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Rumor Spreading Model Considering Education and Remembering Mechanism

Yujiang Liu, Youquan Luo, Yang Liu

Key Laboratory of Jiangxi Province for Numerical Simulation and Emulation Techniques, Gannan Normal University, China

Abstract: *An rumor spreading model with education and forgetting is proposed to understand the effect of the capacity for education. We explored exists of the equilibria, local stability and globally asymptotical stability, and obtained the propagation threshold of rumor spreading. It is found that a backward bifurcation occurs under certain conditions.*

Keywords: *Rumor Propagation, the Basic Reproductive Number, Backward Bifurcation, Global Stability.*

1. Introduction

Globalization and the development of communication make the rumor spread more and more quickly. In recent years, the development of online social networks, such as Facebook, Live Journal, Twitter, YouTube and other social networks, has further accelerated the spread of rumors. Therefore, it becomes increasingly important to study the mechanism and process of the spread of rumors.

The classical rumor model was introduced by Daley and Kendall [1-2]. In the DK model, The population is divided according to rumor status: ignorant, spreader, and stifler. Since then, many efficient models were developed by scholars. The influence of network topology on the spread of rumor has gradually become the focus of rumor modeling, such as small world network [3-4], scale free network[5-8].

In recent years, many scholars have studied the influence of psychological factors on the spread of rumors [9-14]. [12-14] developed vary rumor propagation models, which considered the forgetting mechanism and which is conducive to the termination of the rumor. In this paper, we study a rumor spreading model in a homogeneous network take into account forgetting mechanism and impact of education on the population, which is described in Section 2. Afterwards, the basic reproductive number and the stability of rumor-free equilibrium are conducted in Section 3. in Section 4, rumor-spread equilibria and the stability are performed. The globally stable is deduced in Section 5, Finally, conclusions are given in the last section.

2. Model

Let us consider the total human population which divided into three different compartments name ignorants, educatees, spreaders, noted with $S(t)$, $E(t)$, $I(t)$ respectively.

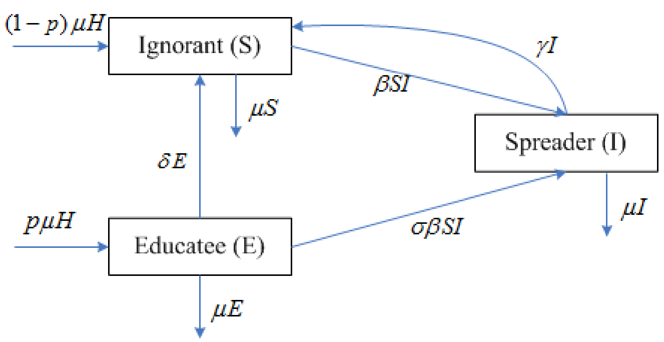

Fig1. *The Flow Diagram of the Rumor Propagation Model*

According to the dynamic interact between individuals, we established the SEI rumor spreading model based on the above assumptions. The model is described as follows:

$$\begin{aligned}\dot{S}(t) &= (1-p)\mu H - \beta k S(t)I(t) - \mu S(t) + \delta E(t) + \gamma I(t)\\ \dot{E}(t) &= p\mu H - \sigma\beta k E(t)I(t) - (\mu+\delta)E(t)\\ \dot{I}(t) &= \beta k S(t)I(t) + \sigma\beta k E(t)I(t) - (\mu+\gamma)I(t)\end{aligned} \quad (1)$$

The parameters of the model (1) descript as follows:

μ , the natural removal rate;

μH , the constant recruitment rate of the population, with $1-p$ proportion enter compartment S, and p proportion enter compartment E;

β , measure the force of rumor propagation against S, $\sigma\beta$ measure the force of rumor propagation against E, where $0<\sigma\le 1$;

δ , the forgetting rate of E, it presents educatee is not permanent.

γ , the forgetting rate from I to S, it presents the spreaders have lost the interest.

k , presents the average degree of the network.

The initial condition of 1 is given as $S(0)>0, E(0)\ge 0, I(0)\ge 0$. Note $N(t)=S(t)+E(t)+I(t)$, we have $\dot{N}(t)=\mu(H-N(t))$ then $\lim_{t\to\inf} N(t)=H$. Substitute $S(t)=H-E(t)-I(t)$ into the second and the third equation of (1), we have the limit system of (1) as follows:

$$\begin{aligned}\dot{E}(t) &= p\mu H - \sigma\beta k E(t)I(t) - (\mu+\delta)E(t)\\ \dot{I}(t) &= \beta k (H - E(t) - I(t))I(t) + \sigma\beta k E(t)I(t) - (\mu+\gamma)I(t)\end{aligned} \quad (2)$$

with initial conditions $E(0)\ge 0, I(0)\ge 0$. Denote

$$\Gamma = \{ (E,I)\in R^2 \mid E\ge 0, I\ge 0, E+I\le H\}$$

According to $\dot{N}(t)=\mu(H-N(t))$, it is easy to show that Γ is positively invariant in system (2).

3. THE BASIC REPRODUCTIVE NUMBER AND RUMOR-FREE EQUILIBRIUM LOCAL STABILITY

The basic reproductive number has played a central role in epidemiological theory for infectious diseases because it it provides an index of transmission intensity and establishes threshold criteria. In this section, we will calculate the basic reproduction number of system (2).

It is easy to see that system (2) always has a rumor-free equilibrium (the absence of spreader node, that is, $I=0$, $P_0(E_0,0)$. where $E_0=\dfrac{p\mu H}{\mu+\delta}$. From the second equation of system (2), we obtain that

$$R_0=\frac{\beta k H(\mu+\delta-(1-\sigma)p\mu)}{(\mu+\gamma)(\mu+\delta)}$$

Theorem1. *The rumor-free equilibrium* P_0 *of system (2) is locally asymptotically stable for* $R_0<1$ *and unstable for* $R_0>1$.

Proof. Let us calculate the Jacobian matrix at P_0

$$J_0=\begin{pmatrix} -(\mu+\delta) & -\dfrac{\sigma\beta k p\mu H}{\mu+\delta}\\ 0 & \beta k H(\mu+\delta-(1-\sigma)p\mu)-(\mu+\gamma)\end{pmatrix}$$

The corresponding characteristic equation of J_0 is

$$\lambda^2 - Tr(J_0)\lambda + Det(J_0) = 0, \tag{3}$$

where

$$\begin{aligned} Tr(J_0) = & -(\mu+\delta) + \beta k H(\mu+\delta-(1-\sigma)p\mu) - (\mu+\gamma) \\ & -(\mu+\delta) - (\mu+\gamma)(1-R_0) \\ Det(J_0) = & -(\mu+\delta)\beta k H(\mu+\delta-(1-\sigma)p\mu) - (\mu+\gamma) \\ & -(\mu+\delta)(\mu+\gamma)(1-R_0) \end{aligned}$$

If $R_0 < 1$, we have $Tr(J_0) < 0, Det(J_0) > 0$ It follows that the two roots of the equation (3) have negative real parts. By the Hurwitz criterion, E_0 is locally asymptotically stable.

If $R_0 > 1$, we have $Det(J_0) < 0$. It mean that the equation (3) have two roots $\lambda_1 > 0, \lambda_2 < 0$, E_0 is unstable. The proof is complete. □

4. ENDEMIC EQUILIBRIA

The first we deduce the exists of equilibrium of system (2).

Let the right of each equation of (2) equal zero, The endemic equilibria are solutions of the following

$$\begin{aligned} & p\mu H - \sigma\beta k E I - (\mu+\delta)E = 0 \\ & \beta k(H - E - I)I + \sigma\beta k E I - (\mu+\gamma)I = 0 \end{aligned} \tag{4}$$

We can eliminate E using the first equation of (4), and substitute into the second equation, we can draw an equation of the form

$$f(I) = AI^2 + BI + C = 0 \tag{5}$$

with

$$\begin{aligned} & A = \sigma\beta k \\ & B = -(H\beta k - \gamma - \mu)\sigma + \mu + \delta \\ & C = \frac{(\mu+\delta)(\mu+\gamma)}{\beta k} - ((p\sigma - p + 1)\mu + \delta)H \end{aligned}$$

Now introduce symbol $\tilde{R}_0 = \frac{\beta k H}{\mu+\gamma}$, Then we can rewrite the expressions for B and C above as

$$B = (\mu+\gamma)(1-\tilde{R}_0) + (\mu+\delta) \text{ and } C = \frac{(\mu+\delta)(\mu+\gamma)}{\beta k}(1-R_0)$$

Let $\Delta = B^2 - 4AC$, to completely determine the existence of the positive equilibria of (2.4), we must consider three possible

a) If $R_0 > 1$, then $C < 0$ and thus $\Delta > 0$, then $f(I)$ has a simple positive root I_2, system (2) has a unique endemic equilibrium P_2, accordingly.

b) If $R_0 = 1$, then clearly $C = 0$ and $\tilde{R}_0 > 1$ and $f(I)$ has the two roots 0 and $-B/A$, system (2) therefore has a unique endemic equilibrium P_2 if and only if $B < 0$.

c) If $R_0 < 1$ then $C > 0$ and there exists two possible subcase. if $\Delta > 0$ and $B < 0$ then $f(I) = 0$ has two positive roots I_1 and I_2, otherwise, $f(I) = 0$ does not have any positives roots.

We can summarize the previous discussion in the following theorem

Theorem2.*The rumor-spread equilibrium of system (2) is no more than two, more precisely*

1. If $R_0 < 1$, $\Delta \ge 0$ and $B < 0$, system (2) has two rumor-spread equilibria P_1 and P_2, especially $\Delta = 0$ will result in P_1 and P_2 coincidence;
2. If $R_0 > 1$ or $R_0 = 1$ and $B < 0$, system (2) has a unique rumor-spread equilibrium P_2;
3. If $R_0 = 1$ and $B \ge 0$, or $R_0 < 1$, $\Delta < 0$, or $R_0 < 1$, $B \ge 0$, system (2) has no rumor-spread equilibrium.

Then, we discuss the stability of rumor-spread equilibria.

Theorem3. *The rumor-spread equilibrium P_1 of system (2) is unstable whenever it exists, while the rumor-spread equilibrium P_2 of system (2) is locally asymptotically stable whenever it exists.*

Proof. Let us calculate the Jacobian matrix of system (2) at a rumor-spread equilibrium, we have

$$\begin{pmatrix} -(\beta k I\sigma + \delta + \mu) & -\sigma\beta k E \\ -(1-\sigma)\beta k I & -\beta k I + \beta k(H - E - I) + \sigma\beta k E - \mu - \gamma \end{pmatrix}$$

From the equilibrium condition (4), the matrix can be simplified as

$$J^* = \begin{pmatrix} -(\beta k I\sigma + \delta + \mu) & -\dfrac{\sigma(H\beta k - \beta k I - \gamma - \mu)}{1-\sigma} \\ -(1-\sigma)\beta k I & -\beta k I \end{pmatrix}$$

We have

$$\begin{aligned} Tr(J^*) &= -(\beta k I\sigma + \delta + \mu) - \beta k I \\ &< 0 \end{aligned} \tag{6}$$

$$\begin{aligned} Det(J^*) &= (\beta k I\sigma + \delta + \mu)\beta k I - \frac{\sigma(H\beta k - \beta k I - \gamma - \mu)}{1-\sigma}(1-\sigma)\beta k I \\ &= 2\beta^2 k^2 I^2 \sigma - \beta k(H\beta k\sigma - \gamma\sigma - \mu\sigma - \delta - \mu) \\ &= 2A\beta k\left(I + \frac{B}{2A}\right) \end{aligned} \tag{7}$$

If P_1 exists, from Theorem 2, P_2 also exists, from equation (5), $I_1 + I_2 = -\dfrac{B}{A}$, then $I_1 < -\dfrac{B}{2A}$ which result in $Det(J^*) < 0$, Based on Routh-Hurwitz Theorem, P_1 is unstable. otherwise, $I_2 > -\dfrac{B}{2A}$ which result in $Det(J^*) > 0$ then P_2 is locally asymptotically stable. □

Remark: From Theorem 1 and Theorem 2.1, we have if $R_0 < 1$, $\Delta \ge 0$ and $B < 0$, the rumor-free equilibrium P_0 and rumor-spread equilibrium P_2 are locally asymptotically stable simultaneously, which means that the system (2) exists backward bifurcation if some conditions are satisfied.

Using as bifurcation parameter the force ratio of rumor propagation σ, we can use Theorem 4.1 in [15] to determine the system (2) undergoes either a forward or a backward bifurcation when $R_0 = 1$.

$$\frac{H\beta k}{\mu+\gamma} > 1 \text{ and } \frac{H\beta k}{\mu+\gamma} - \frac{p\mu}{(\mu+\gamma)(\mu+\delta)} < 1 \tag{8}$$

Let $\bar{\sigma} = 1 - \dfrac{\mu\beta k(\mu+\delta) - (\mu+\gamma)(\mu+\delta)}{pH\mu\beta k}$, It is obviously $0 < \bar{\sigma} < 1$ if (1) is satisfied. Then, we have the following theorem:

Theorem4. *If the condition (1) is satisfied, Let* $\bar{\beta}=\frac{1}{kH\mu p\bar{\sigma}(1-\bar{\sigma})}$ *, then system (2) at* $R_0=1$ *undergoes a forward bifurcation if* $\beta<\bar{\beta}$ *, and a backward bifurcation if* $\beta>\bar{\beta}$ *.*

Proof. To apply Theorem 4.1 of[15], the right and left eigenvectors of the Jacobian $J(P_0)$ are

$$w=\left[-\frac{H\beta k\mu p\bar{\sigma}}{(\mu+\delta)^2},1\right]^T \text{ and } v=[0,1],$$

Let $\phi=\sigma-\bar{\sigma}$, $x_1=E-E_0$, $x_2=I$, the Taylor expansion of system (2) are represented by the $f_i(x,\phi), i=1,2$, and we have

$$\frac{\partial^2 f_1}{\partial x_1\partial x_2}(0,0)=\frac{\partial^2 f_1}{\partial x_2\partial x_1}(0,0)=-\bar{\sigma}\beta k$$

$$\frac{\partial^2 f_2}{\partial x_1\partial x_2}(0,0)=\frac{\partial^2 f_2}{\partial x_2\partial x_1}(0,0)=-(1-\bar{\sigma})\beta k,\frac{\partial^2 f_2}{\partial x_2^2}(0,0)=-2\beta k$$

$$\frac{\partial^2 f_1}{\partial x_2\partial\phi}(0,0)=-\frac{H\beta k\mu p}{\mu+\delta},\frac{\partial^2 f_2}{\partial x_2\partial\phi}(0,0)=\frac{H\beta k\mu p}{\mu+\delta}$$

and all other derivatives equal zero.

$$a=\sum_{k,i,j=1}^{n}v_k u_i u_j\frac{\partial^2 f_k}{\partial x_i\partial x_j}(0,0)=2\beta k^2H\mu p\bar{\sigma}(1-\bar{\sigma})(\beta-\bar{\beta}) \tag{9}$$

$$b=\sum_{k,i=1}^{n}v_k u_i\frac{\partial^2 f_k}{\partial x_i\partial\phi}(0,0)=\frac{H\beta k\mu p}{\mu+\delta}>0 \tag{10}$$

So we conclude that if $\beta<\bar{\beta}$, then $a<0$, the system (2) at $R_0=1$ gets the forward bifurcation, if $\beta>\bar{\beta}$, then $a>0$, which arise a backward bifurcation of the system (2). □

5. GLOBAL STABILITY

Theorem5. *For system (2), the closed orbits and limit cycles in* Γ *does not exist.*

Proof. Construct Dulac function $B(E,I)=\frac{1}{EI}$, we have

$$\frac{\partial(Bf_1)}{\partial E}+\frac{\partial(Bf_2)}{\partial I}=-\left(\frac{\beta k}{E}+\frac{H\mu p}{E^2 I}\right)<0$$

Thus, from Bendixson- Dulac criterion that system (2) has no periodic orbit in Γ. Combine Theorem (2), we derive

Theorem6. *If* $R_0<1$ and $\Delta<0$ *or* $R_0<1$ and $B\ge 0$ *, the rumor-free equilibrium* P_0 *is globally asymptotically stable, and If* $R_0>1$ *, the rumor-spread equilibrium* P_2 *is globally asymptotically stable.*

6. CONCLUSION

Rumor propagation has been investigated through vary types of mathematical models. In our study, we consider a rumor propagation model with education and forgetting mechanism and determine the basic reproductive number. In general, the basic reproductive number less than one is the condition which rumor will die down. But in our model, this condition is not enough owing to a backward bifurcation occur. That is to say, driving the basic reproduction number below one is not enough to eradicate the rumor diffusion. we need yet to further reduce the basic reproduction number, and the initial number of spreader must be controlled to a lower level. This requires the government and related organizations to further develop the corresponding work.

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