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Analytical Solution for Heat and Mass Transfer of two-phase Nanofluid Flow with Magnetic Field in a rotating System using Adomian Decomposition Method

Liberty Ebiwareme[1*], Kubugha Wilcox Bunonyo[2], Onengiyeofori Anthony Davies[3]

[1] *Department of Mathematics, Rivers State University, Nkpolu-Oroworukwo, Port Harcourt, Nigeria*

[2]*Department of Mathematics and Statistics, Federal University Otuoke, Bayelsa, Nigeria.*

[3] *Department of Physics, Rivers State University, Nkpolu-Oroworukwo, Port Harcourt, Nigeria*

Corresponding Author:** ***Liberty Ebiwareme, *Department of Mathematics, Rivers State University, Nkpolu-Oroworukwo, Port Harcourt.*

Abstract: *The numerical investigation of a two-phase nanofliud flow between horizontal plates under the influence of transverse magnetic field and gravity in a rotating system is considered in this present work. It is assumed the velocity and distance between the parallel plates is constant. The governing flow equations of continuity, momentum, mass conservation and energy are converted into a non-linear system of ordinary differential equations by suitable similarity transformation and solved using semi-analytical Adomian decomposition technique. Effects of viscosity parameter, rotation parameter, magnetic field parameter, Brownian motion parameter, thermophoresis parameter, Schmidt number and Prandtl number on the velocity, gravitational acceleration, temperature, and concentration profiles are analysed and presented graphically and in tables. The efficiency, flexibility and accuracy of the proposed method is validated by comparing the result obtained with those in literature using fourth order Runge-Kutta-Fehlberg (RKFM), least square method (LSM) and feed-forward neutral networks (FFNN) with a back-propagated Levenberg-Marquardt (BLM) algorithm and are found to be in excellent accord, which demonstrates that the proposed solution technique is correct, computationally convenient, robust, and acceptable.*

Keywords: *Transverse magnetic field, two-phase nanofluid flow, rotating system, Adomian decomposition method (ADM), Similarity transformation*

1. INTRODUCTION

Nanofluid is defined as an engineered colloidal suspension of manometer-sized particles (nanoparticles) added to a base fluid usually with poor thermal conductivity [1]. The term was first conceived by Choi in 1995 of the Argonne National Laboratory in the United States of America [2]. The choice of the base fluid and nanoparticles depends on the application for which the nanofliud is intended to be used for, however, nanofluid does not occur naturally, but are synthesized in the laboratory [3-6]. The effect of the nanofluid to the base fluid is to increase the thermal conductivity and enhance the heat transfer ability of the nanofluid by considerable percentage.

The flow of nanofluid have myriads of practical imporce in many areas of modern science, engineering and technology, chemical, biomechanics, and nuclear industries. Buoyed by the assumption that most common base fluids have limited heat transfer capabilities, whereas nanoparticles such as Oxide Ceramics (Al_2O_3, CuO), metal carbides (SiC), nitrides (AlN, SiN) or metals (Al, Cu) have very high thermal conductivity in comparison to the common fluids [7]. Pioneering studies on the convective heat transport on nanofluids and its many applications to technology was first conducted by Buongiorno [8-9]. The findings in his study showed that the sum of base fluid velocity and the relative slip velocity gives the absolute velocity of the nanoparticles. Using seven slip mechanism, he established the relative velocity between the nanoparticle and the base fluid incorporating inertia, thermophoresis, Brownian diffusion, magnus effect, fluid drainage and gravity [10]. It was observed that, in the absence of turbulent effect, only Brownian diffusion and thermophoresis remain the key mechanisms in the nanofluids. Similarly, extensive research has been done by so many authors on the magnetohydrodynamics heat transfer analysis in nanofluids. Hatami et al. [11] employed the least square method to analyse the two-phase nanofluid condensation flow for industrial applications under magnetic field and external gravitational force. The study made comparison between the result obtained and that using fourth order Runge-kutta-Fehlberg and the error was found to be minimal. The study considered the Nusselt and Sherwood numbers for the condensation fluid under the same condition. The Magnetohydrodynamic flow and heat transfer of a hybrid nanofluid in a rotating system among two horizontal surfaces in the presence of thermal radiation and joule heating has been studied by Chamka et al. [12] using modified Duan-Rach approach. It was assumed that the lower and upper plates are stretchable and penetrable. Using self-similar transformations, the governing equations were transformed into a system of nonlinear ordinary differential equations and solved analytically. The study established the correlation between the Nusselt numbers for variation in the assorted parameters. [13] used a stochastic-driven approach called feed-forward neutral network (FFNN) with back-propagated Levenberg-Marquardt (BLM) algorithm to investigate the heat transfer analysis of nanofluid flow in a rotating system with magnetic field. The variations in the non-dimensional parameters of rotation, radiation, magnetic field, Schmidt, Prandtl, thermophoresis, Brownian motion and viscosity with the velocity, gravitational acceleration, temperature, and concentration profiles were examined. The study found that, the design algorithm is correct and robust with minimal mean percentage error which agree with established literature using LSM and RKFM.

[14] utilised numerical method to conduct a comparative study of convective heat transfer results between single phase and two-phase nanofluids in a circular tube. It was found that average relative error between experimental and CFD for a single-phase flow model is 16%, whereas for two-phase model, it was fond to be 8%. The comparison of single and two-phase models for nanofluid convection at the entrance of a uniformly heated tube has been considered by [15]. The study confirmed earlier result in literature by [14] and validate the accuracy of two-phase modelling. Mohyud-Din et al. [16] analytically studied the three-dimensional heat and mass transfer flow of nanofluid between two parallel plates in a rotating system with magnetic field effect. The study reveals that, heat transfer is enhanced by the thermophoresis and Brownian motion parameters but have opposite effect to the concentration profile. Also, they reported the temperature boundary layer thickness is decreased in the presence of the Coriolis force.

[17] conducted a numerical study of heat and mass transfer analysis of unsteady MHD nanofluid flow through a channel with moving porous walls in the presence of metallic nanoparticles. Two cases of thermal conductivity were used in the analysis of the H-C model. They computed and examined the effects of permeable Reynold numbers and relaxation/contraction parameter on the velocity, temperature, and concentration profiles. Their finding showed, the flow turns close to the wall of the boundary layer when the contraction is put together with suction. Conversely, when both relaxation and injection parameters are coupled together, the porous walls adjacent the flow decreased. Haider [18] has studied the impact of Stefan blowing in the presence of heat radiation, Arrhenius activation energy and chemical reaction of the unsteady MHD nanofluid using the optimal homotopy analysis technique. Sobamowo et al. [19] have considered an MHD nanofluid squeezing flow analysis method under the influence of slip boundary conditions using variation of parameter method (VPM). The three-dimensional flow of an Oldrolyd-B nanofluid over a bidirectional stretching sheet using combination of differential transformation method (DTM) and Pade approximation has been investigated by Gupta. [20]. The turbulent flow of an MHD Couette nanofluid has been examined by Mosayebidorcheh using differential transformation method [21-22]. Semi-analytical Akbari-Ganji (AGM) has been proposed to analyse heat and mass transfer of nanofliud in the presence of magnetic field. [23]. A Wakif-Galerkin weighted residual method (WGWRM) has been implemented to investigate the convection of nanofluid in a horizontal layer of finite depth [24]. Hassani [25] explored the homotopy analysis method (HAM) to study the analytical solutions for the boundary layer flow of a nanofluid. Hosseinzadeh [26] solved the problem of hybrid fluid in a porous octagonal medium under the effect of magnetic field and radiation. The effect of magnetic field on stagnation flow of hybrid TiO_2-Cu water nanofluid spanning an expanding surface has been investigated by Ghadikolaei [27]. Their findings reveal that, the skin friction coefficient decreased with positive increase in the magnetic field parameter and an increase in the Prandtl number. Mondal and Pal [28] has studied the influence of viscous-Ohmic dissipations and magnetic field on the convection-

radiative boundary layer flow of nanofluids caused by non-linear stretching/shrinking sheet. The finding showed, decrease in magnetic field lead to an increase in the skin friction and volume fraction coefficient. Sheikholeslami and Sadoughi [29] used Mesoscopic method for MHD nanofluid flow inside a porous cavity considering various shapes of nanoparticles.

Since the advent of portable computers and availability of symbolic computing software, novel semi-analytical methods have been developed by academics to find approximate solutions to problems where closed form solutions are not easy to come by or whose analytical solution are difficult to obtain. Some of these innovative methods includes Differential transformation method (DTM), Homotopy analysis method (HAM), Abkari-Ganji method (AGM), Adomian decomposition method (ADM), Homotopy perturbation method (HPM), Variational iteration method (VIM), Differential Quadrature method (DQM) and Exp-function method. Similarly, hybridization of two or more these methods were equally conceived. Examples of some of these hybrid methods are Laplace Adomian decomposition method (LADM), Optimal Homotopy asymptotic method (OHAM), Variational Homotopy perturbation method (VHPM), Homotopy perturbation transform method (HPTM), Optimal variational iteration method (OVIM), Laplace variational iteration method and others [30]

The primary motivation of this study is to solve the resulting system of nonlinear ordinary differential equations for the heat and mass analysis of a two-phase nanofluid flow between two horizontal plates with magnetic field in a rotating system using Adomian decomposition method. Successful application of this method in solving diverse problems in science, engineering and technology can be found in [31-55]. The convergence of the obtained solution for various values of the flow parameters is explicitly discussed and validated in comparison with literature using RKFM and FFNN-BLM. The rest of the study is organized as follows: Section 1 & 2 presents an exhaustive literature of mass and heat transfer of nanofliud under the influence of magnetic between parallel plate and mathematical formulation. The basics of the solution technique and mathematical analysis of the problem wherein ADM is applied to solve the system of nonlinear ordinary differential equations is given in section 3 & 4. The discussion of the numerical result by variation in the pertinent parameters entering the problem and their correlation with the velocity, gravitational acceleration, temperature and concentration in tables and graphs is discussed in section 5, whereas Section 6 gives the conclusion stating the major findings of the study.

2. Mathematical Formulation of the Problem

We consider a two-phase nanofluid flow which is rotating between horizontal plate around the $y-$axis under the influence of transverse magnetic field of intensity B through the plates where the fluid rotates with uniform velocity, Ω. Assuming the distance between the horizontal plates is constant. Let u and v be the velocities in the x and y directions respectively. Hence, motivated by [11], the governing boundary layer equations of continuity, momentum, mass, energy, and heat transfer are given as

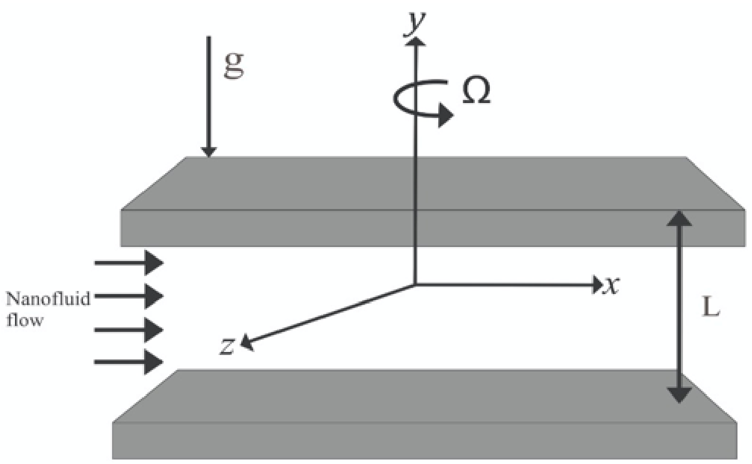

Figure 1: Configuration of a two-phase Nanofluid flow

3. Governing Equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho_f\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+2\Omega\omega\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma B^2 u \tag{2}$$

$$\rho_f\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3}$$

$$\rho_f\left(w\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-2\Omega\omega\right)=\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\sigma B^2 u \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+$$
$$\frac{(\rho C_\rho)_p}{(\rho C_\rho)_f}\left[D_B\left\{\frac{\partial C}{\partial x}.\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}.\frac{\partial T}{\partial y}+\frac{\partial C}{\partial z}.\frac{\partial T}{\partial z}\right\}+\left(\frac{D_T}{T_0}\right)\left\{\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right\}\right] \tag{5}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)+\left(\frac{D_T}{T_0}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{6}$$

subject to the prescribed boundary conditions

$$u=\ v=w=0, T=T_0, C=C_0 \text{ at } y=L$$
$$u=ax, v=w=0, T=T_1, C=C_1 \text{ at } y=0 \tag{7}$$

where T is the temperature, C is the concentration, P is the pressure, ρ_f is the density of the base fluid, μ is the dynamic viscosity, κ is the thermal conductivity, c_ρ is the specific heat capacity of the nanofliud and D_B is the coefficient of the diffusing species.

Next, to aid in our analysis of the governing equations, we introduce the following parameters.

$$\eta=\frac{y}{L}, u=axf'(\eta), v=-aLf(\eta), w=axg(\eta), \theta(\eta)=\frac{T-T_1}{T_0-T_1}, \phi(\eta)=\frac{C-C_1}{C_0-C_1} \tag{8}$$

Putting Eq. (8) into the governing Eqs. (1-5) and eliminating the pressure gradient, we have the resulting nonlinear system as

$$f^{iv}-R(f'f''-ff''')-2Krg'-Mf''=0 \tag{9}$$

$$g''-R(f'g-fg')+2Krf'-Mg=0 \tag{10}$$

$$\theta''+PrRf\theta'+Nb\phi'\theta'+Nt\theta'^2=0 \tag{11}$$

$$\phi''+RScf\phi'+\frac{Nt}{Nb}\theta''=0 \tag{12}$$

subject to the boundary conditions$f(0)=0, f'(0)=1, g(0)=0, \theta(0)=\phi(0)=1$

$$f(1)=0, f'(1)=0, g(1)=0, \theta(1)=\phi(1)=0 \tag{13}$$

where the nondimensional quantities

$R=\frac{aL}{\nu}$ is the viscosity parameter, $Pr=\frac{\mu}{\rho_f\alpha}$ is the Prandtl number, $M=\frac{\sigma B_0^2L^2}{\rho\nu}$ is the magnetic parameter, $Sc=\frac{\mu}{\rho_f D}$ is the Schmidt number, $Kr=\frac{\Omega L^2}{\nu}$ is the rotation parameter, $Nb=\frac{(\rho C)_p D_B(C_L)}{(\rho C)_f\alpha}$ is the Brownian motion parameter, $Nt=\frac{(\rho C)_f D_T(T_L)}{(\rho C)_f\alpha T_L}$ is the thermophoresis parameter

4. Basic Idea of Adomian Decomposition Method (ADM)

To proceed with the analysis, we start by stating the fundamentals of the solution techniques as follows: Let's consider a nonlinear differential equation of the form

$$L\big(u(x)\big) + R\big(u(x)\big) + N\big(u(x)\big) = g(x) \tag{14}$$

where L is the highest order derivative assumed to be invertible, R is the linear differential operator with order less than that of L, N is a nonlinear term and g is the source term

Rewriting Eq. (14) for $L\big(u(x)\big)$, we obtain

$$L\big(u(x)\big) = g(x) - R\big(u(x)\big) - N\big(u(x)\big) \tag{15}$$

Taking the inverse operator, L^{-1} on both sides of Eq. (3), we get

$$u(x) = L^{-1}g(x) - L^{-1}R\big(u(x)\big) - L^{-1}N\big(u(x)\big)$$

$$y(x) = \phi - L^{-1}R\big(u(x)\big) - L^{-1}N\big(u(x)\big) \tag{16}$$

where ϕ is the term arising from the integration of the source term. It is obtained using the following sequence depending on the order of the given equation.

$$\phi = \begin{cases} u(0) & for\ L = \dfrac{d}{dx} \\ u(0) + xu'(0) & for\ L = \dfrac{d^2}{dx^2} \\ u(0) + xu'(0) + \dfrac{x^2}{2!}u''(0) & for\ L = \dfrac{d^3}{dx^3} \\ u(0) + xu'(0) + \dfrac{x^2}{2}u''(0) + \dfrac{x^3}{3!}u'''(0) & for\ L = \dfrac{d^4}{dx^4} \\ \vdots & \\ \vdots & \\ u(0) + xu'(0) + \dfrac{x^2}{2!}u''(0) + \dfrac{x^3}{3!}u'''(0) + \cdots + \dfrac{x^n}{n!}u^{(n)}(0) & for\ L = \dfrac{d^{n+1}}{dx^{n+1}} \end{cases}$$

By the standard Adomian decomposition method, we write the unknown solution as an infinite decomposition series of the form.

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{18}$$

Putting Eq. (18) into Eq. (16), we obtain

$$\sum_{n=0}^{\infty} u_n(x) = \phi - L^{-1}R\left(\sum_{n=0}^{\infty} u_n(x)\right) - L^{-1}N\left(\sum_{n=0}^{\infty} u_n(x)\right) \tag{19}$$

Matching both sides of Eq. (19), we obtain the zeroth order component given by

$$u_0 = \phi$$

Then the recursive relation is given by

$$u_{n+1}(x) = -L^{-1}R(u_n) - L^{-1}N(u_n), \quad n \geq 0 \tag{20}$$

The solution of the problem in Eq. (14) is obtain as limit of the decomposing series

$$u(x) = \lim_{n\to\infty} u_n(x) = u_0(x) + u_1(x) + u_2(x) + \cdots \tag{21}$$

Similarly, the nonlinear term can be determined by an infinite series of the Adomian polynomials given by

$$N(u_0, u_1, u_2, \dots, u_n) = \sum_{n=0}^{\infty} A_n \tag{22}$$

Then the $A_n^{\prime s}$ are obtained from the relation

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty} \lambda^k u_k\right)\right]_{\lambda=0}, n = 0,1,2,3 \tag{23}$$

Using Eq. (22), the first five Adomian polynomials are given as

$$A_0 = N(u_0)$$

$$A_1 = u_1 N'(u_0)$$

$$A_2 = u_2 N'(u_0) + \frac{1}{2!} u_1^2 N''(u_0)$$

$$A_3 = u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{1}{3!} u_1^3 N'''(u_0)$$

$$A_4 = u_4 N'(u_0) + \frac{1}{2} N''(u_0)(2u_1u_3 + u_2^2) + \frac{1}{2} N'''(u_0) u_1^2 u_2 + \frac{1}{4!} N^{(iv)}(u_0) u_1^4$$

$$A_5 = u_5 N'(u_0) + \frac{1}{2} N''(u_0)(2u_1u_4 + 2u_2u_3) + \frac{1}{3!} N'''(u_0)(3u_1^2u_3 + 3u_1u_2^2) + \frac{4}{4!} N^{(iv)}(u_0)(u_1^3u_2) + \frac{1}{5!} N^{(v)}(u_0) u_1^5$$

$$A_6 = u_6 N'(u_0) + \frac{1}{2!} N''(u_0)(2u_1u_5 + 2u_1u_4 + u_3^2) + \frac{1}{3!} N'''(u_0)(3u_1^2u_4 + u_2^3 + 6u_1u_2u_3) + \frac{1}{4!} N^{(iv)}(u_0)(4u_1^3u_3 + 6u_1^2u_2^2) + \frac{5}{5!} N^{(v)}(u_0) u_1^4 u_2 + \frac{1}{6!} N^{(vi)}(u_0) u_1^6$$

5. Solution Procedure Via Adomian Decomposition Method (ADM)

Rearranging Eqs. (9-12) gives the equivalent expressions

$$f^{iv}(\eta) = R(f'f'' - ff''') + 2Krg' + Mf'' \tag{24}$$

$$g''(\eta) = R(f'g - fg') - 2Krf' + Mg \tag{25}$$

$$\theta''(\eta) = -PrRf\theta' - Nb\phi'\theta' - Nt\theta'^2 \tag{26}$$

$$\phi''(\eta) = -RScf\phi' - \frac{Nt}{Nb}\theta'' \tag{27}$$

Writing the Eqs. (24-27) in operator forms gives

$$L_1(f) = R(f'f'' - ff''') + 2Krg' + Mf'' \tag{28}$$

$$L_2(f) = R(f'g - fg') - 2Krf' + Mg \tag{29}$$

$$L_3(f) = -PrRf\theta' - Nb\phi'\theta' - Nt\theta'^2 \tag{30}$$

$$L_4(f) = -RScf\phi' - \frac{Nt}{Nb}\theta'' \tag{31}$$

where $L_1(.) = \frac{d^4}{d\eta^4}(.)$ and $L_2(.) = L_3(.) = L_4(.) = \frac{d^2}{d\eta^2}(.)$ is a fourth order and second order differential operators respectively. Suppose the inverse operators of the above, then we have

$$L_1^{-1}(.) = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta (.) d\eta d\eta d\eta d\eta$$

$$L_2^{-1}(.) = L_3^{-1}(.) = L_4^{-1}(.) = \int_0^\eta \int_0^\eta (.) d\eta d\eta$$

Now applying L_1^{-1}, L_2^{-1}, L_3^{-1} and L_4^{-1} to both sides of Eqs. (28) $-$(31), we get the expression

$$f(\eta) = f(0) + \eta f'(0) + \frac{\eta^2}{2} f''(0) + \frac{\eta^2}{2} f''(0) + L_1^{-1}\left[R\left(N_1(f) - N_2(f)\right) + 2Krg' + Mf''\right] \tag{32}$$

$$g(\eta) = g(0) + \eta g'(0) + L_2^{-1}\left[R\left(N_3(f,g) - N_4(f,g)\right) - 2Krf' + Mg\right] \tag{33}$$

$$\theta(\eta) = \theta(0) + \eta\theta'(0) - L_3^{-1}\left[PrR\left(N_5(f,\theta)\right) + Nb\left(N_6(\phi,\theta)\right) + Nt\left(N_7(\theta)\right)\right] \tag{34}$$

$$\phi(\eta) = \phi(0) + \eta\phi'(0) - L_4^{-1}\left[RSc\left(N_8(f,\phi)\right) - \frac{Nt}{Nb}\theta''\right] \tag{35}$$

Imposing the given boundary conditions at $\eta = 0$, Eqs. (32) $-$(35) reduced to the form

$$f(\eta) = \eta + \frac{\alpha_1}{2}\eta^2 + \frac{\alpha_2}{3!}\eta^3 + L_1^{-1}\left[R\left(N_1(f) - N_2(f)\right) + 2Krg' + Mf''\right] \tag{36}$$

$$g(\eta) = \alpha_3\eta + L_2^{-1}\left[R\left(N_3(f,g) - N_4(f,g)\right) - 2Krf' + Mg\right] \quad (37)$$

$$\theta(\eta) = 1 + \alpha_4\eta - L_3^{-1}\left[PrR\left(N_5(f,\theta)\right) + Nb\left(N_6(\phi,\theta)\right) + Nt\left(N_7(\theta)\right)\right] \quad (38)$$

$$\phi(\eta) = 1 + \alpha_5\eta - L_4^{-1}\left[RSc\left(N_8(f,\phi)\right) - \frac{Nt}{Nb}\theta''\right] \quad (39)$$

where $f''(0) = \alpha_1, f'''(0) = \alpha_2, g'(0) = \alpha_3, \theta'(0) = \alpha_4, \phi'(0) = \alpha_5$ are to be determined later using the boundary condition at $\eta = 1$ and $(f), N_2(f), N_3(f,g), N_4(f,g), N_5(f,\theta), N_6(\phi,\theta), N_7(\theta)$ and $N_8(f,\phi)$ are the nonlinear terms.

By the Adomian decomposition method, the unknowns $f(\eta), g(\eta), \theta(\eta)$ and $\phi(\eta)$ are assumed to be series of the form.

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta), g(\eta) = \sum_{n=0}^{\infty} g_n(\eta), \theta(\eta) = \sum_{n=0}^{\infty} \theta_n(\eta), \phi(\eta) = \sum_{n=0}^{\infty} \phi_n(\eta) \quad (40)$$

Similarly, the nonlinear terms are decomposed as series polynomials in the form

$$N_1(f) = \sum_{n=0}^{\infty} A_n, N_2(f) = \sum_{n=0}^{\infty} B_n, N_3(f,g) = \sum_{n=0}^{\infty} C_n, N_4(f,g) = \sum_{n=0}^{\infty} D_n, N_5(f,\theta) = \sum_{n=0}^{\infty} E_n, N_6(\phi,\theta) = \sum_{n=0}^{\infty} F_n, N_7(\theta) = \sum_{n=0}^{\infty} G_n, N_8(f,\phi) = \sum_{n=0}^{\infty} H_n \quad (41)$$

where $A_n, B_n, C_n, D_n, E_n, F_n, G_n$ and H_n are the Adomian polynomials defined as follows

$N_1(f) = f'f'', A_0 = f_0'f_0'', A_1 = f_0'f_1'' + f_1'f_0'', A_2 = f_0'f_2'' + f_1'f_1'' + f_2'f_0''$
$N_2(f) = ff''', B_0 = f_0f_0''', B_1 = f_0f_1''' + f_1f_0''', B_2 = f_0f_2''' + f_1f_1''' + f_2f_0'''$
$N_3(f,g) = f'g, C_0 = f_0'g_0, C_1 = f_0'g_1 + f_1'g_0, C_2 = f_0'g_2 + f_1'g_1 + f_2'g_0$
$N_4(f,g) = fg', D_0 = f_0g_0', D_1 = f_0g_1' + f_1g_0', D_2 = f_0g_2' + f_1g_1' + f_2g_0'$
$N_5(f,\theta) = f\theta', E_0 = f_0\theta_0', E_1 = f_0\theta_1' + f_1\theta_0', E_2 = f_0\theta_2' + f_1\theta_1' + f_2\theta_0'$
$N_6(\phi,\theta) = \phi'\theta', F_0 = \phi_0'\theta_0', F_1 = \phi_0'\theta_1' + \phi_1'\theta_0', F_2 = \phi_0'\theta_2' + \phi_1'\theta_1' + \phi_2'\theta_0'$
$N_7(\theta) = (\theta')^2, G_0 = (\theta_0')^2, G_1 = 2\theta_0'\theta_1', G_2 = 2\theta_0'\theta_2' + (\theta_1')^2$
$N_8(f,\phi) = f\phi', H_0 = f_0\phi_0', H_1 = f_0\phi_1' + f_1\phi_0', H_2 = f_0\phi_2' + f_1\phi_1' + f_2\phi_0'$

Substitution of Eqs. (40) and (41) into Eqs. (35−(38), we have

$$\sum_{n=0}^{\infty} f_n(\eta) = \eta + \frac{\alpha_1}{2!}\eta^2 + \frac{\alpha_2}{3!}\eta^3 + L_1^{-1}[R(\sum_{n=0}^{\infty} A_n - \sum_{n=0}^{\infty} B_n) + 2Kr\sum_{n=0}^{\infty} g_n' + M\sum_{n=0}^{\infty} f_n''] \quad (42)$$

$$\sum_{n=0}^{\infty} g_n(\eta) = \alpha_3\eta + L_2^{-1}[R(\sum_{n=0}^{\infty} C_n - \sum_{n=0}^{\infty} D_n) - 2Kr\sum_{n=0}^{\infty} f_n' + M(\sum_{n=0}^{\infty} g_n)] \quad (43)$$

$$\sum_{n=0}^{\infty} \theta_n(\eta) = 1 + \alpha_4\eta - L_3^{-1}[PrR(\sum_{n=0}^{\infty} E_n) + Nb(\sum_{n=0}^{\infty} F_n) + Nt(\sum_{n=0}^{\infty} G_n)] \quad (44)$$

$$\sum_{n=0}^{\infty} \phi_n(\eta) = 1 + \alpha_5\eta - L_4^{-1}\left[RSc(\sum_{n=0}^{\infty} H_n) - \frac{Nt}{Nb}(\sum_{n=0}^{\infty} \theta_n'')\right] \quad (45)$$

From the integral Equations (42) − (45), the zeroth order components and the recursive schemes for the approximate analytical solution of system (9-12) are given as follows.

$$f_0(\eta) = \eta + \frac{\alpha_1}{2!}\eta^2 + \frac{\alpha_2}{3!}\eta^3,\ g_0(\eta) = \alpha_3\eta,\ \theta_0(\eta) = 1 + \eta\alpha_4,\ \phi_0(\eta) = 1 + \alpha_5\eta \quad (46)$$

$$f_{n+1}(\eta) = \int_0^{\eta}\int_0^{\eta}\int_0^{\eta}\int_0^{\eta}(R(A_n - B_n) + 2Krg_n' + Mf_n'')d\eta d\eta d\eta d\eta,\ n \geq 0 \quad (47)$$

$$g_{n+1}(\eta) = \int_0^{\eta}\int_0^{\eta}(R(C_n - D_n) - 2Krf_n' + Mg_n)d\eta d\eta,\ n \geq 0 \quad (48)$$

$$\theta_{n+1}(\eta) = \int_0^{\eta}\int_0^{\eta}(PrRE_n + NbF_n + NtG_n)d\eta d\eta,\ n \geq 0 \quad (49)$$

$$\phi_{n+1}(\eta) = \int_0^{\eta}\int_0^{\eta}\left(RScH_n + \frac{Nt}{Nb}\theta_n''\right)d\eta d\eta,\ n \geq 0 \quad (50)$$

The approximate solutions of Eqs. (47)−(50) are obtained using the partial sum as follows

$$f(\eta) = \sum_{n=0}^{\infty} f_k(\eta), g(\eta) = \sum_{n=0}^{\infty} g_k(\eta), \theta(\eta) = \sum_{n=0}^{\infty} \theta_k(\eta),\ \phi(\eta) = \sum_{n=0}^{\infty} \phi_k(\eta) \quad (51)$$

$$f(\eta) = \eta + \frac{\alpha_1}{2!}\eta^2 + \frac{\alpha_2}{3!}\eta^3 + \frac{1}{24}(M\alpha_1 + R\alpha_1 - 2\mathrm{Kr}\alpha_3)\eta^4 + \frac{1}{120}(R\alpha_1^2 + 2\alpha_2)\eta^5 + \frac{1}{180}R\alpha_1\alpha_2\eta^6 + \frac{1}{630}R\alpha_2^2\eta^7 \quad (52)$$

$$g(\eta) = \alpha_3\eta - 2\mathrm{Kr}\eta^2 + (-2\mathrm{Kr}\alpha_1 + M\alpha_3)\eta^3 + (-2\mathrm{Kr}\alpha_2 + \frac{R\alpha_1\alpha_3}{2})\eta^4 + \frac{2}{3}R\alpha_2\alpha_3\eta^5 \quad (53)$$

$$\theta(\eta) = 1 + \alpha_4\eta - \frac{1}{6}\mathrm{Pr}R\eta^3 - \frac{1}{24}\mathrm{Pr}R\alpha_1\alpha_4\eta^4 - \frac{1}{60}\mathrm{Pr}R\alpha_2\alpha_4\eta^5 + \eta^2(-\frac{\alpha_4\mathrm{Nt}}{2} - \frac{\mathrm{Nb}\alpha_4\alpha_5}{2}) \quad (54)$$

$$\phi(\eta) = 1 + \alpha_5\eta - \frac{\alpha_5}{6}RSc\eta^3 - \frac{\alpha_5}{24}RSc\alpha_1\eta^4 - \frac{\alpha_5}{60}RSc\alpha_2\eta^5 \quad (55)$$

Using the given boundary conditions at $\eta = 1$, we obtained the values of the constants $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and α_5. Substituting these constants into Eqs. (52) −(55) gives the complete solution of the problem.

6. Results and Discussion

In this section, we present the numerical results obtained for the non-dimensional velocity, gravitational acceleration, temperature, and concentration profiles under the influence of various parameters. The parameters that arise in this study are viscosity parameter (R), rotation parameter (Kr), magnetic field parameter (M), Brownian motion parameter (Nb), thermophoresis parameter (Nt), Schmidt number (Sc), and Prandtl number $(\Pr)$. Using symbolic software, COMSOL Multiphysics, the flow parameters, and their effects are analyzed, and the result presented in graph and tables.

Table1. *Comparison between ADM result with numerical, LSM and FFNN-BLM methods for velocity profile when* $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$.

η	$f(\eta)$						
	RKFM [11]	LSM [11]	FFNN-BLM [13]	ADM	Error (LSM)	Error (FFNN-BLM)	Error (ADM)
0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.10	0.07656	0.07614	0.07655	0.07613	0.0004	0.00005	0.00003
0.20	0.11453	0.11438	0.11453	0.11451	0.0001	0.00000	0.00002
0.30	0.12540	0.12675	0.12540	0.12630	0.0013	0.00000	0.00001
0.40	0.11850	0.12199	0.11850	0.11850	0.0034	0.00000	0.00000
0.50	0.10077	0.10622	0.10077	0.100760	0.0054	0.00000	0.00001
0.60	0.07719	0.08365	0.07718	0.07718	0.0064	0.00005	0.00005
0.70	0.05145	0.05753	0.05146	0.651440	0.0060	0.00005	0.00001
0.80	0.02705	0.03126	0.02704	0.02704	0.0042	0.00005	0.00000
0.90	0.00799	0.00957	0.00799	0.00798	0.0015	0.00000	0.00000
1.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table2. *Comparison between ADM result with numerical, LSM and FFNN-BLM methods for gravitational acceleration profile when* $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$

η	$g(\eta)$						
	RKFM [11]	LSM [11]	FFNN-BLM [13]	ADM	Error (LSM)	Error (FFNN-BLM)	Error (ADM)
0.00	0.00000	0.00000	0.00000	0.00000	0.000000	0.00000	0.00000
0.10	0.06315	-0.117922	0.06315	0.06313	0.181072	0.00000	0.00002
0.20	0.01530	-0.253173	0.01530	0.01529	0.268473	0.00000	0.00001
0.30	-0.07849	-0.381786	-0.07849	-0.07845	0.303296	0.00000	0.00001
0.40	-0.17525	-0.484581	-0.17525	-0.17525	0.309331	0.00000	0.00000
0.50	-0.24860	-0.546816	-0.24860	-0.24861	0.298216	0.00000	0.00001
0.60	-0.28374	-0.557834	-0.28374	-0.28375	0.274094	0.00000	0.00001
0.70	-0.27380	-0.510716	-0.27380	-0.27382	0.236916	0.00000	0.00002
0.80	-0.21799	-0.401925	-0.21799	-0.21798	0.183935	0.00000	0.00001
0.90	-0.12193	-0.230960	-0.22193	-0.12194	0.109030	0.00000	0.00001
1.00	0.00000	0.000000	0.000000	0.00000	0.000000	0.00000	0.000000

Table3. *Comparison between ADM result with numerical, LSM and FFNN-BLM methods for temperature profile when* $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$

η	$\theta(\eta)$						
	RKFM [11]	LSM [11]	FFNN-BLM [13]	ADM	Error (LSM)	Error (FFNN-BLM)	Error (ADM)
0.00	1.00000	1.00000	0.99969	1.00000	0.00000	0.00031	0.00000
0.10	0.92309	0.92299	0.92309	0.92308	0.00001	0.00000	0.00001
0.20	0.84168	0.84148	0.84168	0.84167	0.00002	0.00000	0.00001
0.30	0.75569	0.75537	0.75569	0.75568	0.00032	0.00000	0.00001
0.40	0.66493	0.66448	0.66494	0.66492	0.00045	0.00000	0.00001
0.50	0.56913	0.56856	0.56913	0.56912	0.00057	0.00000	0.00001
0.60	0.46793	0.46730	0.46793	0.46791	0.00063	0.00000	0.00002

0.70	0.36090	0.36030	0.36091	0.36089	0.00006	0.00005	0.00001
0.80	0.24758	0.24711	0.24759	0.24756	0.00047	0.00005	0.00002
0.90	0.12746	0.12721	0.12746	0.12745	0.00025	0.00000	0.00000
1.00	0.00000	0.00000	0.00001	0.00000	0.00000	0.00001	0.00000

Table4. *Comparison between ADM result with numerical, LSM and FFNN-BLM methods for concentration profile when* $Kr = 10, R = 1, M = 2, Pr = 0.5, Nb = 0.1, Nt = 0.5, Sc = 1$

η	$\phi(\eta)$						
	RKFM [11]	LSM [11]	FFNN-BLM [13]	ADM	Error (LSM)	Error (FFNN-BLM)	Error (ADM)
0.00	1.00000	1.00000	0.99992	0.99989	0.000000	0.000008	0.00011
0.10	0.77832	0.779201	0.77832	0.77831	-0.000088	0.000000	0.00001
0.20	0.58066	0.582439	0.58066	0.58064	-0.001779	0.000000	0.00002
0.30	0.40797	0.410275	0.40797	0.40796	-0.002305	0.000000	0.00001
0.40	0.26111	0.263553	0.26111	0.26110	-0.002443	0.000000	0.00001
0.50	0.14105	0.143394	0.14105	0.14104	-0.002344	0.000000	0.00001
0.60	0.04906	0.051220	0.04906	0.04905	-0.002160	0.000000	0.00001
0.70	-0.01321	-0.011338	-0.01321	-0.01320	-0.001872	0.000000	0.00001
0.80	-0.04375	-0.042262	-0.04375	-0.04374	-0.001488	0.000000	0.00001
0.90	-0.04021	-0.039324	-0.04021	-0.04020	-0.000886	0.000000	0.00001
1.00	0.00000	0.000000	0.00000	0.00000	0.0000000	0.000000	0.00000

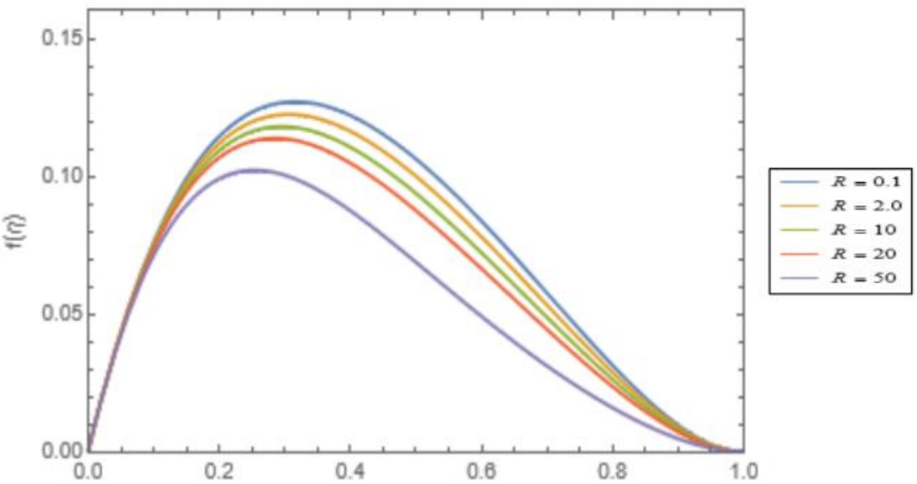

Figure1. *Variation of the temperature profile for variation in rotation parameter*

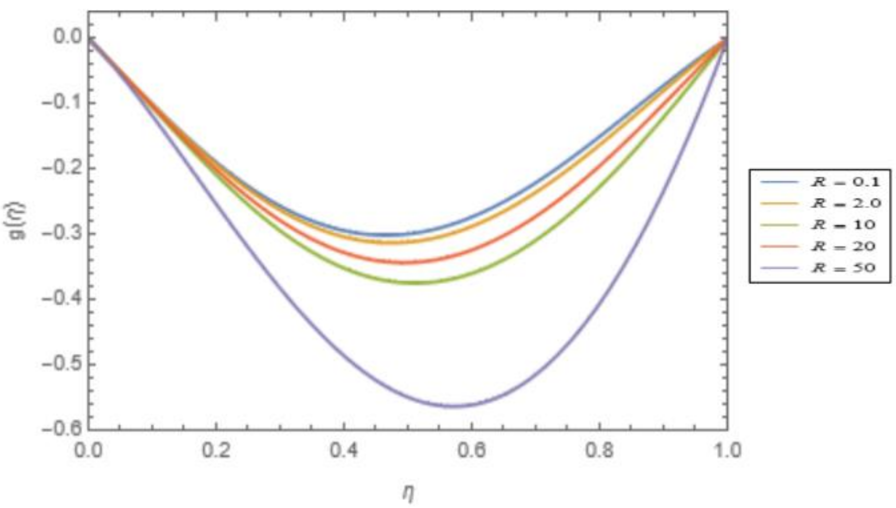

Figure2. *Gravitational profile for variation in rotation parameter and constant values of other parameters*

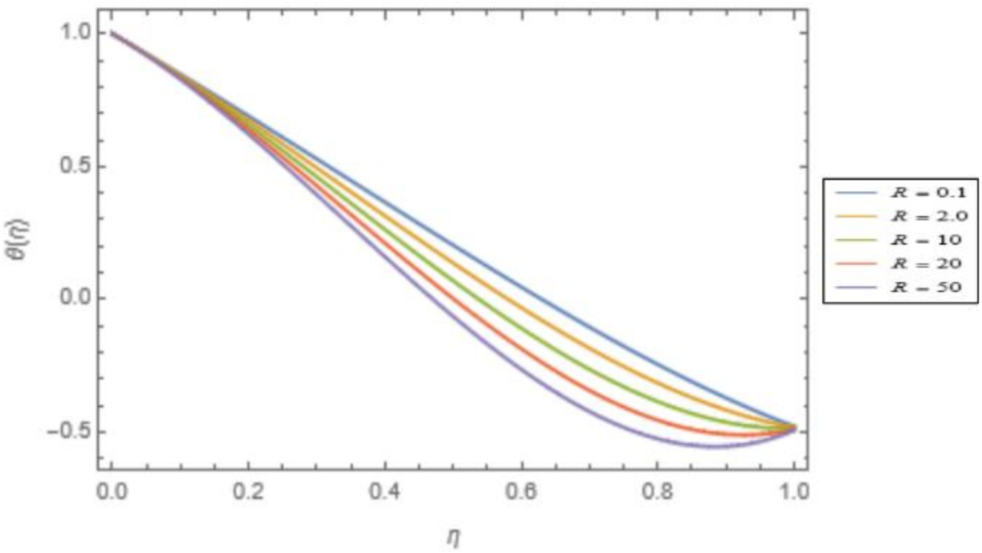

Figure3. *Temperature profile for variation in the radiation parameter when* $M = 2, Pr = 0.71, Sc = 0.1, Nt = 0.1, Nb = 0.5, Kr = 10$

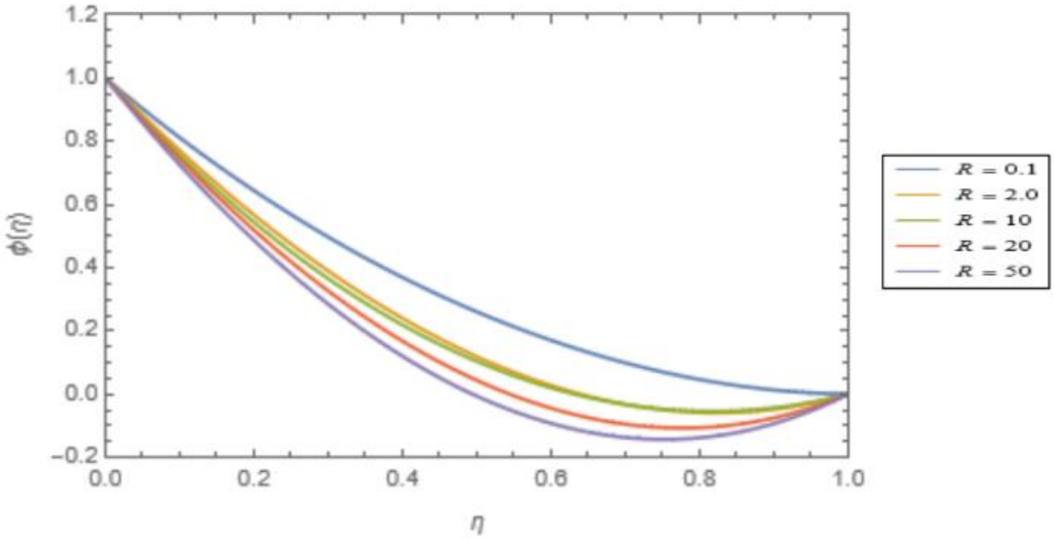

Figure4. *Gravitational acceleration profile for variation in the rotation parameter.*

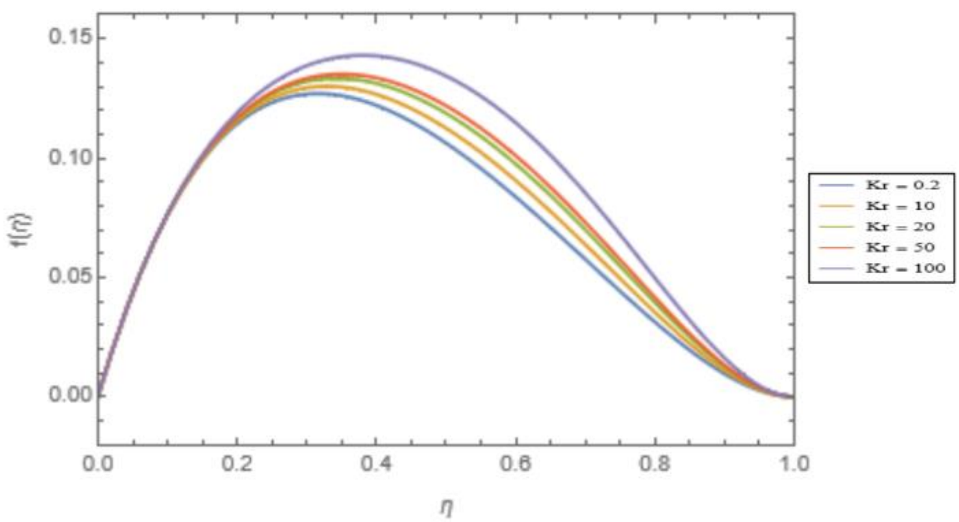

Figure5. *Variation of temperature profiles for increase in rotation parameter.*

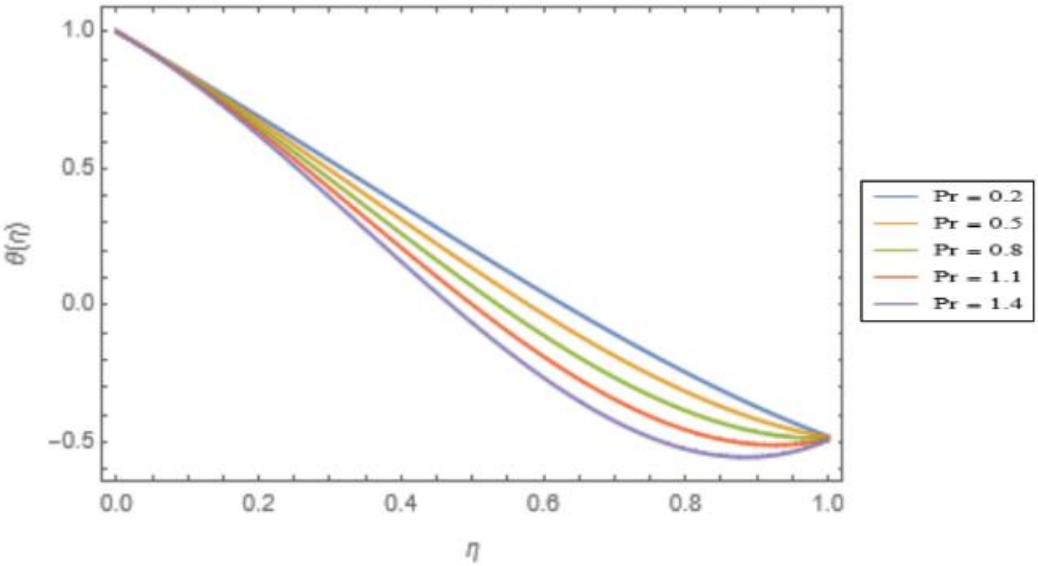

Figure6. *Temperature profile for different values of Prandtl number when* $M = 2, R = 1, Sc = 0.1, Nt = 0.1, Nb = 0{,}5, Kr = 10$

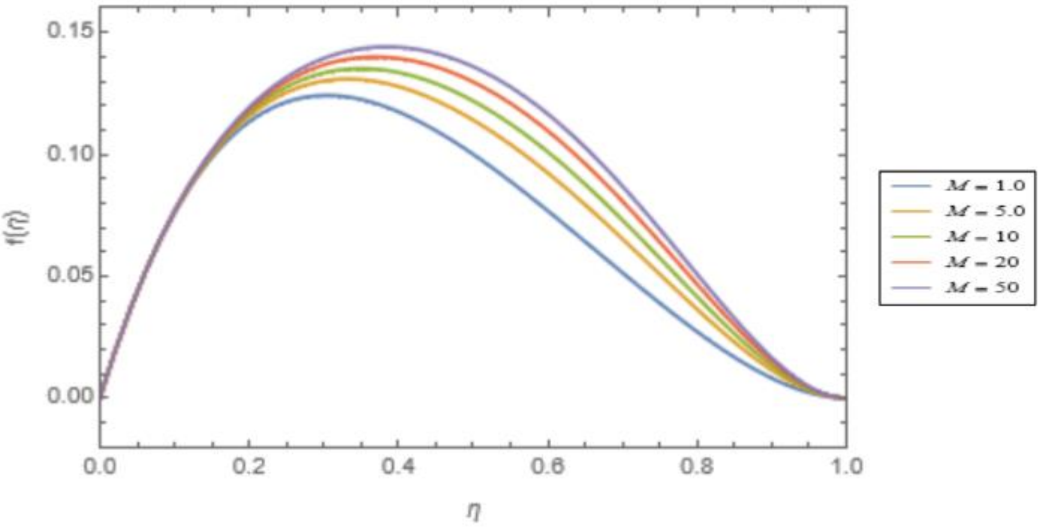

Figure7. *Velocity profile for variation in magnetic field parameter*

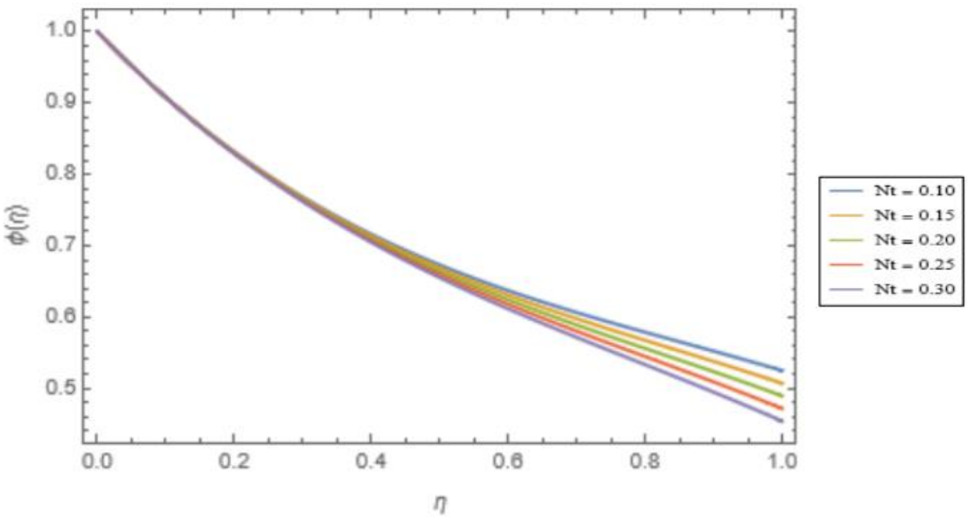

Figure8. *Gravitational acceleration profile for variation in thermophoresis parameter when* $M = 2, R = 1, Sc = 0.1, Nt = 0.1, Nb = 0{,}5, Kr = 10$

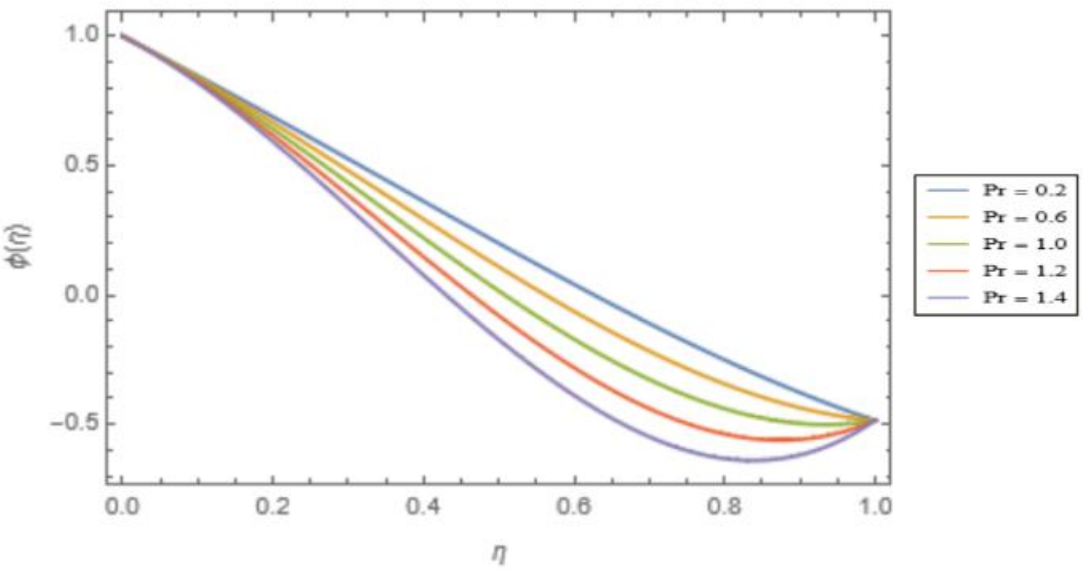

Figure9. *Gravitational acceleration profile for variation in the Prandtl number and constant values of* $M = 2, R = 1, Sc = 0.1, Nt = 0.1, Nb = 0{,}5, Kr = 10$

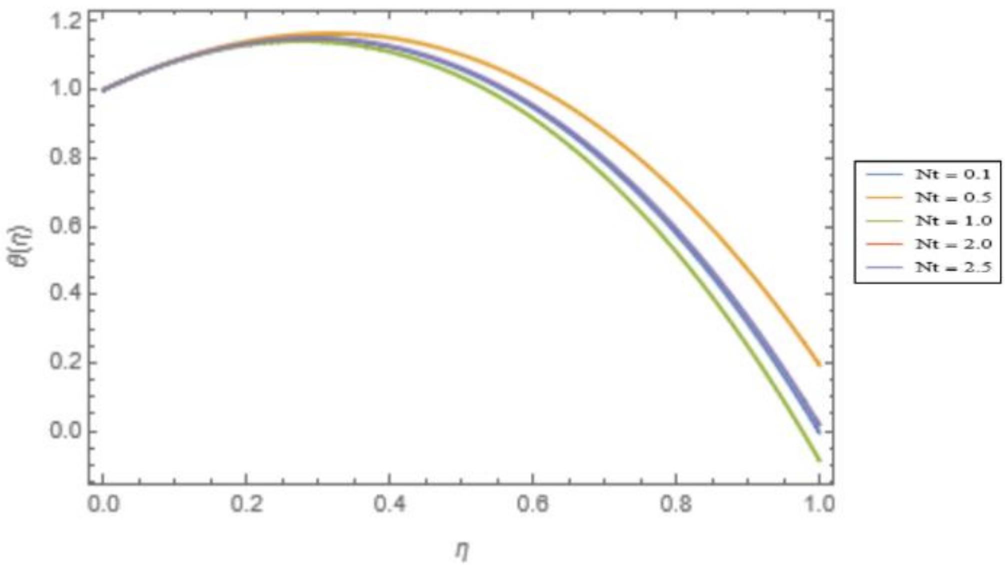

Figure10. *Temperature profile for variation in the thermophoresis Parameter* $M = 2, R = 1, Sc = 0.1, Pr = 0.71, Nb = 0.5, Kr = 10$

7. CONCLUSION

In this paper, we examined the magnetohydrodynamic heat transfer analysis of a two-phase nanofliud flow between horizontal plates under the influence of magnetic field in a rotating system using semi-analytical technique. The system of ordinary differential equations is solved analytically using Adomian decomposition method (ADM). Parametric study was carried out to ascertain the influence of the pertinent flow parameters on the flow velocity $(f(\eta))$, gravitational acceleration, $(g(\eta)$, temperature, $(\theta(\eta))$ and concentration, $(\phi(\eta))$. From our analysis, we draw the following conclusions for the study as follows.

i the velocity profile increased with increase in the viscosity and rotation parameters.

ii the influence of the magnetic field parameters is to decrease the velocity profile of the flow as it produces the Lorentz force

iii the temperature profile and viscosity parameter vary proportionally.

iv the impact of the thermophoresis parameter is to decrease the temperature profile.

v the velocity boundary layer thickness of the flow decreased in the presence of the Prandtl number.

vi Schmidt number and magnetic parameter increases the concentration profile, whereas it is decreased in the presence of thermophoresis parameter.

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NOMENCLATURE

A	Ratio of the thermophoresis effect to the Brownian diffusion effect of the nanofliud
C	Concentration of nanofliud
C_p	Specific heat capacity
g	acceleration due to gravity
κ	thermal conductivity
B	magnetic field parameter
ρ_f	density of the base fluid
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient

GREEK SYSMBOLS

Ω	Angular velocity
η	Similarity variable
θ	Self-similar temperature
ϕ	Nanoparticle concentration
μ	dynamic viscosity
ρ	density
υ	kinematic viscosity
σ	electric conductivity

SUBSCRIPTS

n_f	Nanofluid
f	Nanofluid phase
s	Solid phase
m	medium
0	lower plate
1	upper plate

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