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Analytical Investigation of Transient Heat Transfer in Geothermal Wellbore System

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Abstract: *In borehole heat exchangers, a set of coupled partial differential equations describe heat transfer in the three components of the borehole such as pipe-in, pipe-out and grout. This paper presents an analytical model for understanding coupled conductive-convective heat transfer processes in a borehole heat exchanger subjected to defined initial and boundary conditions. This analytical model introduces a simulation of transient heat transfer in a single U-tube geothermal borehole heat exchanger. The main focus of this research is on the solution technique that can be useful for many other applications, including fluid flow in narrow pipes, high fluid velocities, high fluid viscosities, and pipes made of composite materials. Also, the method can be useful for solving other no homogeneous coupled partial differential equations. Results show that suggested technique is accurate and effective for solving this type of equations.*

Keywords: *Borehole heat exchanger, Heat conduction, Heat Convection, Wellbore*

1. Introduction

Geothermal energy is a form of thermal energy that generates in the core of the earth, about 6000 km below the surface. Basically, geothermal heat pump systems consist of two parts named the ground heat exchanger and the heat pump unit. The ground heat exchanger is a system of pipes, known as a loop which is buried in the ground either vertically or horizontally. In winter, heat from the earth is extracted via a fluid, usually water or a mixture of water and antifreeze, circulating through the pipes at a certain rate and collecting heat from the earth. The heat pump extracts heat from the fluid and pumps it into the building. In summer, that process is reversed, and the heat pump extracts heat from the indoor air and transfers it to the heat exchanger. Heat removed from the indoor air during summer can also be used for heating water, which can be used for cooking and bathing. The vertical system is widely used, especially in areas where the land is scarce. The heat in such a system is extracted by borehole heat exchangers (BHE's), also known as vertical ground heat exchangers or down hole heat exchangers, which consist of plastic pipes, mainly polyethylene or polypropylene, installed in a borehole as U-tubes and fixed by filling the borehole with grout. The U-tube carries a circulating fluid as a working fluid. The U-tube effectively forms two pipes. One pipe receives the circulating fluid from the heat pump and conveys it downward that is called pipe-in and the other pipe collects the circulating fluid at the bottom of pipe-in and brings it out to the surface, to enter the heat pump that is called pipe-out. The heat pump, usually located inside the building, extracts designed amount of heat from the fluid and pumps it back to the BHE. The circulating fluid, usually water with 20%–25% anti-freezing coolant such as Mono Ethylene Glycol, gets into contact with the surrounding soil via the U-tube material and grout. The grout, usually bentonite-cement mix, exchanges heat with the soil and the BHE inner pipes.

Borehole heat exchangers are slender heat pipes with dimensions of the order of 30 mm in diameter for the inner pipes, 150 mm in diameter for the borehole, and 100 m in length for the borehole and the inner pipes. In practice there are different types of BHE. They mainly differ in their configurations. In Fig. 1, a schematic figure of U type BHE's is shown.

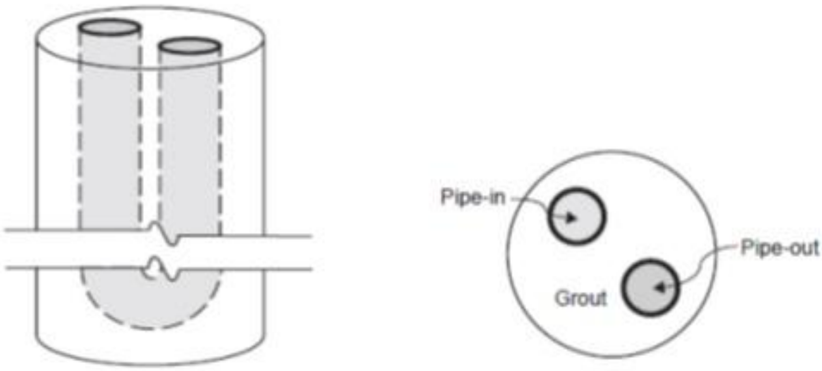

Figure1. *Schematic of U-type borehole heat exchanger*

Physically, the heat flow mechanism in such a system is well understood, but computationally, and in spite of the bulk of existing models, still creeping due to the combination of the slenderness of the boreholes heat exchangers and the involved thermal convection. This combination of geometry and physics constitutes the main source of computational challenges in this field. Consequently, several geometrical and physical simplifications have been introduced in order to circumvent this problem and obtain feasible solutions. All known solution techniques, such as analytical, semi-analytical and numerical, have been utilized for this purpose. Nevertheless, in spite of the versatility of the numerical methods, analytical and semi-analytical solutions are yet preferable because of their comparatively little demands on computational power and ease of use in engineering practice.

Heat flow in geothermal systems models by analytical and semi analytical methods. Most of these models are based on the research of cars law and Jaeger [1] that are applied for heat flow in infinite, semi-infinite and finite domains subjected to point, cylindrical, plane and line heat sources. In these models, the heat transfer mechanism and borehole heat exchanger detailed composition are ignored totally and considered as a constant heat source. In another research Gu and Neal [2] simulated transient heat flow in a composite domain subjected to using an analytical model resembling U-tubes surrounded by grout, a constant heat source and a soil mass bounded by a far field boundary. They solved the governing partial differential equation using the Eigen function expansion. Also Lamarche and Beauchamp [3] utilized Laplace transforms and solve them analytically to obtain a solution of the composite domain problem. Band yopadhyay et al. [4] employed the Gaver–Stehfest numerical algorithm for solving the inverse Laplace transform in dimensionless equations. Eskilson and Claesson [5] introduced a semi-analytical simulation for GSHP that approximates heat flow in the BHEs by two interacting channels conveying an embedded in an ax symmetric soil mass and a circulating fluid in the vertical axis. Heat flow in the soil is assumed transient conductive and in the channels, steady state convective. They applied Laplace transform and the explicit forward difference method to solve the heat equations of the channels and soil mass respectively. Zeng et al. [6] presented a semi-analytical solution of the same problem for dimensionless heat equations of the channels. Marcotte and Pasquire [7] solved transient pseudo convection. They used the fast Fourier transform for discretizing the time domain. In their work, the principle of superposition was applied to simulate the response to multiple heat fluxes. Javeb and Claesson [8] obtained a solution for pseudo convective approach. Beier [9] developed a model for transient heat transfer for a thermal response test (TRT) on a vertical borehole with a U-tube. The model provides an analytical solution for the vertical temperature profiles of the circulating fluid through the U-tube, and the temperature distribution in the ground.

Al-Khoury [10] introduced a semi-analytical model for the simulation of transient heat transfer with friction heat gain in a single U-tube geothermal borehole heat exchanger. They showed that the friction effect appears as a no homogeneous term in the governing equations, which constitutes a set of coupled partial differential equations describing heat flow in the three components of the borehole. The spectral analysis was applied for discretizing the time domain, and the Eigen-function expansion is used for discretizing the spatial domain to solve the governing initial and boundary value problem. The analysis shows the friction is not really significant for the geometry, materials, fluid velocities and viscosities, typically applied in shallow geothermal systems. In summery and according to the last researches it can be seen that the basic difference between analytical, semi- analytical and numerical methods is the rigor in describing the initial and boundary value problems. The analytical and semi-analytical methods are accurate and computationally efficient. However, they are limited in describing the boundary conditions and the geometry. On the other hand, the numerical methods are general, but computationally inefficient. Because of the extreme slenderness of the BHE's and the geometrical aspect ratio of the surrounding soil mass, the numerical analysis is computationally very demanding.

In spite of the computational efficiency of the previous works, analytical and semi-numerical solutions are yet more desirable because of their little requirements on computational power and ease of use in engineering application. So, in this paper, a framework for deriving an analytical model for the simulation of coupled conductive-convective heat transfer processes in a borehole heat exchanger subjected to defined initial and boundary conditions is presented. The analytical method is utilized for solving partial differential equations.

2. Governing Equations

Heat equations of a single U-tube borehole heat exchanger can be expressed as [11]:

$$\left(\rho C_p\right)_f \frac{\partial T_{in}}{\partial t} + \nabla.\left[\left(\rho C_p\right)_f \mathbf{u} T_{in}\right] = \nabla.\left(K_f^{eff}\, \nabla T_{in}\right) + b_{ig}\left(T_{in} - T_g\right) \quad (1a)$$

$$\left(\rho C_p\right)_g \frac{\partial T_g}{\partial t} = \nabla.\left(K_g^{eff}\, \nabla T_g\right) + b_{ig}\left(T_g - T_{in}\right) \quad (1b)$$

$$\left(\rho C_p\right)_f \frac{\partial T_{out}}{\partial t} - \nabla.\left[\left(\rho C_p\right)_f \mathbf{u} T_{out}\right] = \nabla.\left(K_f^{eff}\, \nabla T_{out}\right) + b_{og}\left(T_{out} - T_g\right) \quad (2a)$$

$$\left(\rho C_p\right)_g \frac{\partial T_g}{\partial t} = \nabla.\left(K_g^{eff}\, \nabla T_g\right) + b_{og}\left(T_g - T_{out}\right) \quad (2b)$$

Schematic zones:

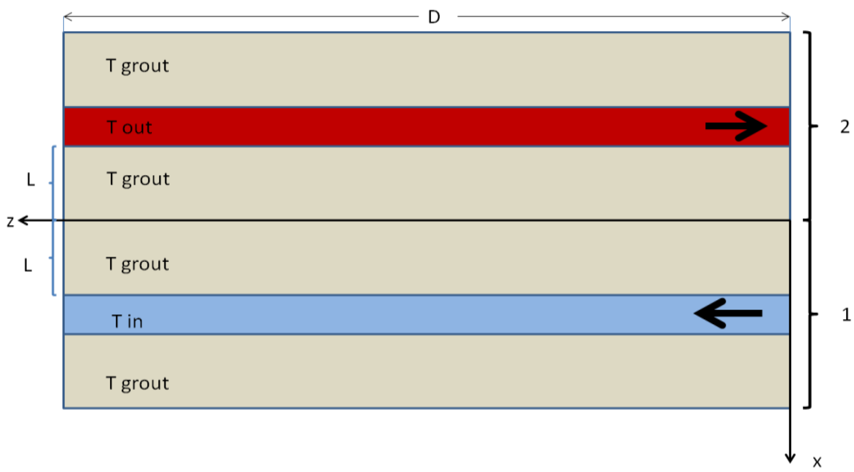

Figure2. *Schematic of the problem*

2.1. For zone 1

Assume the grout is in shape of uniform parallel large slabs with a same thickness of *2L* between two tubes, as shown in Fig. 2. The local thermal conduction occurs mainly in the radial direction of each piece of grout. Then the transient grout thermal conduction in the internal grout can be considered, which is obtained from the energy conservation low and Fourier equation and ignoring the convection term in grout, as follows [12]:

$$\left(\rho C_p\right)_g \frac{\partial T}{\partial t} = K_g \frac{\partial^2 T}{\partial x^2} \quad (3)$$

The appropriate boundary and initial conditions are as written [12-13]:

$$\begin{cases} 0 < x < L,\ t = 0 \ \rightarrow\ T = T_{soil} \\ x = 0,\ t > 0 \ \rightarrow\ \dfrac{\partial T}{\partial x} = 0 \\ x = L,\ t > 0 \ \rightarrow\ T = T_{in} \end{cases} \quad (4)$$

$$q_m = -\frac{K_g}{L}\frac{\partial T}{\partial x}\bigg|_{x=L} = b_{gi}\left(T_g - T_{in}\right) \tag{5}$$

Defining the dimensionless parameters

$$T_D = \frac{T_{soil} - T}{T_{soil} - T_{in}}, \qquad t_D = \frac{K_g}{\left(\rho C_p\right)_g L^2}t, \qquad X_D = \frac{x}{L} \tag{6}$$

The dimensionless partial differential equation and its initial and boundary conditions are

$$\frac{\partial T_D}{\partial t_D} = \frac{\partial^2 T_D}{\partial {X_D}^2}$$

$$\begin{aligned} &0 < X_D < 1,\ t_D = 0 \ \rightarrow\ T_D = 0 \\ &X_D = 0,\ t_D > 0 \ \rightarrow\ \frac{\partial T_D}{\partial X_D} = 0 \\ &X_D = 1,\ t_D > 0 \ \rightarrow\ T_D = 1 \end{aligned} \tag{7}$$

The solution of Eq. (7) in Laplace space is

$$T_D = A_1 cosh\left(\sqrt{S}X_D\right) + A_2 sinh\left(\sqrt{S}X_D\right) \tag{8}$$

After transforming the boundary conditions to Laplace space and applying them for finding A_1 and A_2 in Eq. 8, the following solution is obtained

$$T_D = \frac{cosh\left(\sqrt{S}X_D\right)}{S\, cosh\left(\sqrt{S}\right)} \tag{9}$$

The time domain solution for Eq. 9 is

$$T_D = 1 + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}exp\left(\frac{-(2n-1)^2\pi^2}{4}t_D\right)cos\left(\frac{(2n-1)\pi}{2}X_D\right) \tag{10}$$

Eq. (10) is the solution required to extract for the dimensionless temperature in the grout. Integration over Eq. (10) helps us to obtain the average grout temperature [14]:

$$T_g = \frac{\int T dV_m}{V_m} \tag{11}$$

Therefore the average temperature in grout is written as

$$T_g = T_{soil} - \Delta T_{in}\left\{1 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}exp\left(\frac{-(2n-1)^2\pi^2 K_g}{4\left(\rho C_p\right)_g L^2}t\right)\right\} \tag{12}$$

Where

$$\Delta T_{in} = T_{soil} - T_{in} \tag{13}$$

2.1.1. Determination of the b_{gi}

Using the Eq. 10 the left hand side of Eq. 5 can be calculated as

$$-\frac{K_g}{L}\frac{\partial T}{\partial X}\bigg|_{X=L} = \frac{2K_g}{L^2}\Delta T_{in}\left\{\sum_{n=1}^{\infty}exp\left(\frac{-(2n-1)^2\pi^2 K_g}{4\left(\rho C_p\right)_g L^2}t\right)\right\} \tag{14}$$

The Eq. 12 can be rewritten as

$$T_g - T_{in} = \Delta T_{in}\left\{\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}exp\left(\frac{-(2n-1)^2\pi^2 K_g}{4\left(\rho C_p\right)_g L^2}t\right)\right\} \tag{15}$$

Using Eq. 5 and by truncation at n=1 [15] the grout fluid heat transfer coefficient could be calculated through Eq.16 as follow

$$b_{gi} = \frac{-\frac{K_g}{L}\frac{\partial T}{\partial X}\Big|_{X=L}}{\left(T_g - T_{in}\right)} = \frac{K_g\pi^2}{4L^2} \tag{16}$$

This means grout fluid heat transfer coefficient is constant and depends only on thermal conductivity and height of the grout when the finite acting exists.

2.2. Analytical solution of the temperature distribution for the tube in zone 1

In the simulation of the subsurface flow, heat transfer between grout and fluid with a shape factor has a real critical role. By a constant shape factor during the pseudo steady state transfer, the heat exchange will be proportional to the temperature difference as the following equation (Ozisik, 1993): [16]

$$q_m = \alpha\left(T_g - T_{in}\right) \tag{17}$$

Which q_m is the heat exchange term of the grout–tube in fluid system, α is the proportionality coefficient. If we consider conduction coefficient effect, the new equation will be as:

$$q_m = \sigma\frac{K_g}{\left(\rho C_p\right)_g}\left(T_g - T_{in}\right) \tag{18}$$

Where the heat transfer coefficient is, $\alpha = \sigma\frac{K_g}{\left(\rho C_p\right)_g}$. Here, σ is the shape factor [16]. To incorporate the effects of the temperature transient in the grout into the shape factor, the conduction equation for the grout is solved and then an average value of the grout temperature over the grout volume can be introduced into Eq. (18). Obtained heat transfer between grout and fluid as a function of grout temperature is given in Eq.19

$$q_m = -\left(\rho C_p\right)_g\frac{\partial T_g}{\partial t} \tag{19}$$

The grout-fluid exchange term is related to the rate of heat accumulation in the grout and can be shown by Eq. (19). Combining Eqs. (2) and (19) leads to the definition of the heat transfer shape factor (Abbasi et al. 2017)

$$\left(\rho C_p\right)_g\frac{\partial T_g}{\partial t} = \sigma K_g\left(T_{in} - T_g\right) \tag{20}$$

By differentiation of Eq. 12 respect to the time (t) we obtain the Eq.21

$$\frac{\partial T_g}{\partial t} = -\frac{2K_g\Delta T_{in}}{\left(\rho C_p\right)_g L^2}\sum_{n=1}^{\infty} exp\left(\frac{-(2n-1)^2\pi^2 K_g}{4\left(\rho C_p\right)_g L^2}t\right) \tag{21}$$

Using the Eqs. 21 and 20 and 15 the shape factor can be obtained as

$$\sigma = \frac{\pi^2}{4L^2} \tag{22}$$

In Eq. 1 the term $b_{gi}\,a\left(T_g - T_{in}\right)$ may be considered as a source/sink. In other word this means heat flux can be entered in the fluid of tube in. Therefore, by using the Eqs. 20, 21 and 22 the Eq. 1 in z direction could be rewrite as follows

$$\left(\rho C_p\right)_f\frac{\partial T_{in}}{\partial t} + \left(\rho C_p\right)_f\mathbf{u}\frac{\partial T_{in}}{\partial z} = K_f^{eff}\frac{\partial^2 T_{in}}{\partial z^2} + aq_m$$

$$q_m = \left(\rho C_p\right)_g\frac{\partial T_g}{\partial t} = \sigma K_g\left(T_{in} - T_g\right) \tag{23}$$

The appropriate boundary and initial conditions for fluid of tube in temperature are as follows

$$\begin{cases} 0 < z < \infty,\ t = 0 \ \rightarrow\ T_{in} = T_{soil} \\ z = 0,\ t > 0 \ \rightarrow\ T_{in} = T_{inj} \\ z = \infty,\ t > 0 \ \rightarrow\ T_{in} = T_{soil} \end{cases} \tag{24}$$

Using Eqs. 21, 23 and rewrite the Eq. 25 as

$$\left(\rho C_p\right)_f\frac{\partial T_{in}}{\partial t} + \left(\rho C_p\right)_f\mathbf{u}\frac{\partial T_{in}}{\partial z} = K_f^{eff}\frac{\partial^2 T_{in}}{\partial z^2} + a\left(\rho C_p\right)_g\frac{\partial T_g}{\partial t} \tag{25}$$

$$a\left(\rho C_p\right)_g\frac{\partial T_g}{\partial t} = \frac{\pi^2 K_g}{4L^2}\left(T_{in} - T_g\right) \tag{26}$$

Eqs. 25 and 26 are equivalent to Eqs. 1 and 2 therefore the analytical solution of the Eqs.25 and 26 could be applied as analytical solution of the Eqs. 1 and 2.

2.3. Solution

After solution we add normal distribution shape factor to analyse its effects on heat transfer coefficient. With dimensionless parameters defining as

$$\eta = \frac{(\rho C_p)_f \mathbf{u}}{K_f} z \,, \; \omega = \frac{(\rho C_p)_f}{(\rho C_p)_f - a(\rho C_p)_g} \,, \; \tau = \frac{\left[(\rho C_p)_f \mathbf{u}\right]^2}{K_f\left((\rho C_p)_f - a(\rho C_p)_g\right)} t \,,$$

$$\lambda = \frac{a K_f K_g (\rho C_p)_g}{\left[2L(\rho C_p)_f \mathbf{u}\right]^2} \,, \quad T_{D_n} = \left.\frac{T_{soil} - T_n}{T_{soil} - T_{in}}\right|_{n=in,g} \tag{27}$$

Then the Eqs. 24 – 26 convert to as

$$\frac{\partial^2 T_{D\,in}}{\partial \eta^2} - \frac{\partial T_{D\,in}}{\partial \eta} = \omega \frac{\partial T_{D\,in}}{\partial \tau} + (1-\omega)\frac{\partial T_{D\,g}}{\partial \tau} \tag{28}$$

$$(1-\omega)\frac{\partial T_{D\,g}}{\partial \tau} = \lambda\left(T_{D\,in} - T_{D\,g}\right) \tag{29}$$

With boundary and initial conditions as

$$\begin{cases} 0 < \eta < \infty, \; \tau = 0 \; \rightarrow \; T_{D\,in} = T_{D\,g} = 0 \\ \eta = 0, \; \tau > 0 \; \rightarrow \; T_{D\,in} = 1 \\ \eta = \infty, \; \tau > 0 \; \rightarrow \; T_{D\,in} = 0 \end{cases} \tag{30}$$

The Laplace transform reduces Eqs. (28) And (29) to

$$\frac{\partial^2 \overline{T_{D\,in}}}{\partial \eta^2} - \frac{\partial \overline{T_{D\,in}}}{\partial \eta} = \omega S \overline{T_{D\,in}} + (1-\omega) S \overline{T_{D\,g}} \tag{31}$$

$$S(1-\omega)\overline{T_{D\,g}} = \lambda\left(\overline{T_{D\,in}} - \overline{T_{D\,g}}\right) \tag{32}$$

Solving Eq. (29) for $T_{D\,g}$ and substituting it into Eq. (31), and then, solving Eq. (31) with boundary conditions of Eq. (30) results in Eq. (33).

$$\overline{T_{D\,in}} = \frac{1}{S} exp\left[\left(1 - \sqrt{1 + 4\left(\omega S + f(S)\right)}\right)\frac{\eta}{2}\right] \tag{33}$$

Where

$$f(S) = \lambda - \frac{\lambda^2}{(1-\omega)S + \lambda} \tag{34}$$

The final solution of the Eqs. 28 and 29 with its boundary and initial conditions (Eq. 30) in time domain are as follows (Kocabas, 2011)[17]:

$$T_{D\,in} = T_{D\,1} + T_{D\,2} \tag{35}$$

Where

$$T_{D\,1} = \frac{1}{2}\left[exp\left(\frac{\eta}{2}(1-\theta)\right) erfc\left(\frac{\eta - \theta\tau/\omega}{2\sqrt{\tau/\omega}}\right) + exp\left(\frac{\eta}{2}(1+\theta)\right) erfc\left(\frac{\eta + \theta\tau/\omega}{2\sqrt{\tau/\omega}}\right)\right] \tag{36}$$

Where $\theta = \sqrt{1 + 4\lambda}$ and

$$T_{D\,2} = \int_0^{\tau} \frac{\eta\sqrt{\omega}}{2\sqrt{\pi\xi^3}} exp\left(\frac{(\eta - \xi/\omega)^2}{4\xi/\omega}\right) G_2 \, d\xi$$

$$G_2 = \int_0^{\lambda(\tau-\xi)/(1-\omega)} \left\{exp\left(-\left(\sqrt{\lambda\xi/\omega} - \sqrt{\mu}\right)^2\right) exp\left(-2\sqrt{\mu\xi/\omega}\right) \frac{I_1\left(2\sqrt{\mu\lambda\xi/\omega}\right)}{\sqrt{\mu\omega/(\lambda\xi)}}\right\} d\mu \tag{37}$$

Hear tube-in fluid temperature at depth D can be calculated and then could be considered as input of the tube-out. There for the tube-out fluid temperature can be obtained through next step which is similar to zone 1 solution.

3. RESULTS AND DISCUSSION

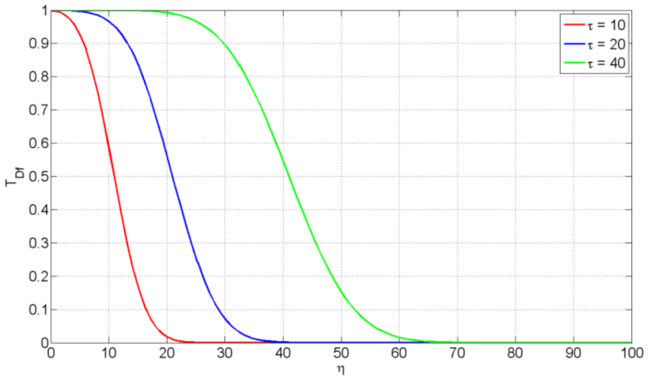

Figure3. *Variation of temperature W.r.t. depth in three time step for* $\lambda = 2520.877$

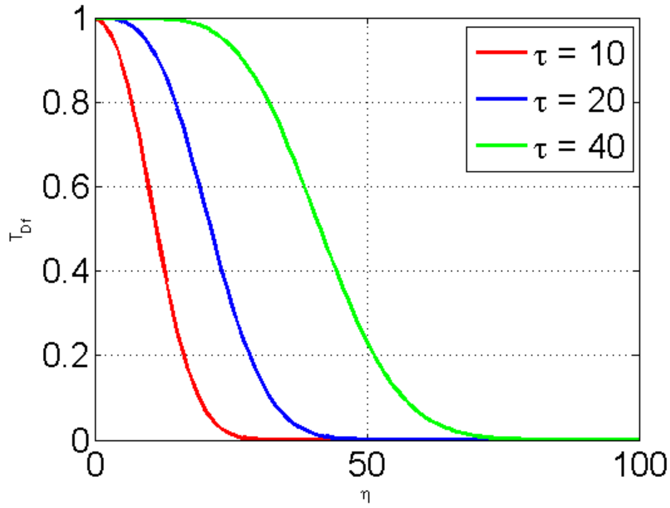

Figure4. *Variation of temperature W.r.t. depth in three time step for* $\lambda = 0.2520877$

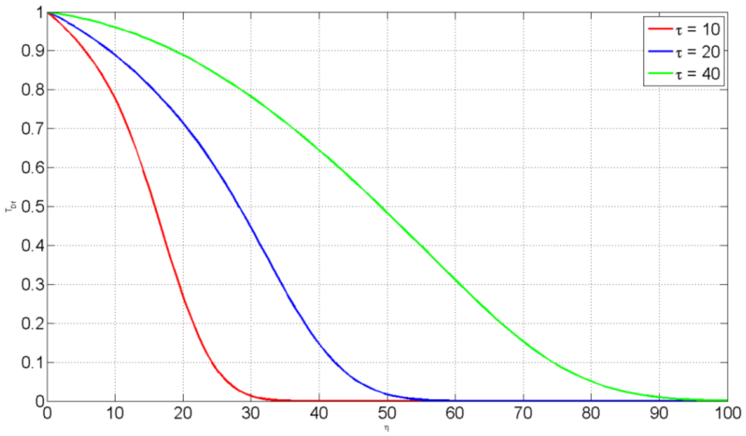

Figure5. *Variation of temperature W.r.t. depth in three time step for* $\lambda = 0.02520877$

Figures 3, 4 and 5 show the temperature distributions along the depth of the BHE in Z direction versus time obtained from the analytical solution. As it can be seen from the figures, effect of time period on temperature variation is shown. According to the figures, with increasing the time period, the temperature increases and the thickness of the thermal boundary layer is fixed in smaller time period. Also the effect of fluid flow velocity on the temperature distribution is shown in different figures. Increasing the velocity causes the decreasing of the lambda parameter and therefore temperature distribution will be in straight line in bigger depth. Also we can list results as below:

- Fluid flow velocity has an important effect on fluid temperature distribution along the depth of borehole heat exchanger.
- Usable time period for a wellbore to extract a geothermal energy plays an important role in the temperature distribution of the fluid flow in borehole heat exchangers.
- Velocities of fluid flow and time period have a clear effect on the thickness of Thermal boundary layer.

4. CONCLUSION

In this study, partial differential equations due to geothermal heat exchanger have been investigated analytically and temperature distribution has been shown with different figures. It is mentioned that analytical and semi-numerical solutions are yet more desirable because of their little requirements on computational power and ease of use in engineering application. So, in this paper, a framework for deriving an analytical model for the simulation of coupled conductive-convective heat transfer processes in a borehole heat exchanger subjected to defined initial and boundary conditions is presented. Now days some practical aspects of the geothermal system are considerable and results of analytical solution can be used for many applications in heating, cooling and another energy production systems.

Nomenclature:

Fluid	f
Tube in	in
Tube out	out
Injection temperature	T_{inj}
Grout	g
Borehole length	D
Thickness of grout between two pipe	2L
Fluid density	$(\rho)_f$
Fluid specific thermal capacity	$(c_g)_f$
Fluid thermal conductivity	k^{eff}_{f}
Grout density	$(\rho)_g$
Grout specific thermal capacity	$(c_g)_g$
Grout thermal conductivity	k^{eff}_{g}
Thermal coefficient between two components	$b_{\alpha\beta}$

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