12
29-38	87	87.498	88	87.498	0.027	95.959	183.457	107.959	195.457	12	0	-12
30-39	89	89.498	90	89.498	0.027	99.959	189.457	103.959	193.457	4	4	0
31-39	91	91.498	92	91.498	0.027	101.959	193.457	101.959	193.457	0	0	0

Table-5. *Scientific Computations of Sixth Level*

Activity	Time Estimates			ET	σ^2	Earliest[E]		Latest[L]		TF	FF	IF
	a	m	b			ES	EF	LS	LF			
32-40	93	93.498	94	93.498	0.027	115.414	208.912	215.457	308.955	100.043	12.001	-88.042
33-40	95	95.498	96	95.498	0.027	125.415	220.913	213.457	308.955	88.042	0	-88.042
34-41	97	97.498	98	97.498	0.027	137.419	234.917	209.457	306.955	72.038	12.002	-60.036
35-41	99	99.498	100	99.498	0.027	147.421	246.919	207.457	306.955	60.036	0	-60.036
36-42	101	101.498	102	101.498	0.027	161.455	262.953	201.457	302.955	40.002	12	-28.002
37-42	103	103.498	104	103.498	0.027	171.455	274.953	199.457	302.955	28.002	0	-28.002
38-43	105	105.498	106	105.498	0.027	183.457	288.955	195.457	300.955	12	12	0
39-43	107	107.498	108	107.498	0.027	193.457	300.955	193.457	300.955	0	0	0

Table-6. *Scientific Computations of Seventh Level*

Activity	Time Estimates			ET	σ^2	Earliest[E]		Latest[L]		TF	FF	IF	CPI
	a	m	b			ES	EF	LS	LF				
40-44	109	109.498	110	109.498	0.027	220.913	330.411	308.955	418.453	88.042	28.006	-60.036	
41-44	111	111.498	112	111.498	0.027	246.919	358.417	306.955	418.453	60.036	0	-60.036	
42-45	113	113.498	114	113.498	0.027	274.953	388.451	302.955	416.453	28.002	28.002	0	
43-45	115	115.498	116	115.498	0.027	300.955	416.453	300.955	416.453	0	0	0	*
44-46	117	117.498	118	117.498	0.027	358.417	475.915	418.453	535.951	60.036	60.036	0	
45-46	119	119.498	120	119.498	0.027	416.453	535.951	416.453	535.951	0	0	0	*

Critical path is obtained as below

$$1 \xrightarrow{3.473} 3 \xrightarrow{11.492} 7 \xrightarrow{27.496} 15 \xrightarrow{59.498} 31 \xrightarrow{91.498} 39 \xrightarrow{107.498} 43 \xrightarrow{115.498} 45 \xrightarrow{119.498} 46$$

Project Length is defined as $\sqrt{\text{Sum of Variances of each Critical activity}}$

i.e Project Length $= \sqrt{0.027+0.027+0.027+0.027+0.027+0.027+0.027+0.027}$
$=0.4647$

The values of TE, TL and SE corresponding to every node are given in table (3).

The slack event time may be positive, negative or zero.

It is also observed that the values of slack event time vanish at each critical activity.

Slack event time is defined as the amount of time in which the event can be retarded with out involving the scheduled completion time for the project. Any activity on the critical path necessitates time in excess of its expected completion time and detains the project completion consequently.

Table-7

Nodes	TE	TL	SE	Nodes	TE	TL	SE
1	**0**	**0**	**0**	24	79.957	123.959	44.002
2	1.442	61.478	60.036	25	81.957	121.959	40.002
3	3.473	3.473	0	26	85.957	117.959	32.002
4	6.926	94.968	88.042	27	87.957	115.959	28.002
5	8.93	68.966	60.036	28	93.951	109.959	16.008
6	12.963	40.965	28.002	29	95.959	107.959	12
7	**14.965**	**14.965**	**0**	30	99.959	103.959	4
8	20.419	120.462	100.043	**31**	**101.959**	**101.959**	**0**
9	22.42	110.462	88.042	32	115.414	215.457	100.043
10	26.424	98.462	72.038	33	125.415	213.457	88.042
11	28.425	88.461	60.036	34	137.419	209.457	72.038
12	34.459	74.461	40.002	35	147.421	207.457	60.036
13	36.459	64.461	28.002	36	161.455	201.457	40.002
14	40.461	52.461	12	37	171.455	199.457	28.002
15	**42.461**	**42.461**	**0**	38	183.457	195.457	12
16	49.915	153.959	104.044	**39**	**193.457**	**193.457**	**0**

17	51.916	151.959	100.043	40	220.913	308.955	88.042
18	55.917	147.959	92.042	41	246.919	306.955	60.036
19	57.917	145.959	88.042	42	274.953	302.955	28.002
20	63.921	139.959	76.038	**43**	**300.955**	**300.955**	**0**
21	65.921	137.959	72.038	44	358.417	418.453	60.036
22	69.922	133.959	64.037	45	416.453	416.453	0
23	71.923	131.959	60.036	**46**	**535.951**	**535.951**	**0**

6. PROJECT ANALASIS

Project analysis is carried out with specific schedule times and the standard normal variables are identified in the possible range of probability from o to 1.The percentage of possibilities of completion of the Project are obtained and specified in the following Table-8.The graphs are also illustrated.

Table-8

SCT	ETC	Z	Probability	Percentage of Possibility (%)
534	535.951	-4.918	0	0
535	535.951	-2.046	0.2068	20.6
536	535.951	0.105	0.53983	53.9
537	535.951	2.257	0.98778	98.7
538	535.951	4.409	1	100

The derived Standard Normal Curves are shown from Fig.2-Fig.6

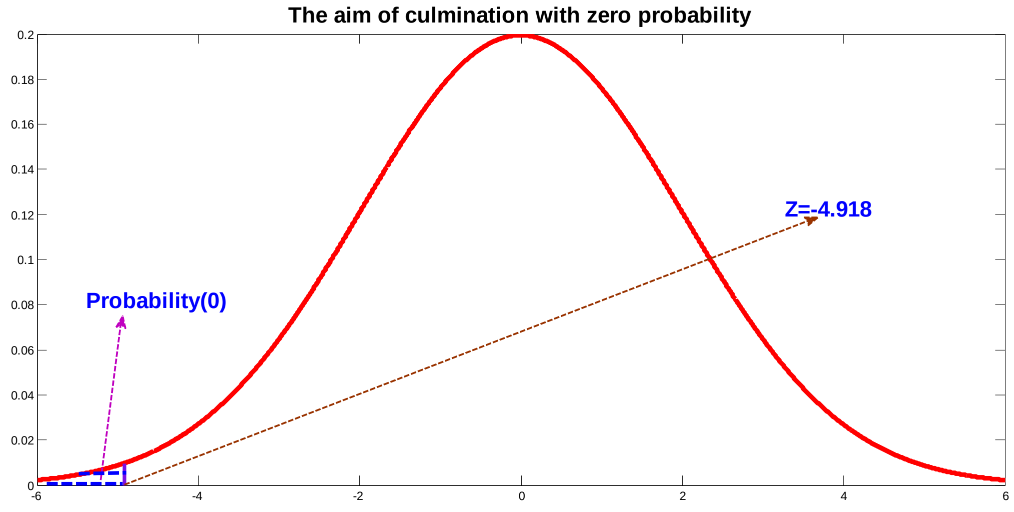

Fig.2

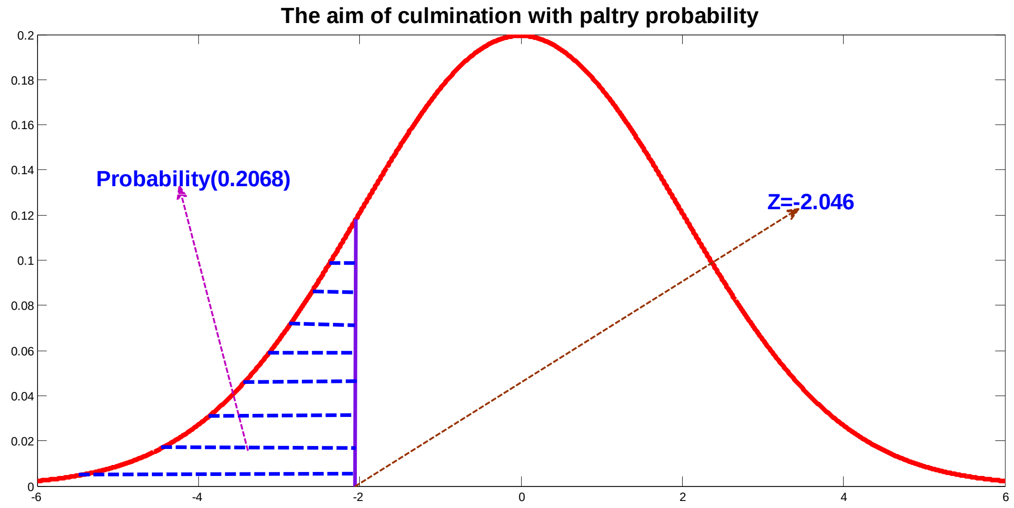

Fig.3

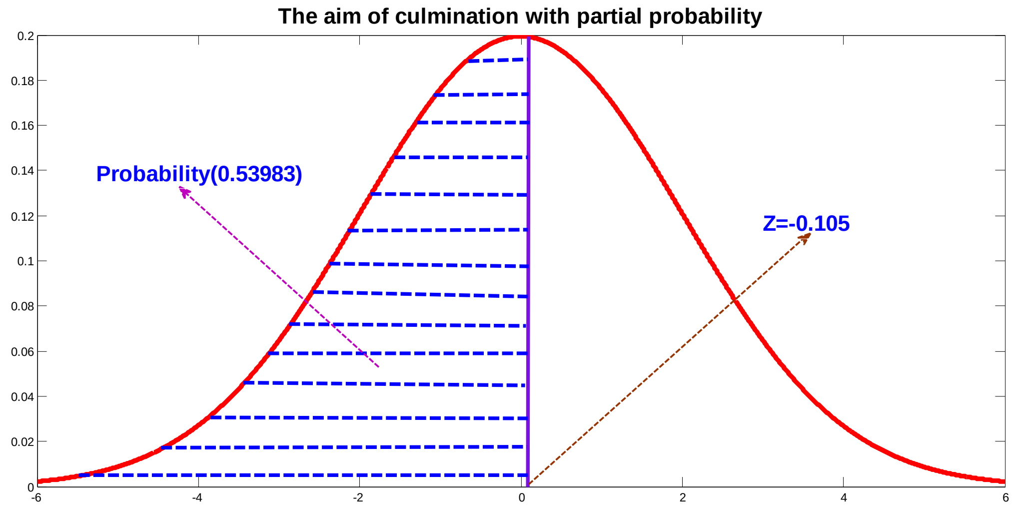

Fig.4

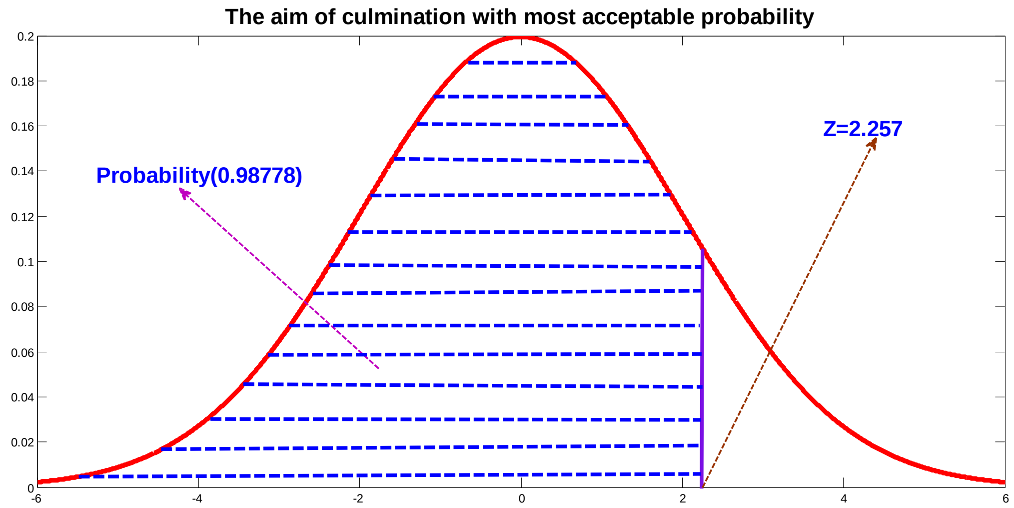

Fig.5

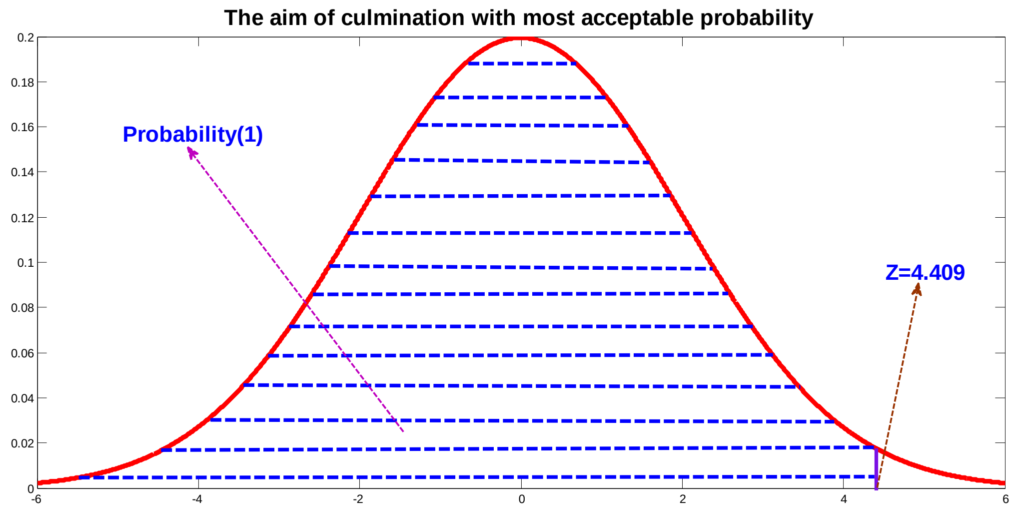

Fig.6

7. Conclusions

The following conclusions are incurred in this investigation of scientific study.

(i).In Critical Path

(a).It is noted that all Total Float values of Critical activities are vanished.

(b).The value of Slack event of each node in critical path has become zero.

(c). TE and TL are same at each node in critical path.

(ii).G.P sustains consistently the Network even though the network has large size.

(iii).Variances are identical at any activity of the Network.

(iv).The case in which G.P is conceived on most likely time estimate, the expected completion time of successive activity is gradually increased.

(v).The influence of G.P in the Network are identified as

(a).G.P supports accurately only when SCT is greater than ETC, except SCT value is nearer to ETC.

(b).G.P does not support effectively when SCT is less than or equal to ETC.

(c). Standard Normal Distribution curves illustrate the percentage of possibilities of the Project.

References

[1] K.V.L.N.Acharyulu and Maddi.N.Murali Krishna,(2013). Some Remarkable Results in Row and Column both Dominance Game with Brown's Algorithm, International Journal of Mathematics and Computer Applications Research,Vol. 3, No.1, pp.139-150.

[2] K.V.L.N.Acharyulu and Maddi.N.Murali Krishna, (2013). A Scientific Computation On A Peculiar Case of Game Theory in Operations Research, International Journal of Computer Science Engineering and Information Technology Research,Vol. 3 , No.1, pp.175-190.

[3] Levin, R., and C.A. Krik Patrick,(1966).Planning and control with PERT/CPM, McGraw-Hill Book company, New York.

[4] Wiest, J.D., and F-Levy,(1969).A management Guide to PERT/CPM, Patrick-mall, Inc. Engle Wood Cliffs, N.J.

[5] Billy E. Gillett,(1979).Introduction to operations Research, Tata McGraw-Hill Publishing Company limited, PP.434-453,New York.

[6] S.D.Sharma,(1999). Operations Research, PP.4.300-4.355, Kedar Nath Ram Nath & Co.

AUTHORS' BIOGRAPHY

Kanduri Venkata Lakshmi Narasimhacharyulu Who is known as Dr.K.V.L.N.Acharyulu is working sas Associate Professor in the Department of Mathematics, Bapatla Engineering College, Bapatla which is a prestigious institution of Andhra Pradesh. He took his M.Phil. Degree in Mathematics from the University of Madras and stood in first Rank,R.K.M. Vivekananda College,Chennai. Nearly for the last thirteen years he is rendering his services to the students and he is applauded by one and all for his best way of teaching. He has participated in some seminars and presented his papers on various topics. More than 70 articles were published in various International high impact factor Journals. He obtained his Ph.D from ANU under the able guidance of Prof. N.Ch.Pattabhi Ramacharyulu,NIT,Warangal. He authored three books. He is a Member of Various Professional Bodies and created three world records in research field. He received so many awards and rewards for his research excellency in the field of Mathematics.

I.Pothuraju, He is working as Assistant professor in Department of Mathematics, Bapatla Engineering College. He has two years of teaching experience. He is doing his M.phil under the guidance of Dr.K.V.L.N.Acharyulu.He did M.Sc(Mathematics) in Bapatla Engineering College. He obtained MBA from Pydah College,Andhra University. He completed his B.Sc(M.P.C) in Bapatla College of Arts & Science. He has a zeal to invent new findings in Mathematics.