

Application of Numerical Methods for the Analysis of Incremental Oil

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Abstract: Incremental oil values of Enhanced Oil Recovery projects have been obtained in the past using the Decline Curve Analysis and the Trapezoidal rule, with the former posing a challenge of inaccuracy in shifting while the latter approximates the area under a curve with a straight line segment. This has substantially undermine the accuracy and significance of the incremental oil values obtained by these methods.

In this work, Cubic spline was used to numerically model rate-time curves and Thomas algorithm was used to solve the resulting tri-diagonal system of equations. Also, Simpson quadrature was applied to the respective rate-time data and the concept of finite difference was introduced to account for the error term. Experimental data was used for the analysis.

Result showed a more accurate predictions of Incremental oil by the Cubic spline and Simpson quadrature as compared to Trapezoidal rule and Decline curve. Since, incremental oil values rightly measures the success of any EOR project by estimating accurate volumes of oil produced via such displacement mechanism, it imply that adopting the Natural cubic spline and Simpson quadrature will aid better estimations of the economic values of projects and effective management decision making.

Keywords: EOR, Cubic spline, Simpson quadrature, Trapezoidal rule and Decline curve

1. INTRODUCTION

A universal technical measure of the success of an Enhanced oil recovery (EOR) project is the amount of the incremental oil recovered. **Figure.1** below defines incremental oil. Imagine a field, reservoir, or a well whose oil rate is declining from **A** to **B**. At **B**, an EOR project was initiated, and if successful, the rate should show a deviation from the projected decline after **B**. Incremental oil been the difference between what was actually recovered, **B** to **D** and what would have been recovered had the process not been initiated, **B** to **C**. is estimated by the area under rate-time curve; the incremental oil recovered is therefore the shaded region.

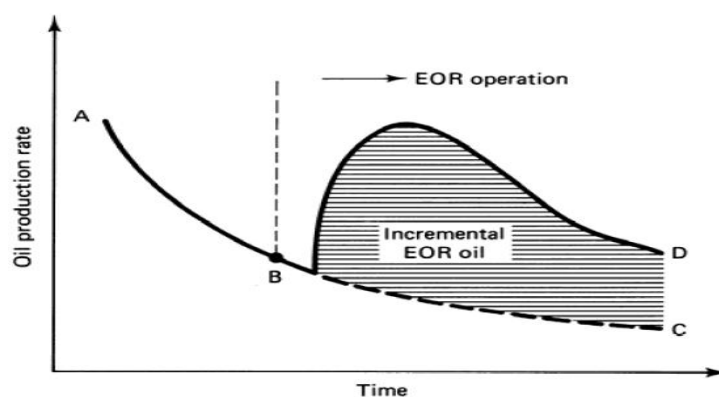


Figure1. Incremental Oil Recovery from Typical EOR Response curve^[1]

Consequently, the additional amount of hydrocarbon in place produced through the successful application of an EOR technique, in the excess of the existing (conventional) process - primary or secondary recovery processes of operation - is referred to, technically as the **Incremental oil**.

Estimating the incremental oil from an EOR process involves integration to evaluate the area under the rate-time curve. Ways of achieving this over the years in the industry ranges from the use of models to graphical procedures. One of the model is the Decline curve method and one of the graphical procedures is the Trapezoidal rule. The Decline curve is an effective tool used in predicting well performance and reservoir life based on real life data. But, its adoption in estimating incremental oil have shown little success in the past, as its results have shown to be highly erroneous. This is attributed to the challenge of inaccuracy in shifting^[2] while, the Trapezoidal rule approximates of the integral under a curve with that under a straight line segment^[3]. These challenges have substantially impair the accuracy of incremental oil predictions obtained from EOR projects.

To minimize these errors, this paper propose to adapt other numerical methods, notably the natural cubic spline and Simpson quadrature in modeling EOR curves and predicting the incremental oil recovered from such displacement mechanism. Furthermore, it seeks to comparatively analyze the performance these numerical techniques with pre-existing methodologies in order to determine their level of accuracy and emerge with a more robust approach to incremental Oil prediction.

Some experimental works where data can be obtained for use includes the works of Onuoha and Olafuyi, 2013, Orivri et al, 2014, Atsenuwa et al, 2014, Yebu et al, Uwari et al, Kharma et al, Taiwo and Olafuyi in 2015^[4-9]. And some other works of Taiwo et al in 2016, Mamudu et al 2016 , Onuoha et al 2017, Ojukwu et al 2018, Madugu et al 2018, and Taiwo et al 2019^[10-15] can were also of great contributions to this work in estimating the incremental oil from EOR.

2. METHODS

2.1. Trapezoidal Rule

The trapezoidal is the first of Newton-Cotes integration formulae. Trapezoidal Rule is an already established numerical integration formula which involves the use of the integral under a straight line to approximate the integral under a curve. The respective mathematical model for the rule is clearly outlined below.

$$I = \int_a^b f(x) dx \tag{1}$$

For a single strip (two ordinates)

$$I = \frac{h}{2} [f(x_0) + f(x_1)] - \frac{h^3}{12} f''(x) \tag{2}$$

For multiple strips ($n > 2$ ordinates)

$$I = \frac{h}{2} [f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) + \dots + f(x_{n-1})) + f(x_n)] - \frac{h^3}{12} f''(x) \tag{3}$$

$$Error = -\frac{h^3}{12} n f''(x) \tag{4}$$

Where I = Area under the graph

2.2. The Simpson's Rule

One drawback of the trapezoidal rule is that its error is related to the second derivation. Therefore more complicated approximations formulas can improve the accuracy for curves, these includes using 2nd and 3rd order polynomial. There are two versions of this rule as exists.

2.3. The Simpson's 1/3th Rule

In this rule, the function is approximated by a second-order degree polynomial between successive points. It corresponds to using second-order Lagrange polynomials in modelling data points.

Mathematically,

$$I = \frac{h}{3} [f(x_0) + 4[f(x_1) + f(x_3) \dots + f(x_{2n-1})] + 2[f(x_2) + f(x_4) \dots + f(x_{2n-2})] + f(x_n)] \tag{5}$$

The error associated with this rule is given by the model;

$$Error = -\frac{nh^5}{90} f^4(x) \tag{6}$$

2.4. The Simpson’s Three-Eight Rules

This rule corresponds to using third-order polynomial to fit four points. Integrating over the four points gives; the model is given as shown below.

$$I = \frac{3h}{8} [f(x_0) + 3[f(x_1) + \dots + f(x_{3n-1})] + 2(f(x_3) \dots + f(x_{3n-3})) + f(x_{3n})] - \frac{nh^5}{80} f^4(x) \quad 7$$

The error associated with this rule is given by the model;

$$Error = -\frac{nh}{80} f^4(x) \quad 8$$

2.5. Numerical Error

Past works on these quadratures, have shown that they have tendencies to under or over predict sometimes, depending on the data architecture. Therefore, in view of this, it becomes pertinent that the differentials for the respective quadrature be accounted for in order to mitigate any of these cases occurring and consequently ensuring a more accurate incremental oil result from displacement mechanisms.

Since laboratory rate –time values mimicking that obtainable from the field were considered, this paper utilizes the numerical differentiation technique, known as the Finite difference to account for the error as shown below

$$\frac{dy}{dx} = \frac{\Delta f_i}{\Delta x} \quad 9$$

This can be extended to the second and fourth differentials in order to fully account for the error in the Trapezoidal and Simpson’s rules respectively.

$$\frac{d^2y}{dx^2} = \frac{f_{i+2}-2f_{i+1}+f_i}{\Delta x^2} \quad 10$$

$$\frac{d^4y}{dx^4} = \frac{\Delta^4 f_i}{\Delta x^4} = \frac{f_{i+5}-4f_{i+3}+6f_{i+2}-4f_{i+1}+f_i}{\Delta x^4} \quad 11$$

Therefore the Newton forward difference table would be drawn for each data.

2.6. Cubic Spline Interpolation

Real world numerical data is usually difficult to analyze. Any function which would effectively correlate the data would be difficult to obtain and highly unwieldy. To this end, the idea of the cubic spline was developed. Using this process, a series of unique cubic polynomials are fitted between each of the data points, with the stipulation that the curve obtained be continuous and appear smooth. The mathematical spline is similar in principle. The points, in this case, are numerical data. The weights are the coefficients on the cubic polynomials used to interpolate the data. These coefficients ‘bend’ the line so that it passes through each of the data points without any erratic behavior or breaks in continuity^[16]

2.7. Cubic Spline Model Formulation

Figure 2 below depicts a cubic spline that spans n knots. Denoting $f_{i,i+1}(x)$ as the cubic polynomial that spans the interval between knots i and $i + 1$, we note that the spline is a piecewise cubic curve, assembled together to form the $n - 1$ cubics, $f_{1,2}(x), f_{2,3}(x), \dots, f_{n-1,n}(x)$, all of which have different coefficients.

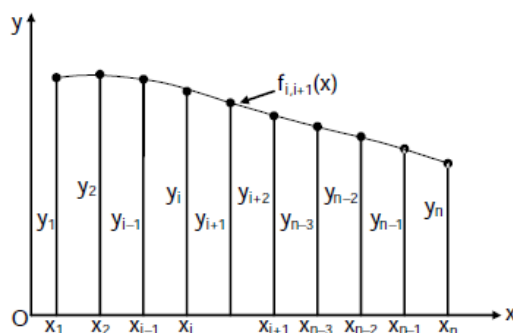


Figure2. Notation used to derive quadratic splines.

Notice that there are n interval and $n + 1$ data points.

Using Lagrange's two-point interpolation, the second derivative which is a linear function can be expressed as:

$$f''(x) = \left(\frac{x_{i+1}-x}{x_{i+1}-x_i}\right) f''(x_i) + \left(\frac{x-x_i}{x_{i+1}-x_i}\right) f''(x_{i+1}) \quad \text{for } x_i \leq x \leq x_{i+1} \quad 12$$

Denoting the second derivative of the knot at i by k_i and with the condition of its continuity (from one spline to the successive), it's required that;

$$f''_{i-1,1}(x_i) = f''_{i+1,1}(x_i) = k_i \quad 13$$

Integrating twice with respect to x and with the condition that $f(x)$ is required to pass through (x_i, y_i) and (x_{i+1}, y_{i+1}) gives:

$$\begin{aligned} & f_{i, i+1}(x) \\ &= \frac{k_i}{6} \left[\frac{(x-x_{i+1})^3}{x_i-x_{i+1}} - (x-x_{i+1})(x_i-x_{i+1}) \right] - \frac{k_{i+1}}{6} \left[\frac{(x-x_i)^3}{x_i-x_{i+1}} - (x-x_i)(x_i-x_{i+1}) \right] \\ &+ \frac{y_i(x-x_{i+1}) - y_{i+1}(x-x_i)}{x_i-x_{i+1}} \end{aligned} \quad 14$$

The above model derives the cubic relationship between two successive data in a given data set. Therefore, this can be adapted to rate-time data to give the interpolant between two rates in successive time interval by the model;

$$\begin{aligned} f_{i,i+1}(t) &= \frac{k_i}{6} \left[\frac{(t-t_{i+1})^3}{t_i-t_{i+1}} - (t-t_{i+1})(t_i-t_{i+1}) \right] - \frac{k_{i+1}}{6} \left[\frac{(t-t_i)^3}{t_i-t_{i+1}} - (t-t_i)(t_i-t_{i+1}) \right] \\ &+ \frac{q_i(t-t_{i+1}) - q_{i+1}(t-t_i)}{t_i-t_{i+1}} \end{aligned} \quad 15$$

Consequent upon this, there must be some formulations to determine the second derivatives (k_i) at the interior knots. This leads to yet unused condition of continuity of the first derivative with which the equations of curvatures are formulated.

2.8. Equations of Curvatures

Here, we note that the second derivatives (k_i) of the spline at the interior knots are found from the slop continuity conditions:

$$f'_{i,i-1}(x_i) = f'_{i,i+1}(x_i) \quad i = 1, 2, 3, \dots, n-1 \quad 16$$

Applying the conditions given by continuity between successive splines results in the following simultaneous equations:

$$k_{i-1}(x_{i-1} - x_i) + 2k_i(x_{i-1} - x_{i+1}) + k_{i+1}(x_i - x_{i+1}) = 6 \left[\frac{y_{i-1} - y_i}{x_{i-1} - x_i} - \frac{y_i - y_{i+1}}{x_i - x_{i+1}} \right] \quad i = 2, 3, \dots, n-1 \quad 17$$

But if the data are evenly spaced at interval (h), then we can write:

$$h = x_{i-1} - x_i = x_i - x_{i+1} \quad 18$$

Then, Eq. (3.9) becomes:

$$k_{i-1} + 4k_i + k_{i+1} = \frac{6}{h^2} [y_{i-1} - 2y_i + y_{i+1}] \quad i = 2, 3, \dots, n-1 \quad 19$$

Eq. (19) is a set of $n - 2$ linear simultaneous equations for the n derivatives (k_i), and when these are known, the spline function approximation formed by the set of functions in Eq.(15) defined over the consecutive intervals $x_i \leq x \leq x_{i+1}$ can be constructed. It is crucial to the practical use of splines that this system of equations being nonsingular, and that an extremely efficient algorithm be available to solve it. In this research, Thomas Algorithm was the solution method for the Triadiagonal system of equation resulting from Eqn 19 above. ^[17]

As the values of k_1 and k_n cannot be found from the condition that $f'(x)$ is continuous across the nodes x_1 and x_n , these values must be specified as additional conditions. The choice of values for k_1 and k_n prescribed as end conditions must be made intuitively, based on the way the data points

indicate the interpolated curve is most likely to behave (be extrapolated) beyond the end points of the interval $x_1 \leq x \leq x_n$. We used linear boundary condition in this research ($k_1 = k_n = 0$).^[18]

2.9. End or Boundary Conditions

Natural or linear spline end conditions.

This choice of end condition involves setting

$$k_1 = k_n = 0. \tag{20}$$

APPLICATION

Therefore ,Incremental oil is defined mathematically as given below;

$$\text{Incremental oil} = \int_a^b (\text{Composite function})dt - \int_a^b f(t)dt \tag{21}$$

Where the Composite function= Cubic spline/Simpson rule/Trapezoidal rule.

$$\int_a^b f(t)dt = \text{Equation of projected decline}$$

The equation for projected decline was determine by obtaining trendline for each cases study using Excel software.The resulting estimation from the composite functions were subtracted from that of the trendline in order to ascertain the incremental oil.Furthermore,in predicting the Incremental oil using the Decline curve,the economic limit for each experimental process was assumed to be the last rate prior to the EOR process execution.

The rate – time curves considered are shown below:

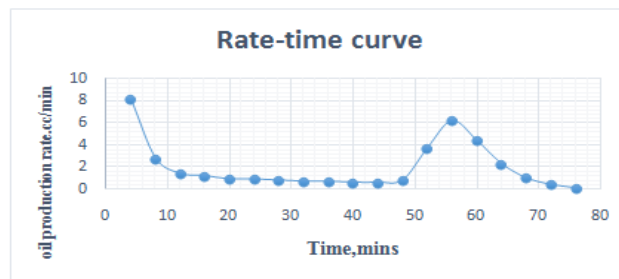


Figure3. Shows the rate-time profile for the polymer flooding process

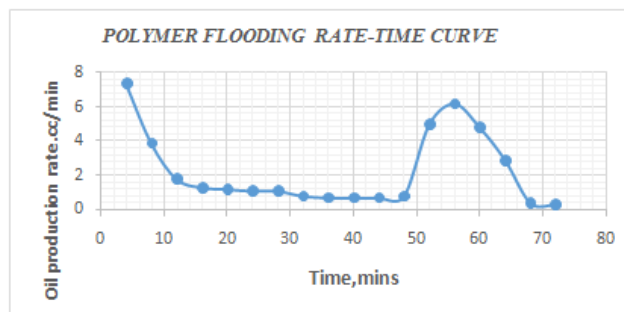


Figure4. Shows the rate-time profile for the polymer flooding process

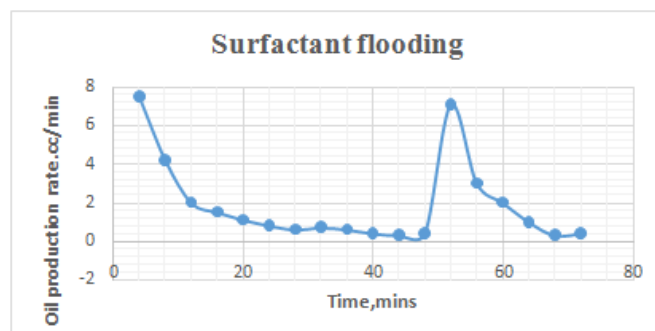


Figure5. Shows the rate-time profile for the surfactant flooding process

3. RESULT AND ANALYSIS

This paper seeks to obtain accurate incremental oil prediction from successful EOR processes. Shown below is the outcome of our applied models (Natural cubic spline and Simpson rule) and their comparative performance with exiting methodologies (Decline curve and Trapezoidal)

Case One: Polymer Flooding

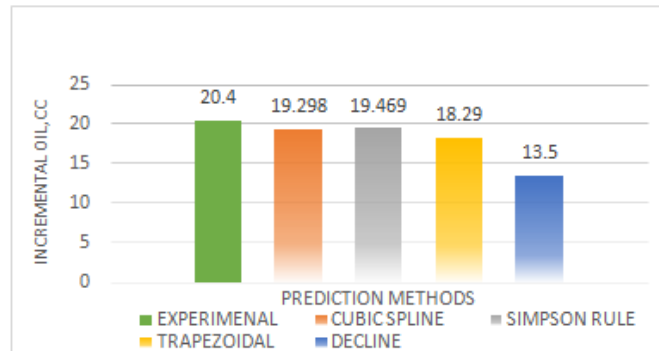


Figure6. Incremental predictions for the polymer flooding

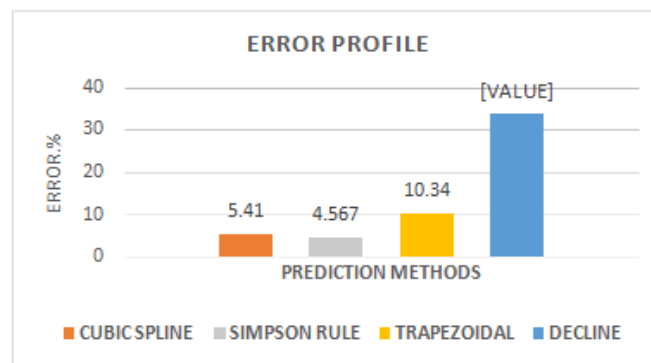


Figure6b. Error profile for polymer flooding case study

Case Two: Polymer Flooding

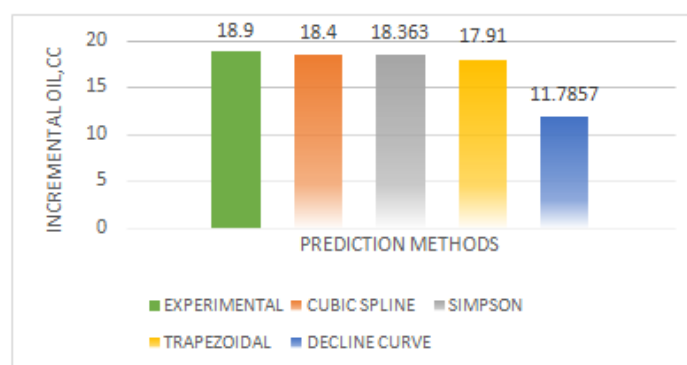


Figure7. Incremental oil prediction for the polymer flooding case study

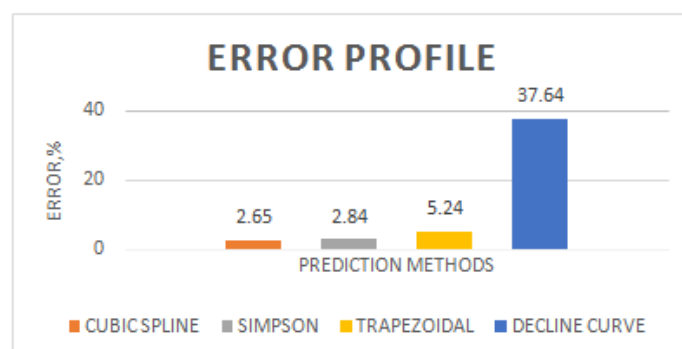


Figure7b. Error profile for polymer flooding case study

Case Three: Alkaline Surfactant Polymer Flooding

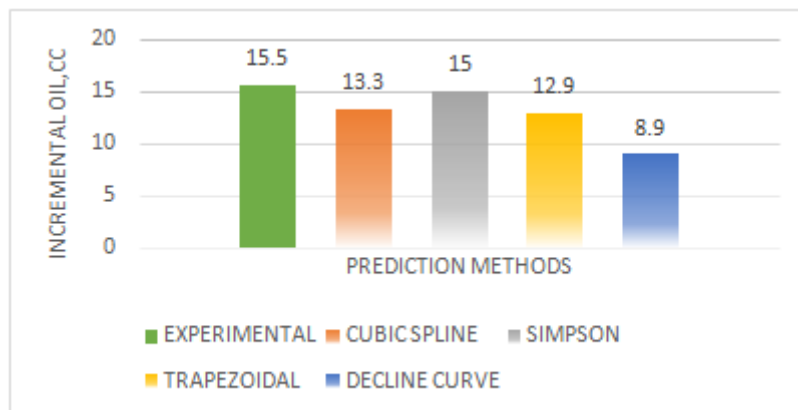


Figure8. Incremental oil prediction for the alkaline surfactant polymer flooding case study

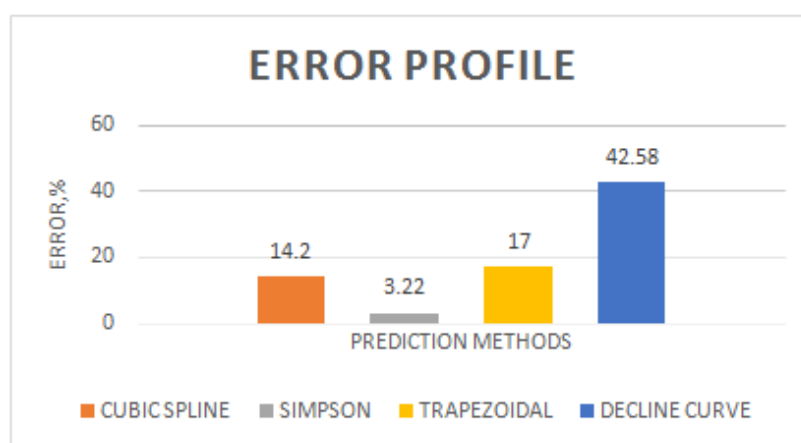


Figure8b. Error profile for alkaline surfactant polymer flooding case study.

The figures 6(a and b), 7(a and b) and 8(a and b) clearly shows the superiority of the Natural cubic spline and Simpson quadrature in predicting incremental oil from successful EOR processes compared to the existing methodologies. This is shown clearly in the “error profile” plots where the Natural cubic spline and Simpson rule out-performed the Decline curve and the Trapezoidal rule.

One of the reasons for this better performance of the Numerical methods can be attributed to the adoption of higher degree polynomials in modelling data points. In Natural cubic spline, a cubic polynomial is used to numerically model between two knot points while the Simpson’s has been shown to be exactly accurate for some cubic polynomials. This better model data points compare with Trapezoidal rule and totally different from the concept Decline curve adopts

It is also worthy of note that the introduction of Finite difference, in this research work has effectively helped in accounting for the error term in both the Simpson’s rule and trapezoidal rule and has improved the result of both when compare to earlier work especially for the Trapezoidal rule.

4. CONCLUSION

From the analysis of the results, the following conclusion can be drawn;

1. The amount of incremental oil predicted by the Natural cubic spline and Simpson’s rule were more accurate than that of the Trapezoidal rule and Decline curve.
2. The error due to trapezoidal can be attributed to its assumption of linear relationship between data in approximation given data points, which is hardly obtainable in real field data
3. The introduction of Finite difference effectively helps account for the error terms in both trapezoidal rule and Simpson’s rule respectively and consequently enhanced their accuracies.
4. The Cubic spline and Simpson’s rule both represent a viable Numerical tool and gives a reliable prediction of incremental oil regardless of the number of data points considered

5. The difference between the integral under the incremental oil recovery curve and the integral under the corresponding curve of the projected decline represents the incremental oil.
6. Using higher degree polynomial, as adopted by the Natural cubic spline and Simpson rule to approximate data points effectively and therefore when integrated gives a better incremental oil.

NOMECLATURE

EOR	Enhanced oil recovery
x	Independent variable
x_i	Node
$f_{i,i+1}$ and $f_{i,i-1}$	Interpolant or cubic
$f'_{i,i+1}$	First derivatives
$f''_{i,i+1}$	Second derivatives
Δf_i	First forward difference
N_p	Cumulative production(cc)
q	Recovery rate(cc/min)
y_i Knot	
k_i	Coefficient of the second derivatives
i, I, J	Position vectors
M	Position vector
h	Height (mins)
$\frac{d^2y}{dx^2}$	Second order differentials
$\frac{d^4y}{dx^4}$	Fourth order differentials

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