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Representation of Discrete Images in an Infinite Binary Tree Language: A Recursive Analysis Model on the Stern-Brocot Tree

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Abstract: *This article is primarily about discrete geometry, represents discrete images in the infinite binary tree language, i.e. we start from the Stern-Brocot tree which is a particularly interesting representation of all strictly positive rationals in the form of a binary tree. By constructing this tree iteratively, we obtain a complete and ordered representation of all strictly rational positive. Each fraction in the tree is an irreducible fraction, that is to say that the numerator and the denominator do not have a common divisor other than 1 verifying certain properties. The article also presents several properties of this representation. These properties allow us to: Determine the position of a fraction in the tree, Determine the fraction corresponding to a given position in the tree and Number all positive fractions.*

Keywords: *Stern–Brocot tree, square matrices of order 2, discrete geometry, irreducible fraction, fraction associated with the matrix.*

1. Introduction

Constructing the Stern-Brocot tree using square matrices of order 2 in discrete geometry is an alternative approach to generating the Stern-Brocot tree. This method uses linear transformations applied to the coordinates of the tree nodes to obtain the child nodes [3, 4].

The Stern-Brocot tree method is used to determine the optimal division of the image into subregions. It is based on the notion of median fraction that we mentioned previously. In this context, each node of the Stern-Brocot tree represents a median fraction which indicates the relative position of the corresponding region with respect to its neighbors [16].

It should be noted that this method of constructing the Stern-Brocot tree using square matrices of order 2 is specific to discrete geometry and constitutes one alternative approach among others to generate this tree. Other methods, such as the iterative construction we discussed previously, are also used.

Also recalling the basic concept in graph theory on the tree, we start from divisibility in discrete geometry to construct an infinite binary tree called "Stern – Brocot tree", which is a method for generating all irreducible fractions positive $\frac{m}{n}$ from integers and therefore to construct $\mathbb{Q}$ from $\mathbb{Z}$ [20, 22].

The construction of images to represent the Stern – Brocot tree, by positive irreducible rational fractions $\frac{m}{n}\, et \,\frac{m'}{n'}$ comprising between 0 and 1.

The addition of these 2 adjacent fractions allows us to find a median fraction $\frac{m}{n} \oplus \frac{m'}{n'}$ which allows us to construct an image that we insert between the 2 starting fractions $\frac{0}{1}, \frac{1}{0}$

Our objective in this article is to show that any fraction associated with a square matrix of order 2 of the Stern–Brocot tree is irreducible [9, 16].

In addition the following properties are verifiable

- Equality $\Delta_M = 1$ (the difference of the diagonal products of the associated square matrix).
- Each positive square matrix is represented once and only once in the constructed tree.
- The irreducible fractions associated with these matrices of the tree are strictly positive.

2. Reminders on Generalities

2.1. Integers

Integers are a type of number used in mathematics and computer science to represent integer quantities. That is to say, are the numbers which do not allow a digit after the decimal point [1, 18]. i.e. without decimal part

For example

- From 1 to 11
- 5; 1; 8; 97; 304; 3043; 0 (Zero) is also a whole number because there is nothing to count.

2.2. Tree

A tree is a special type of connected and acyclic graph. A graph is said to be connected when there is a path between each pair of vertices, and it is said to be acyclic when it contains no cycle (a cycle is a sequence of vertices in which the starting vertex is also the ending vertex). A tree is therefore a connected and acyclic graph.

In a tree there is a special vertex called a "root", from which all other vertices are reachable by a single path. Trees are often used to represent hierarchical structures or dependency relationships [13].

In other words a treeis the datum of a set E and of a symmetric relation R on E such that any two distinct points X and Y of E are connected by a single finite injective pathi.e. $n+1$ points $\mathbb{Z}_0, \dots, \mathbb{Z}_n$. that is to say the distinct points of E satisfying X= $\mathbb{Z}_0, \dots, \mathbb{Z}_n \mathbb{Z}_0, \mathbb{Z}_i \,\mathcal{R}\, \mathbb{Z}_{i+1}$ for $i < n$, $\mathbb{Z}_n = y$.

A tree is a connected graph G = (X, U) without cycles [2, 19].

2.3. Binary Tree in Computing

In computer science, a binary tree is a hierarchical data structure where each node has at most two child nodes, usually called "left children" and "right children". This structure is called "binary" because each node has at most two children [3].

A binary tree is often used to represent kinship or classification relationships. The top node is called the “root” of the tree, and nodes that do not have children are called “leaves”. The other nodes are called "internal nodes". Each node can have a direct link to its parent node, except the root which has no parent node.

Binary trees can be used to implement different efficient data structures, such as binary search trees, binary heaps, expression trees, etc [17].

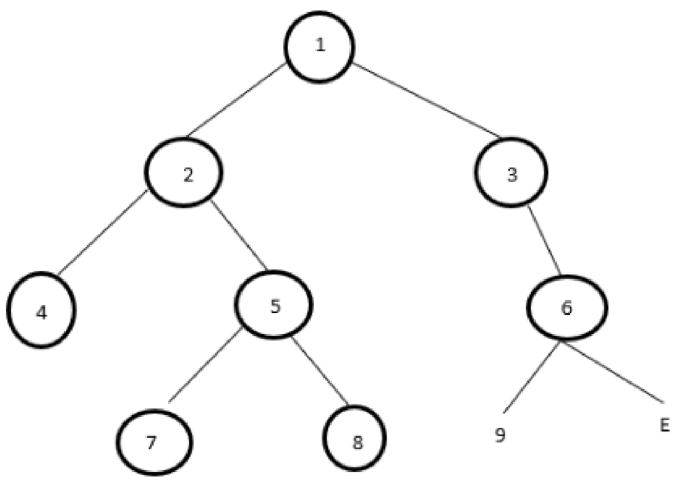


Fig1. *Representation of a binary tree*

To build a tree, we start from an origin that we call the root of the tree, then we construct the branches which lead to the leaves called nodes, ie to all possible events [21].

2.4. Irrational Fraction

An irreducible fraction, also called a simplified fraction, is a fraction whose numerator and denominator have no common factor other than 1. In other words, a fraction is irreducible when its numerator and denominator have no common divisor other than 1 [14].

So to make a fraction irreducible, we simplify the numerator and the denominator by their common divisor.

For example: make irreducible:

1) $\frac{68}{51}$, we decompose 68 and 51 into the product of prime factors.

   68 = 2 x34 = 2 x2 x17 = $2^2 \times 17$

   $$51 = 3 \times 17$$

   on one therefore $\frac{68}{51} = \frac{2^2 \times 17}{3 \times 17} = \frac{4}{3}$ is an irreducible fraction.qui

2) $\frac{67}{15}$ is an irreducible fraction because 67 and 15 have no common divisor other than 1.

2.5. Rational Fraction

A rational fraction, also known as an algebraic fraction, is a mathematical expression in which the numerator and denominator are polynomials. More precisely, a rational fraction is the quotient of two polynomials [7].

The general form of a rational fraction is as follows:

$$f(x) = P(x) / Q(x),$$

where $P(x)$ et $Q(x)$ are the polynomials. We can still say that a rational fraction is a quotient of two polynomials constructed using an indeterminate ie making the quotient of two formal polynomials or it is a fraction whose numerator and denominator are polynomials [4, 15].

For example .$\frac{2x+3}{x-2}$

2.6. Image Processing

Image processing focuses on the mathematical techniques used to analyze and process images. It uses mathematical concepts and algorithms to solve specific problems related to digital images, i.e., image processing is a science of information and applied mathematics that studies digital images and their transformation in order to improve their quality or extract information or it is all the operations carried out on the image, in order to improve readability and facilitate interpretation [5, 11].

2.7. Digital Image

A digital image is a visual representation of a scene or object in the form of digital data. So a digital image is any image (drawing, icon, photograph) acquired, created, processed and stored in binary form [8].

2.8. Construction of the Stern-Brocot tree using FAREY Sequences

It is about building from i at this stage [10].

We start from two fractions $\frac{0}{1}$, $\frac{1}{0}$ and then repeat the following operation as many times as we want.

Insert .$\frac{m+m'}{n+n'}$ between two adjacent fractions $\frac{m}{n}$ and $\frac{m'}{n'}$, the operation $\frac{m}{n} \oplus \frac{m'}{n'}$ is a fraction called "median" of $\frac{m}{n} \oplus \frac{m'}{n'} = \frac{m+m'}{n+n'}$ is to check the property m'n – mn' = 1

The "median" fraction allows us to construct an infinite binary tree between $\frac{0}{1}$ et $\frac{1}{0}$

At the first step between $\frac{0}{1}et\frac{1}{0}$, we construct the image$\frac{1}{1}$ because the median of$\frac{0}{1}and\frac{1}{0}$ is$\frac{0}{1} \oplus \frac{1}{0} = \frac{0+1}{1+0} = \frac{1}{1}$ = whichwe insert between the two fractions hence the image$\left(\frac{0}{1},\frac{1}{1},\frac{1}{0}\right)$

In the second step we have to insert two images from:

$$\frac{0}{1} \oplus \frac{1}{1} = \frac{0+1}{1+1} = \frac{1}{2} \text{ and } \frac{1}{1} \oplus \frac{1}{0} = \frac{1+1}{1+0} = \frac{2}{1} = \text{ hence the images} \left(\frac{0}{1},\frac{1}{2},\frac{1}{1},\frac{2}{1},\frac{1}{0}\right)$$

In the third step we will insert 4 images as follows:

$$\frac{0}{1} \oplus \frac{1}{2} = \frac{0+1}{1+2} = \frac{1}{3}$$
$$\frac{1}{2} \oplus \frac{1}{1} = \frac{1+1}{2+1} = \frac{2}{3}$$
$$\frac{1}{1} \oplus \frac{2}{1} = \frac{1+2}{1+1} = \frac{3}{2}$$
$$\frac{2}{1} \oplus \frac{1}{0} = \frac{2+1}{1+0} = \frac{3}{1}$$

Hence the images

$$\left(\frac{0}{1},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{1}{1},\frac{3}{2},\frac{2}{1},\frac{3}{1},\frac{1}{0}\right)$$

Let us now construct an infinite tree called the “Stern – Brocot tree”.

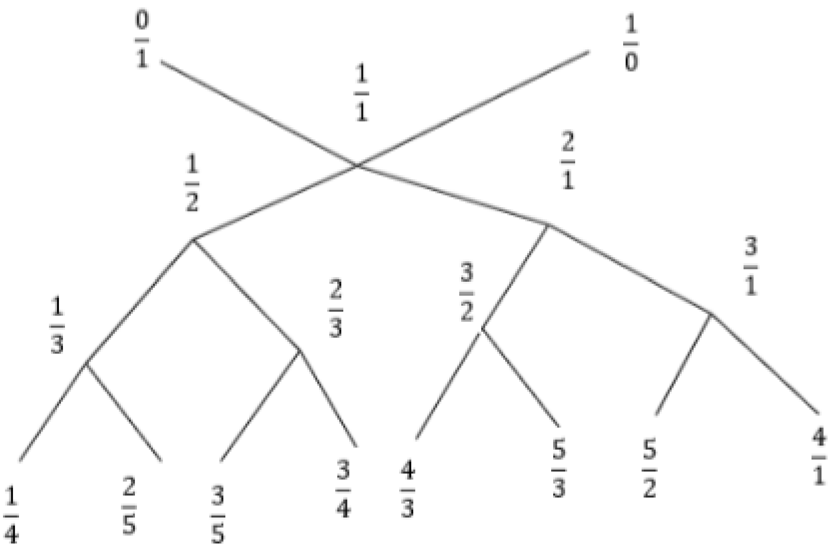


Fig2. *Representation of an infinite binary tree*

Checking the property m'n – mn' = 1

Let's carry consecutive images above

- $\left(\frac{0}{1},\frac{1}{3}\right)$ here $m = 0, n = 1, m' = 1, n' = 3$

  $m'n - mn' = 1$becomes $1.1 - 0.3 = 1 - 0 = 1$
- $\left(\frac{1}{3},\frac{1}{2}\right)$ here $m = 1, n = 3, m' = 1, n' = 2$

  $m'n - mn' = 1$becomes $1.3 - 1.2 = 3 - 2 = 1$
- $\left(\frac{1}{2},\frac{2}{3}\right)$ here $m = 1, n = 2, m' = 2, n' = 3$

  $m'n - mn' = 1$becomes $2.2 - 1.3 = 4 - 3 = 1$

- $\left(\frac{2}{3},\frac{1}{1}\right)$ here $m=2, n=3, m'=1, n'=1$

  $m'n-mn'=1$becomes $1.3-2.1\ =3-2=\ 1$
- $\left(\frac{1}{1},\frac{3}{2}\right)$ here $m=1, n=1, m'=3, n'=2$

  $m'n-mn'=1$becomes $3.1-1.2\ =3-2=\ 1$
- $\left(\frac{3}{2},\frac{2}{1}\right)$ here $m=3, n=2, m'=2, n'=2$

  $m'n-mn'=1$becomes $2.2-3.1\ =4-3=\ 1$
- $\left(\frac{2}{1},\frac{3}{1}\right)$ here $m=2, n=1, m'=3, n'=1$

  $m'n-mn'=1$becomes $3.1-2.1\ =3-2=\ 1$
- $\left(\frac{3}{1},\frac{1}{0}\right)$ here $m=3, n=1, m'=1, n'=0$

  $m'n-mn'=1$becomes $1.1-3.0\ =1-0=\ 1$

Statistical

The property$m'n-mn'=1$ is indeed respected.

2.9. FAREY Suites

Farey sequences are an ordered sequence of irreducible fractions that lie between 0 and 1. Farey sequences are nothing other than the subtrees of the Stern-Brocot tree, which are an increasing sequence of fractions rational. irreducible positives include between 0 and 1 whose denominator is less than or equal to n [16].

Example $F_5 = \left\{\frac{0}{1},\frac{1}{5},\frac{1}{4},\frac{1}{3},\frac{2}{5},\frac{1}{2},\frac{3}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{1}{1}\right\}$

The construction of the FAREY sequences is done recursively from $F_1 = \left\{\frac{0}{1},\frac{1}{1}\right\}$

It is a sequence of irreducible fractions $\in\ [0,1]$ allowing all the irreducible positive rational fractions to be listed and represented in an ordered manner.

The FAREY sequence of order is calculated from the FAREY sequence of order n-1 by adding the medians of denominators less than or equal to n, calculated from the consecutive fractions of.

The property m'n – mn' =1 is verified for two consecutive fractions of

3. Construction of the Stern−Brocot Tree using Square Matrices of Order 2

Consider two fractions and a square matrix of order 2, M=$\frac{a}{b}\ ,\frac{c}{d}\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$$\phi : \mathbb{Q} \times \mathbb{Q} \rightarrow \mathbb{Q} \cup \{\infty\}$$

$\begin{pmatrix} a & c \\ b & d \end{pmatrix} \rightarrow \frac{a+c}{b+d}\mathbb{Q}$ a fraction associated with the matrix M. this fraction is strictly positive and that each element of can be represented by an irreducible fraction.

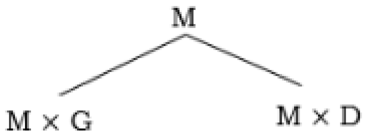
Consider two square matrices

G= and D=$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Let's construct a descending tree from an initial matrix as follows:

From each square matrix M of the parent tree two new branches towards two other matrices MG (left) and MD (right).××

The two new matrices are called the “daughter matrices” of M.



In this method considered, we take as initial matrix, the matrix I=$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Let us determine by matrix calculation the matrices of the 3rd line as well as the 4th line of the Stern – Brocot tree.

In the first line 1, we have M= I = i.e. 2^0= 1 image.$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

In the 2nd line we have

G = and = D i.e. =2^1=2 images obtained in such a way $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

GM $=\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} et \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 0+0 \\ 1+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

MD $=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 1+0 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

In the 3rd row of the tree we will have 4 matrices as follows

IGG $=\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 0+0 \\ 1+1 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

IGD $=\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 0+1 \\ 1+0 & 1+1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

IDG $=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+1 & 0+1 \\ 0+1 & 0+1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

IDD $=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

Images Hence the four

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}; \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}; \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}; \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

In the 4th line, we will have either 8 images.2^3

IGGG $=\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 0+0 \\ 2+1 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$

IGGD $=\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 1+0 \\ 2+0 & 2+1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$

IGDG $=\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+1 & 0+1 \\ 1+2 & 0+2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

IGDD $=\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 1+1 \\ 1+0 & 1+2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$

IDGG $=\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2+1 & 0+1 \\ 1+1 & 0+1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

IDDG $=\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2+0 & 2+1 \\ 1+0 & 1+1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$

IDDG d$=\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+2 & 0+2 \\ 0+1 & 0+1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

IDDD $=\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

Hence the representation of part of an infinite binary tree constructed by square matrices of order 2.

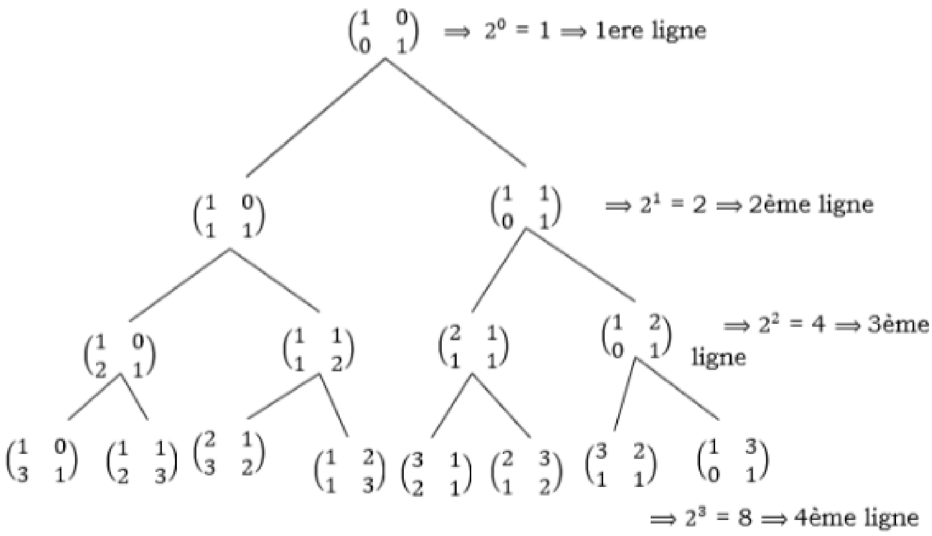


Fig3. *Representation of the Stern – Brocot tree constructed by square matrices of order 2.*

Statistics: On this tree constructed by raster images

- All images are represented once and only once in the tree.
- All matrices are positive.

The property is well respected for each matrix of the tree

- Each matrix is associated with an irreducible fraction as follows:
  - For $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \frac{1+0}{0+1} = \frac{1}{1}$
  - For $\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix} = \frac{1+0}{1+1} = \frac{1}{2}$
  - For= $\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix} = \frac{1+1}{0+1} = \frac{2}{1}$
  - For $\begin{pmatrix}1 & 0\\2 & 1\end{pmatrix} = \frac{1+0}{2+1} = \frac{1}{3}$
  - For $\begin{pmatrix}1 & 1\\1 & 2\end{pmatrix} = \frac{1+1}{1+2} = \frac{2}{3}$
  - For $\begin{pmatrix}2 & 1\\1 & 1\end{pmatrix} = \frac{2+1}{1+1} = \frac{3}{2}$
  - For $\begin{pmatrix}1 & 2\\0 & 1\end{pmatrix} = \frac{1+2}{0+1} = \frac{3}{1}$
  - For $\begin{pmatrix}1 & 0\\3 & 1\end{pmatrix} = \frac{1+0}{3+1} = \frac{1}{4}$
  - For $\begin{pmatrix}1 & 1\\2 & 3\end{pmatrix} = \frac{1+1}{2+3} = \frac{2}{5}$
  - For $\begin{pmatrix}2 & 1\\3 & 2\end{pmatrix} = \frac{2+1}{3+2} = \frac{3}{5}$
  - For $\begin{pmatrix}1 & 2\\1 & 3\end{pmatrix} = \frac{1+2}{1+3} = \frac{3}{4}$
  - For $\begin{pmatrix}3 & 1\\2 & 1\end{pmatrix} = \frac{3+1}{2+1} = \frac{4}{3}$
  - For $\begin{pmatrix}2 & 3\\1 & 2\end{pmatrix} = \frac{2+3}{1+2} = \frac{5}{3}$
  - For $\begin{pmatrix}3 & 2\\1 & 1\end{pmatrix} = \frac{3+2}{1+1} = \frac{5}{2}$
  - For $\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix} = \frac{1+3}{0+1} = \frac{4}{1}$

Generally speaking, we will admit that for any square matrix of order 2 M= of the Stern – Brocot tree where the numbers a, b, c, d are the integers verifying b + d = 0

We associate the irreducible fraction with a matrix M= of the Stern – Brocot tree.

If ad – bc which is the difference of the diagonal products of the matrix denoted by = ad – bc = 1 or d (a + c) – c (b + d) = 1.

Evidence

- $\triangle_M = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \Rightarrow$ ici $a = 2, b = 3, c = 1, d = 2$

?

$\Rightarrow ad - bc = 1$

$\Rightarrow$ 2.2 – 3.1 = 4 – 3 = 1

Ou $d\,(a + c) - c\,(b + d) = 1$

2 (2 + 1) – 1 (3 + 2) = 1

2 (3) – 1 (5)

6 – 5 = 1.

- $\triangle_M = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \Rightarrow$ ici $a = 1, b = 0$, c = 3, $d = 1$

?

$\Rightarrow ad - bc = 1$

$\Rightarrow$ 1.1 – 0.3 = 1 = 1 – 0 = 1

Ou $d(a + c) - c\,(b + d)$

$\Rightarrow$ 1 (1 + 3) – 3 (0 + 1)

$\Rightarrow$ 1 (4) – 3 (1)

4 – 3 = 1

So, for the other matrices.

Then and = $1\triangle_M \times G = 1\,\triangle_M \times D$

We admit that all the other matrices N of the Stern – Brocot tree verifying equality, we deduce that any fraction associated with a matrix of the Stern – Brocot tree is irreducible. $\triangle_N = 1$

Let M = a matrix of the Stern – Brocot tree so a, b, c, d are four natural integers such that b + d 0 and ad – bc = 1, we will admit that$\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ ≠a + c 0, then is irreducible.≠ $\frac{a+c}{b+d}$

As we ad (a + c) – c (b + d) = 1, a non-zero natural number, common divisor to a + c and b + d is therefore equal to 1.

This shows that the non-zero natural numbers a + c and b + d are relatively prime or that the fraction is irreducible. $\frac{a+c}{b+d}$

4. Conclusion

In the overall approach of this article, we matrixed by induction on the Stern–Brocot tree the representation of discrete images in an infinite binary tree language.

The properties derived from the representation (construction) of the Stern–Brocot tree for a fraction also apply on square matrices of order 2. The Stern–Brocot algorithm allows us to construct an infinite binary tree consisting of the images in the form of matrices which appear once and only once.

And these matrices are associated with positive irreducible fractions.

We have also given in this article a method using matrix calculation to determine a fraction knowing its position in the Stern – Brocot tree.

We have deduced a process for numbering all positive matrices. that is to say a bijection of positive rationals onto positive natural numbers.

In this paper, we have taken a comprehensive approach to representing discrete images in an infinite binary tree language using a recursive matrix representation based on the Stern-Brocot tree.

The properties of the Stern-Brocot tree representation (construction) for a fraction also apply to 2x2 square matrices. The Stern-Brocot algorithm makes it possible to construct an infinite binary tree made up of images in the form of matrices, each of which appears only once.

In addition, each of these matrices is associated with a unique positive irreducible fraction.

We also presented in this article a method that uses matrix calculation to determine a fraction given its position in the Stern-Brocot tree.

Finally, we derived a procedure for numbering all positive matrices, which establishes a bijection between positive rational numbers and positive integers.

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