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Effects of Pertinent Parameters on Pressure and Derivative Behaviour of a Layered Reservoir System with Crossflow

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Abstract: *This paper studies the effects of some pertinent well and reservoir parameters on dimensionless pressure (P_D) and derivative (P_D') behaviour of horizontal wells in a two layered reservoir subject to a bottom water drive. Pressure and derivative values were generated using layered reservoir models with a permeable interface. Results showed the emergence of the typical constant pressure boundary trend(constant pressure and zero derivative values) at late times. It was observed that this characteristic phenomenon shielded the effect of the parameters examined at late times. Both layers displayed some similar pressure and derivative behaviour with the varied parameters. The dimensionless pay thickness (h_D), dimensionless wellbore radius (r_{wD}) and crossflow coefficient (β) influenced the pressure behaviour of the wells while the modification factor (E), well width(y_D) flow point(x_D), well location(z_{WD}) showed negligible effects.*

Results from this study will find use in efficient well planning and effective depletion strategy as well as enhancing the overall performance of layered crossflow reservoir system with bottom water drive.

Keywords: *Permeable interface, layered reservoir, dimensionless pressure, pressure derivative, constant pressure boundary horizontal well*

1. Introduction

Layered reservoirs usually consist of two groups; crossflowsystem where layers can hydro dynamically communicate at the contact places and no crossflow (commingled system) where communication is only via the wellbore [1-4]. Research has shown that crossflow reservoir system experience lower pressure than no crossflow system. And crossflow direction is always from areas of low permeability to that of higher permeability [5]. Researches done in layered crossflow system include development of well test analysis models,horizontal welltechnology[6-10],enhanced oil recoveryand unconventional reservoir modelling[11-13]. Research on pressure behaviour of single and layered reservoir systemsshowed that they behave differently [14].Some factors like well completion, reservoir parameters, rock and fluid properties play vital role in shaping their characteristics profile and impact on productivity [15,16]. This paper aims to investigate the effects of changes in well / reservoir parameters (radius, pay thickness, modification factor, well width, crossflow coefficient and well location) on layer pressure and derivative response for a cross flow layered reservoir system using horizontal wells.

2. Methodology / Model Description

This study extends the work done by Akpobi et al. [17].They developed mathematical models of a two layered reservoir with bottom water drive to study the performance of horizontal wells placed in each layer as shown in Figure1a. The Source and Greens function method was utilized to formulate the mathematical model equations. This work utilized the pressure and derivative models under the condition of crossflow interface to study the effects of parameters likeradius, pay thickness, modification factor, well width, crossflow coefficient and well location. Summary of theory is in Appendix. Model equations were numerically solved [17,18]. Plots of dimensionless pressure and derivative versus dimensionless time were developed and used for the analysis.

Table1. *Example Parameters*

Parameter	Well Length L(ft)	Height h.(ft)	Well Standoff, Z_W (ft)	Reservoir Length, X_e (ft)	Reservoir Width, Y_e (ft)	Elevation Allowance(ft)	Permeability Ratio K_2:K_1 (md)
Value	1000	200	100	10000	6000	20	10

3. RESULTS /DISCUSSION

Data in Table1 was used in mathematical models(rw = 0.375ft, C_t =1.0E-6 psi^{-1}, φ=0.2, μ=0.3cp, α =2, flow rate is assumed constant at q = 400 STB/day).Same data was used for layer1 and 2 computations. The reservoir layer is isotropic (k_x= k_y= k_z), E=1 and β=1[17]

3.1. Effects of Dimensionless Pay Thickness (h_D)

Figure 1 provides the result of pressure and derivative response of changes in h_D in a crossflow system. Using varying values of h_D in the range of $0.05 \le h_D \le 1$. It was observed that pressure and derivative response increased with larger $h_D$values. At late times constant pressure was observed and derivative values collapsed to zero. Results showed similar trend for both layers of the system.

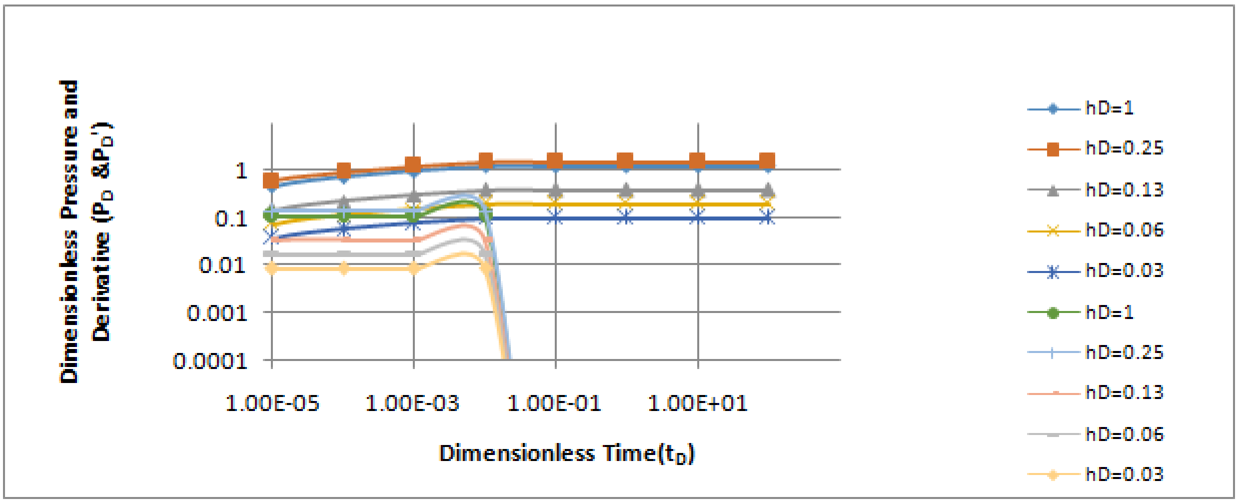

Figure1. *Effects of dimensionless pay thickness (h_D)*

3.2. Influence of Z_{WD}

To study the influence of well location four values of z_{WD}(0.0, 0.125, 0.3 and 0.5) were used to compute pressure and derivative response. The single line for pressure and derivative in Figure 2 showed that pressure and derivative response were insensitive to changes in vertical well location.

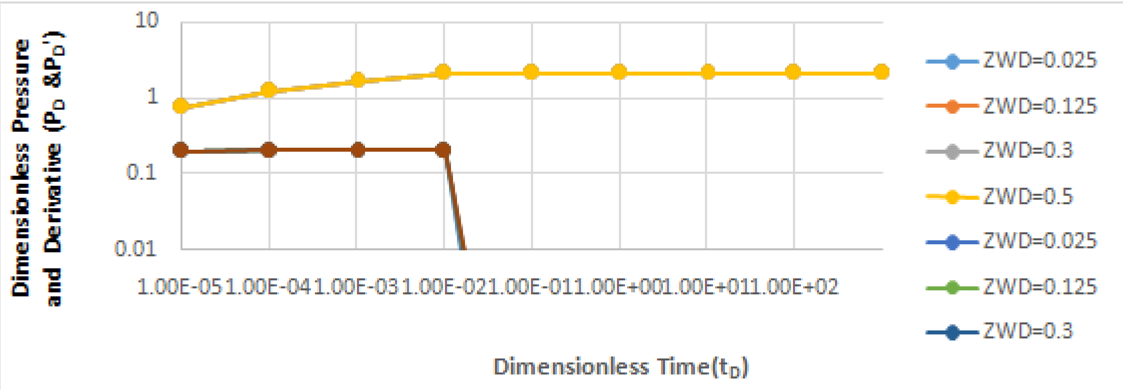

Figure2. *Influence of Z_{WD}*

3.3. Influence of r_{WD}

Influence of changes in the wellbore radius on pressure and derivative response, was investigated using r_{WD} values in the range of 1E-6 $\leq r_{WD} \leq$1E-3. From Figure 3, it was observed that for larger r_{WD} pressure decreases, and this change was more pronounced during the early time flow period. Derivative was insensitive to changes in r_{WD}. Same results were obtained from layer two of the crossflow system. This result will find use in well completion for the system.

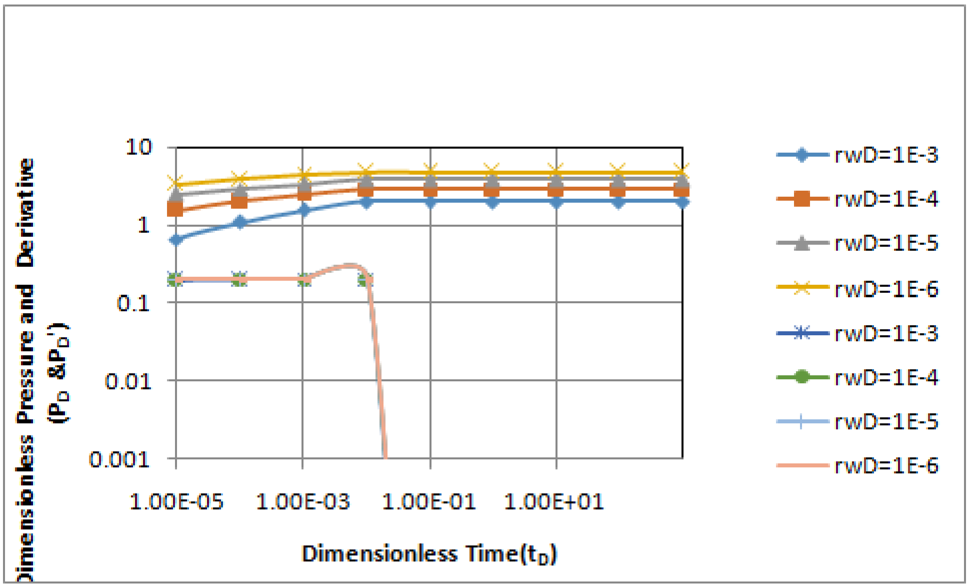

Figure3.*Influence of R_{WD}*

3.4. Influence of Y_D

Influence of changes in the well width on pressure and derivative response, was investigated usingy_D values in the range of 5E-6 $\leq y_D \leq$3E-3. As can be observed from Figure 4 changes in well width has no effect on pressure and derivative response for this cross flow layered system.

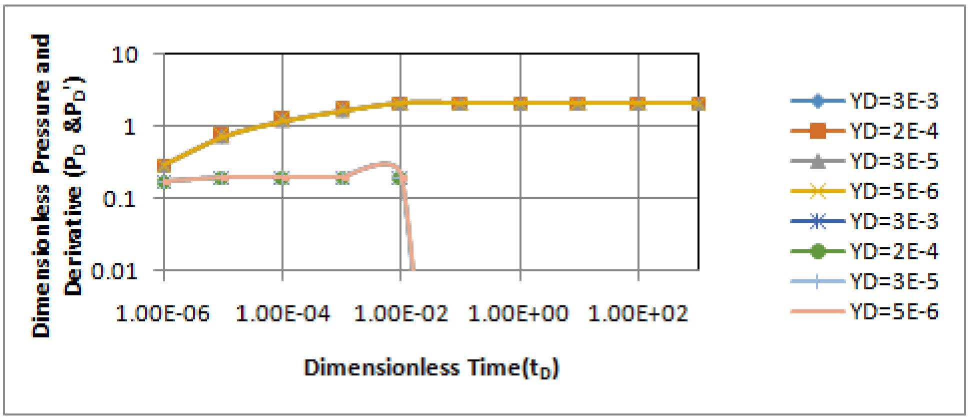

Figure4.*Influence of Y_D*

3.5. Influence of X_{WD}

Also varying flow point (x_{wD}) did not influence pressure and derivative as can be observed from Figure 5

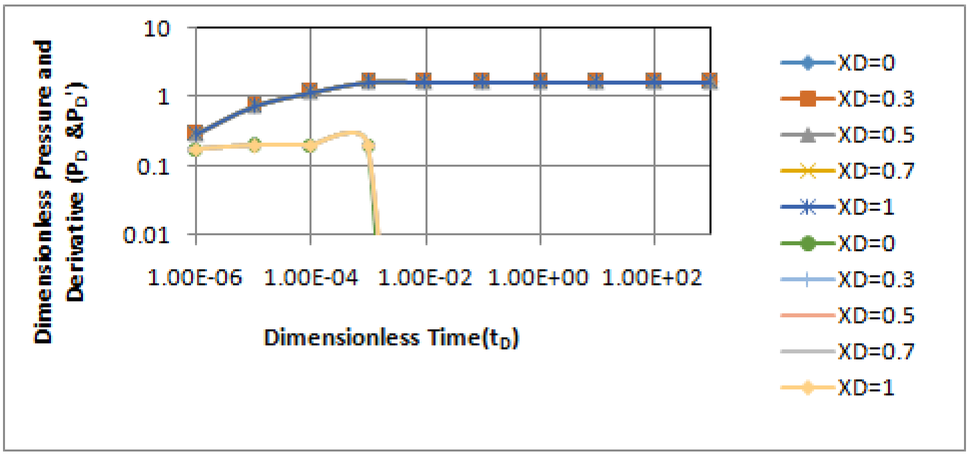

Figure5.*Influence of X_D*

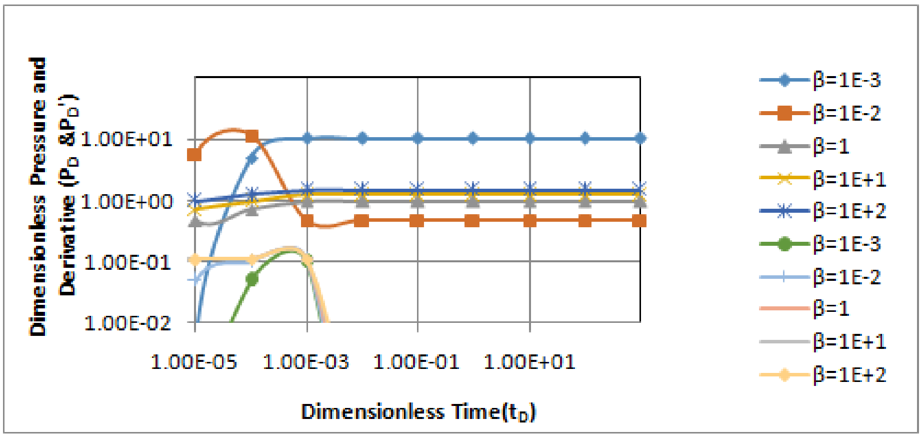

Figure6.*Influence of β*

3.6. Influence of β (Crossflow Coefficient)

Values of 1E-3 ≤ β ≤1E2 were used to investigate its influence on pressure and derivative response and from figure 6, it was observed that for values of β≤1 the early time pressure response was unstable but stable at late time. For β≥1 there is little difference in pressure and derivative values although the values were stable. Hence the stable and favourable values of β are those greater than one (permeability of layer 2 must be higher). The crossflow direction is always from area of low permeability to that of higher permeability. Hence in the example, when there is drawdown, formation fluids start moving from layer with low k (layer 1) to that with a higher k (layer 2) as a result there is increased pressure response in layer two. Well should be completed in the more permeable layer.

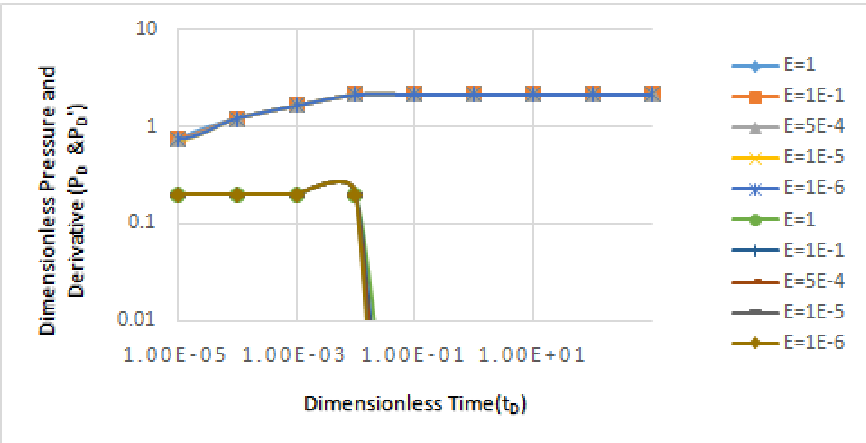

Figure7.*Influence of Modification Factor E.*

3.7. Influence of Modification Factor E

Using values of $1E\text{-}6 \leq E \leq 1$. No effect was noticed on pressure and derivative response as a result of changes in the modification factor, as seen in Figure 7. E usually takes effect when early time flow period has elapsed, hence this may be attributed to constant pressure boundary effect. A late time trend of bottom water drive system where pressure stabilizes and derivatives collapses to zero. This observation was previously made by Ozkan [19] in his study of bottom water drive with horizontal wells for single systems.

4. CONCLUSION

Information on the effects of well and reservoir parameters on pressure and derivative behaviour can be utilized to improve performance of crossflow layered reservoir system. In this study the model equations for a layered crossflow reservoir system were presented. The effect of some pertinent well and reservoir parameters on the behaviour of pressure and pressure derivative of horizontal wells placed in each layer of the system were also examined. The following conclusions were made.

P_D and P_D' response depicted the characteristic late time trend of a reservoir with constant pressure boundary. Using well one as the reference well, P_D and P_D' increased with increasing dimensionless pay thickness, while it decreased with increasing r_{WD}, more during the early time flow period. Early time response was unstable for values of β <1, derivative was observed to show negligible changes. P_D and P_D' were insensitive to changes in Y_D, X_D and E.

This study showed that pressure derivative is not a suitable tool to test for the sensitivity of these parameters. Both layers gave near similar results, but the study recommends that well should be completed in the more permeable layer of the system. Results from this study will find use in efficient well planning, completion and depletion strategy for layered system with bottom water drive.

5. NOMENCLATURE

C_t Total compressibility Psi^{-1}

h pay thickness Ft

h_{Dt} total dimensionless pay thickness

i distance in x,y or z direction

K permeability md

L well length ft

M mobility ratio

P pressure psi

P_D dimensionless pressure

P_D' dimensionless Pressure derivative

psi pound per square inch

r_W well radius ft

t time hrs

t_D dimensionless time

β crossfow coefficient

φ porosity

μ viscosity cp

6. SUBSCRIPT

bt breakthrough

D dimensionless

e external

t total

w wellbore

REFERENCES

[1] U. Chaudhry (2004) Oil Well Testing Handbook, Gulf Professional Publishing

[2] D. Bourdet(2002)Well Test Analysis: The Use of Advanced Interpretation Models. Elsevier.

[3] E.D Akpobi(2024)Parametric Study in a Layered Bottom Water Drive Reservoir with an Impermeable Interface. Journal of Scientific and Engineering Research, 2024, 11(3):pp121-131

[4] M.B.Vasquez and M.A. Pedro (2021) Rate transient Analysis in two layered reservoir without cross flow Paper presented at the SPE Trinidad and TobagoSection Energy Resource Conferencedoi:https://doi.org/10.2118/200906-MS

[5] N. Kodhelaj, Bozgo, S.(2016) Role of Well Testing and Information in Petroleum Industry :Testing in Multilayered Reservoir. Int. Journal of Eng Science and Computing July pp 1647-1652.

[6] N.M. Al- Ajmi, H. Kazemi and E. Ozkan (2008) Estimation of Storativity Ratio in a layered reservoir with Crossflow. SPE Reservoir Evaluation and Engineering 11(2)267 doi: https://doi.org/10.2118/84294-PJ.

[7] R. PrijambodoRagha, Van R., Reynolds A.C. (1985) Well test analysis for well producing layered reservoirs with crossflow. SPEJ. 25(03) 380-396 Paper number SPE-10262-PA

[8] Viana, I.V., Bela, R.V., Grave, M. (2024). An analytical model for injectivitytests in multilayered reservoir with formation crossflow. *Int J Geomath* **15**, 6 (2024).

[9] E.D Akpobi (2017) *Performance of Horizontal Wells in Layered Reservoir Subject to a Bottom Water Drive* [PhD Thesis, University of Benin Edo State, Nigeria.

[10] F.J.Kuchuck (1996) Pressure Behaviour of Horizontal Well in Multilayer Reservoir withCrossflow SPE Formation Evaluation 11.1. pp55-65

[11] L.Haishan, L. M. Zhitao, Tagaviferand H.Razal (2017) Modelling Polymer Flooding with in Layered Reservoir Considering Viscous Fingering. SPE paper presented at the SPE Canada Heavy Oil Technical Conference;doi: https://doi.org/10.2118/185017-MS

[12] S. Jin, H. Guoqing, X. Liang, M. He (2023) A method for Matching Unconventional Wells and reservoir based on Semi analytic Models. Energies 16(7) 3207

[13] W.Liang, J. Wang, C.Leung,S. Goh, P. Li. (2023)Impact of crossflow and two-phase flow on gas production from stacked deposits Energy and Fuels. 37(13) 8935-8948.

[14] K. Fiktri (1995) Well testing and interpretation for Horizontal wells. JPT 36-44

[15] J.O.Oloro,E.SAdewole,O.AOlafuyi(2014). Pressure distribution of horizontal wells in a layered reservoir with simultaneous gas cap and bottom water drive. American Journal of Engineering Research 3(12)pp41–53.

[16] Butter, M., Loyd, E., Kuvshinov, I. (2006) Potential of Multiple Fractured Horizontal Well in Layered Reservoir. SPE paper presented at Russian Oil and Gas Technical Conference and Exhibition Moscow

[17] E.D. Akpobi, E.S. Adewole and O.A Olafuyi. (2016) Model for Layered Reservoir Subject to a Bottom Water Drive Using Horizontal Wells. Journal of the Nigerian Association of Mathematical Physics (J.NAMP) 35. pp 311-322.

[18] E.D Akpobi and E. Ebojoh, Computer Aided Pressure Transient Pressure Analysis of a Layered Reservoir System with a Constant Pressure Boundary. Int. Journal of Computational and Experimental Science and Engineering (IJCESEN) 6, 35 DOI: 10.22399/ijcesen.619912

[19] Ozkan, E. and Raghavan, R. (1990) Performance of horizontal well subject to bottom water drive.SPERE 5(3):375-383. August DOI: 10.2118/18559- PA.

[20] A.C ,Gringarten, Ramey H J (1973). The use of Source and Green's functions in solving unsteady – flow problems in reservoirs. Soc Pet Eng J. 1973;13(5):285–296

[21] E.S Adewole. (2009)InstantaneousSourceFunctionof a Permeable Interface. Journal of the Nigerian association of mathematical Physics Vol.4,pp.189-196.

APPENDIX

This section gives a brief summary of the theory and mathematical derivation involved in the model formulation as presented in the research done byAkpobietal[9,17]. Dimensionless parameters used include dimensionless well and reservoir distances, time and pressure

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}} \quad (1a)$$

(wherei = x, y, z)

$$t_D = \frac{0.0010548Kt}{\varphi \mu C_t L^2} \text{ (2a)}$$

$$P_D = \frac{kh\Delta p}{141.2 q \mu B} \text{ (3a)}$$

The equation governing the flow of a slightly compressible fluid in a porous media is the diffusivity equation expressed in dimensionless form as

$$\frac{\partial^2 P_D}{\partial_{x^2 D}} + \frac{\partial^2 P_D}{\partial_{y^2 D}} + \frac{\partial^2 P_D}{\partial_{z^2 D}} = \frac{\partial P_D}{\partial t_D} \text{ (4a)}$$

Fig 1a. Schematic of the model for a two layered Crossflow reservoir with horizontal wells

The pressure drop caused by production from a continuous source [20] is expressed as

$$\Delta p(x, y, z, t) = \frac{1}{\varphi C_t} \int_0^t q_l . S(x, y, z) . \partial t \text{ (5a)}$$

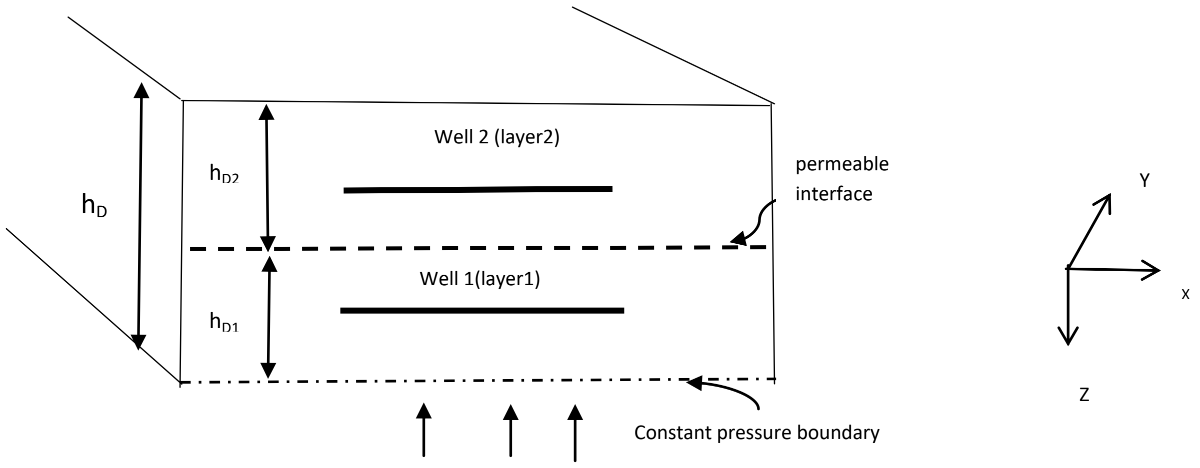

Figure1a.*Schematic of reservoir model*

Where $q_L$represents flow rate per unit length of the source, s (x, y, z, t) represents the instantaneous source function (ISF) for the particular reservoir and well configuration**[21]**

$$P_D(X_D, Y_D, Z_D, \tau) = 2\pi h_D E_j \int_0^{t_D} S(X_D, \tau).S(Y_D, \tau).S(Z_D, \tau)\partial\tau \text{ (6a)}$$

Dimensionless Pressure (P_{Dj}) for each layer is developed as

$$P_{Dj} = P_{D1} + P_{D2} + P_{D3} + \ldots\ldots P_{Dn} \text{ (7a)}$$

where j depicts the layer and n is the last flow period.

1. **Modification Factor** E that accounts for the contributions of both layers at the interface as a result of the dual nature of the z-axis

$$EJ = \frac{W_1 + W_2}{2\pi(W_1{}^2 + W_2{}^2)} \text{(8a)}$$

$$W_1 = h_{D1} \int_{X_{wD}}^{X_{eD}} \int_0^{y_{eD}} \int_0^{h_{D1}} \left[\int_0^{t_D} s_1(x_D, \tau), s_1(y_D, \tau), s_1(z_D, \tau) d\tau\right] d_{zD}\, d_{yD}\, d_{xD} \text{(9a)}$$

$$W_2 = h_{D2} \int_{X_{wD}}^{X_{eD}} \int_0^{y_{eD}} \int_{h_{D2}}^{h_D} \left[\int_0^{t_D} s_2(x_D, \tau), s_2(y_D, \tau), s_2(z_D, \tau) d\tau\right] d_{zD}\, d_{yD}\, d_{xD}. \text{(10a)}$$

2. **Crossflow Coefficient (β);** is a factor used to correct for differences in response time of the layers to the same transient regime for crossflow systems.

$$\beta = \frac{\phi_i \mu_i C_{ti} L_i^{\,2} K_{i+1}}{\phi_{i+1}\mu_{i+1}C_{ti+1}L_{i+1}^{\,2}K_i} \tag{11a}$$

3. **Total Permeability of the system**

$$K_t = \frac{K_1h_1 + K_2h_2 ....K_nh_n}{h_1h_2......h_n} \quad (12a)$$

4. **Fluid mobility ratio**

$$M = \frac{K_1h_1/\mu_1}{K_2h_2/\mu_2} \tag{13a}$$

5. **Total pay thickness of the system**

$$h_{Dt} = h_{D1} + h_{D2} \quad (14a)$$

Dimensionless pressure (P_{D1}) for layer one crossflow systems

$$
P_{D1} = \left[
\begin{aligned}
&\left[\frac{\alpha}{8L_D}\sqrt{\frac{K}{K_y}}Ei\left(-\frac{r_{WD}^{\,2}}{4\tau_D}\right)\right] + \\
&\left[\sqrt{\pi}E_1\int_{t_{De1}}^{t_{De2}}\left\{
\begin{aligned}
&erf\left(\frac{\sqrt{\frac{K}{K_X}}+X_{D1}}{2\sqrt{\tau_D}}\right)+erf\left(\frac{\sqrt{\frac{K}{K_X}}-X_{D1}}{2\sqrt{\tau_D}}\right). \\
&\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}^{\,2}}}\cos\frac{n\pi y_{wD}}{Y_{eD}}\cos\frac{n\pi Y_W}{Y_{eD}}\right). \\
&\frac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(Z_{D1}-Z_{WD1})^2}{4\tau_D}}
\end{aligned}
\right\}\partial\tau\right] + \\
&\left[\sqrt{\pi}E_1\int_{t_{De1}}^{t_{De2}}\left\{
\begin{aligned}
&erf\left(\frac{\sqrt{\frac{K}{K_X}}+X_{D1}}{2\sqrt{\tau_D}}\right)+erf\left(\frac{\sqrt{\frac{K}{K_X}}-X_{D1}}{2\sqrt{\tau_D}}\right).\frac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(Y_{D1}-Y_{WD1})^2}{4\tau_D}}. \\
&\left(\sum_{n=1}^{\infty}\exp^{\left[--\frac{n^2\Pi^2\tau_D}{h_{D1}^{\,2}}\right]}\sin\frac{n\pi Z_{wD1}}{h_{D1}}\sin\frac{n\pi z_{D1}}{h_{D1}}\right)
\end{aligned}
\right\}\partial\tau\right] + \\
&\left[\frac{4\pi E_1}{X_{eD}Y_{eD}}\int_{t_{De3}}^{t_D}\left\{
\begin{aligned}
&\left(1+\frac{4X_{eD}}{\pi}\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\pi^2\tau_D}{X_{eD}^{\,2}}\right]}\sin\frac{m\pi x_f}{2X_{eD}}\cos\frac{m\pi x_{wD1}}{X_{eD}}\cos\frac{m\pi x_{D1}}{X_{eD}}\right). \\
&\left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}^{\,2}}}\cos\frac{n\pi y_{wD}}{Y_{eD}}\cos\frac{n\pi Y_W}{Y_{eD}}\right). \\
&\left(\sum_{n=1}^{\infty}\exp^{\left[--\frac{n^2\pi^2\tau_D}{h_{D1}^{\,2}}\right]}\sin\frac{n\pi Z_{wD1}}{h_{D1}}\sin\frac{n\pi z_{D1}}{h_{D1}}\right)
\end{aligned}
\right\}\partial\tau\right]
\end{aligned}
\right] \tag{15a}
$$

Dimensionless pressure derivative for layer one (crossflow)

$$
P_{D1}{}' = \left[\begin{array}{l}
\left[\left[\frac{\alpha}{8L_D} \sqrt{\frac{K}{K_y}} \frac{K}{K_Z} \frac{1}{\tau_D} \exp\left(-\frac{r_{WD}{}^2}{4\tau_D} \right) \right] + \right. \\
\left[\sqrt{\pi} E_1 \left\{ \begin{array}{l} erf\left(\frac{\sqrt{\frac{K}{K_X}} + X_{D1}}{2\sqrt{\tau_D}} \right) + erf\left(\frac{\sqrt{\frac{K}{K_X}} - X_{D1}}{2\sqrt{\tau_D}} \right) . \left(1 + 2\sum_{n=1}^{\infty} \exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}{}^2}} \cos\frac{n\pi y_{wD}}{Y_{eD}} \cos\frac{n\pi Y_W}{Y_{eD}} \right) . \\ \frac{1}{2\sqrt{\tau_D}} \exp^{-\frac{(Z_{D1}-Z_{WD1})^2}{4\tau_D}} \end{array} \right\} \right] \Bigg] + \\
\sqrt{\pi} E_1 \left\{ \begin{array}{l} erf\left(\frac{\sqrt{\frac{K}{K_X}} + X_{D1}}{2\sqrt{\tau_D}} \right) + erf\left(\frac{\sqrt{\frac{K}{K_X}} - X_{D1}}{2\sqrt{\tau_D}} \right) . \frac{1}{2\sqrt{\tau_D}} \exp^{-\frac{(Y_{D1}-Y_{WD1})^2}{4\tau_D}} . \\ \left(\sum_{n=1}^{\infty} \exp^{\left[-\frac{n^2\pi^2\tau_D}{h_{D1}{}^2} \right]} \sin\frac{n\pi Z_{wD1}}{h_{D1}} \sin\frac{n\pi z_{D1}}{h_{D1}} \right) \end{array} \right\} + \\
\left[\frac{4\pi E_1}{X_{eD} Y_{eD}} \left\{ \begin{array}{l} \left(1 + \frac{4X_{eD}}{\pi} \sum_{m=1}^{\infty} \exp^{\left[-\frac{m^2\pi^2\tau_D}{X_{eD}{}^2} \right]} \sin\frac{m\pi x_f}{2X_{eD}} \cos\frac{m\pi x_{wD1}}{X_{eD}} \cos\frac{m\pi x_{D1}}{X_{eD}} \right) . \\ \left(1 + 2\sum_{n=1}^{\infty} \exp^{-\frac{n^2\pi\Pi^2\tau_D}{Y_{eD}{}^2}} \cos\frac{n\pi y_{wD}}{Y_{eD}} \cos\frac{n\pi Y_W}{Y_{eD}} \right) . \\ \left(\sum_{n=1}^{\infty} \exp^{\left[-\frac{n^2\pi^2\tau_D}{h_{D1}{}^2} \right]} \sin\frac{n\pi Z_{wD1}}{h_{D1}} \sin\frac{n\pi z_{D1}}{h_{D1}} \right) \end{array} \right\} \right]
\end{array} \right] \qquad (16a)
$$

Dimensionless pressure (P_{D2}) expression for layer two crossflow system

$$P_{D2} = \left[\begin{array}{l} \left[\dfrac{\alpha}{8L_D}\sqrt{\dfrac{K}{K_y}\dfrac{K}{K_Z}}Ei\left(-\dfrac{r_{WD}{}^2}{4\tau_D}\right)\right] + \\ \left[\sqrt{\pi}\beta E_2 \displaystyle\int_{t_{De1}}^{t_{De2}} \left\{ \begin{array}{l} erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}}+X_{D2}}{2\sqrt{\beta\tau_D}}\right)+erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}}-X_{D2}}{2\sqrt{\beta\tau_D}}\right). \\ \left(1+2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\beta\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\pi y_{wD2}}{Y_{eD}}\cos\dfrac{n\pi Y_{D2}}{Y_{eD}}\right) \\ \dfrac{1}{2\sqrt{\beta\tau_D}}\exp^{-\frac{(Z_{D2}-Z_{WD2})^2}{4\beta\tau_D}}. \end{array} \right\} \partial\tau \right] + \\ \left[\sqrt{\pi}\beta E_2 \displaystyle\int_{t_{De2}}^{t_{De3}} \left\{ \begin{array}{l} erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}}+X_{D2}}{2\sqrt{\beta\tau_D}}\right)+erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}}-X_{D2}}{2\sqrt{\beta\tau_D}}\right).\dfrac{1}{2\sqrt{\beta\tau_D}}\exp^{-\frac{(Y_{D2}-Y_{WD2})^2}{4\beta\tau_D}}. \\ \left(\displaystyle\sum_{n=1}^{\infty}\exp^{\left[-\frac{(2n-1)^2\pi^2\beta\tau_D}{4h_{D2}{}^2}\right]}\sin\dfrac{(2n-1)\pi z_{wD2}}{2h_{D2}}\sin\dfrac{(2n-1)\pi z_{D2}}{2h_{D2}}\right) \end{array} \right\} \partial\tau \right] + \\ \left[\dfrac{4\pi\beta E_2}{X_{eD}Y_{eD}} \displaystyle\int_{t_{De3}}^{t_D} \left\{ \begin{array}{l} \left(1+\dfrac{4X_{eD}}{\pi}\displaystyle\sum_{m=1}^{\infty}\exp^{\left[-\frac{m^2\pi^2\beta\tau_D}{X_{eD}{}^2}\right]}\sin\dfrac{m\pi x_f}{2X_{eD}}\cos\dfrac{m\pi x_{wD2}}{X_{eD}}\cos\dfrac{m\pi x_{D2}}{X_{eD}}\right). \\ \left(1+2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\beta\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\pi y_{wD2}}{Y_{eD}}\cos\dfrac{n\pi Y_{D2}}{Y_{eD}}\right). \\ \left(\displaystyle\sum_{n=1}^{\infty}\exp^{\left[-\frac{(2n-1)^2\pi^2\beta\tau_D}{4h_{D2}{}^2}\right]}\sin\dfrac{(2n-1)\pi z_{wD2}}{2h_{D2}}\sin\dfrac{(2n-1)\pi z_{D2}}{2h_{D2}}\right) \end{array} \right\} \partial\tau \right] \end{array} \right] \quad (17a)$$

Dimensionless pressure derivative for layer two (crossflow systems) is given by the expression:

$$
\begin{aligned}
P_{D2}{}' = \Bigg[& \left[\frac{\alpha}{8L_D} \sqrt{\frac{K}{K_y}\frac{K}{K_z}} \frac{1}{\tau_D} \exp\left(-\frac{r_{WD}{}^2}{4\tau_D} \right) \right] + \\
& \left[\sqrt{\pi}\beta E_2 \left\{ \begin{array}{l} erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} + X_{D2}}{2\sqrt{\beta\tau_D}} \right) + erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} - X_{D2}}{2\sqrt{\beta\tau_D}} \right) . \\ \left(1 + 2\displaystyle\sum_{n=1}^{\infty} \exp^{-\frac{n^2\pi^2\beta\tau_D}{Y_{eD}{}^2}} \cos\dfrac{n\pi y_{wD2}}{Y_{eD}} \cos\dfrac{n\pi Y_{D2}}{Y_{eD}} \right) . \\ \dfrac{1}{2\sqrt{\beta\tau_D}} \exp^{-\frac{(Z_{D2}-Z_{WD2})^2}{4\beta\tau_D}} \end{array} \right\} \right] + \\
& \left[\sqrt{\pi}\beta E_2 \left\{ \begin{array}{l} erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} + X_{D2}}{2\sqrt{\beta\tau_D}} \right) + erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} - X_{D2}}{2\sqrt{\beta\tau_D}} \right) . \dfrac{1}{2\sqrt{\beta\tau_D}} \exp^{-\frac{(Y_{D2}-Y_{WD2})^2}{4\beta\tau_D}} . \\ \left(\displaystyle\sum_{n=1}^{\infty} \exp^{\left[-\frac{(2n-1)^2\pi^2\beta\tau_D}{4h_{D2}{}^2} \right]} \sin\dfrac{(2n-1)\pi z_{wD2}}{2h_{D2}} \sin\dfrac{(2n-1)\pi z_{D2}}{2h_{D2}} \right) \end{array} \right\} \right] + \\
& \left[\frac{4\pi\beta E_2}{X_{eD}Y_{eD}} \left\{ \begin{array}{l} \left(1 + \dfrac{4X_{eD}}{\pi} \displaystyle\sum_{m=1}^{\infty} \exp^{\left[-\frac{m^2\pi^2\beta\tau_D}{X_{eD}{}^2} \right]} \sin\dfrac{m\pi x_f}{2X_{eD}} \cos\dfrac{m\pi x_{wD2}}{X_{eD}} \cos\dfrac{m\pi x_{D2}}{X_{eD}} \right) . \\ \left(1 + 2\displaystyle\sum_{n=1}^{\infty} \exp^{-\frac{n^2\pi^2\beta\tau_D}{Y_{eD}{}^2}} \cos\dfrac{n\pi y_{wD2}}{Y_{eD}} \cos\dfrac{n\pi Y_{D2}}{Y_{eD}} \right) . \\ \left(\displaystyle\sum_{n=1}^{\infty} \exp^{\left[-\frac{(2n-1)^2\pi^2\beta\tau_D}{4h_{D2}{}^2} \right]} \sin\dfrac{(2n-1)\pi z_{wD2}}{2h_{D2}} \sin\dfrac{(2n-1)\pi z_{D2}}{2h_{D2}} \right) \end{array} \right\} \right] \Bigg]
\end{aligned}
\qquad (18a)
$$

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