International Journal of Humanities Social Sciences and Education (IJHSSE)
Volume 5, Issue 4, April 2018, PP 20-45
ISSN 2349-0373 (Print) & ISSN 2349-0381 (Online)
http://dx.doi.org/10.20431/2349-0381.0504003
www.arcjournals.org

The Real, the Infinite, Categories, and Cardinal Numbers: The Problem of Metaphysics in the *Critique of Pure Reason*

Kazuhiko Yamamoto*

Megumi Institute of Ethics and Philosophy, Japan

***Corresponding Authors:** *Kazuhiko Yamamoto, Megumi Institute of Ethics and Philosophy, Japan*

Abstract: *The metaphysical exploration – an attempt to rectify the foundation of Kant's "transcendental philosophy" – led us to the findings that while "appearances in general are nothing outside our representations," appearances themselves are things that exist outside our representations. In other words, appearances themselves, i.e., categories, while serving "only for the possibility of empirical cognition," would last forever in virtue of the "transcendental truth" or the "transcendental ideality of appearances," irrespective of the existence of humans on this planet. This insight opens the way to whole new realms of mathematics and philosophy, enabling us to find a solution to the conundrums in mathematics, such as inductive numbers, Cantor's continuum hypothesis, Zermelo's axiom, and the infinite or transfinite cardinal numbers.*

Keywords: *Real, Infinite, Pure Concept, Pure Image, Categories, Cardinal Numbers*

1. INTRODUCTION

In the *Critique of Pure Reason*, there are many discourses which seem difficult to comprehend or rather we may say that the whole path of the discourse is stuffed with enigmatic, twisted metaphysics, which appear almost refusing to be understood. We have tried to decipher the enigmatic code in Kant's metaphysics by means of finding the law of nature in regard to humans, and applying it to his metaphysics in transcendental analytic (YAMAMOTO 2016: 87-100, YAMAMOTO 2017a: 19-37, YAMAMOTO 2017b: 72-81, YAMAMOTO 2017c: 57-70, YAMAMOTO 2017d: 19-29). When it is found that the law of nature signifies nothing but the utter dismemberment or "decomposition" (A525/B553[1]) of humans, i.e., death itself, which could be extended to all appearances – the living things – in the world of sense by means of categorical syllogism (YAMAMOTO 2016: 87-100, YAMAMOTO 2017c: 57-70, YAMAMOTO 2017d: 19-29), we feel that the enigmatic discourse in the *Critique of Pure Reason* becomes comprehensible, which, in turn, would enable us to solve conundrums in regard to mathematics or physics. Why do we feel that way? It is because we believe that the conundrums in the *Critique of Pure Reason* parallel the conundrums in mathematics (HILBERT 1902: 437-479) or of those in quantum mechanics (EINSTEIN et al. 1935: 777-780). If we succeed in solving the conundrum in regard to Kant's metaphysics, it might lead us to a clue to solve the conundrums in mathematics (YAMAMOTO 2017b: 72-81, YAMAMOTO 2017c: 57-70, YAMAMOTO 2017d: 19-29) or in physics. Why? We have already made remarks which could be the answer, saying, "What is the major premise? It is nullity in space-time – the product of our 'metaphysical deduction' in which 'the origin of the *a priori* categories in general was established through their complete coincidence with the universal logical functions of thinking' (B159), namely categorical syllogisms. Since 'the categorical syllogisms, whose major premise, as a principle, states the relation of a predicate to a subject' (A406-B433) correspond to general logic which 'abstracts from all content of the predicate' (A72), our metaphysical axioms all converge into nullity in space-time – space-time itself. Seeing that this nullity in space-time or space-time itself is equivalent to 'the *a priori* categories in general' (B159), we say that our metaphysical axioms, homogeneous with categorical syllogisms, are applicable to appearances themselves, indicating that it can go to explain not only 'the possibility of things in the world of sense' (A677/B705) but as far as 'the possibility of a

[1] A525/B553 designates the pagination of the standard German edition of Kant's works, as indicated by means of marginal numbers in the Critique of Pure Reason (Kant, Immanuel, *Critique of Pure Reason*, Cambridge University Press, 1999). All citations are the same.

world-whole itself' (A677/B705) – the universe. Therefore, the 'transcendental ideality of appearances' (A506/B534) can be thought to mean that appearances themselves are to exist irrespective of humans in virtue of 'things in the world of sense' (A677/B705) or 'a world-whole itself' (A677/B705). In other words, it designates that while 'appearances in general are nothing outside our representations' (A507), appearances themselves are things to exist outside our representations" (YAMAMOTO 2017c: 57-70). Furthermore, in order to make it more explicit, we would add here, saying as follows: 1) the law of nature which edicts that the utter dismemberment or decomposition of humans is inevitable or unavoidable leads them to attain to the "synthetic *a priori* cognition" (A14/B28) in regard to death itself, i.e., space-time itself, since it is to signify "necessity" (B111) and "universality" (B378) among humans, 2) the "synthetic *a priori* cognition" in regard to the utter dismemberment or decomposition is commensurate with the "pure concept of the understanding" (A143) or "a concept of reason" (B377) – the category – 3) humans constitute *Homo sapiens*, since they, as appearances themselves which are to be accompanied by the utter dismemberment or decomposition of themselves, cognize it through pure intuition affected by the sensation of nullity – empirical intuition – and the "synthesis of apprehension" (A99), 4) since a human "stands under the original unity of consciousness" (B140) in virtue of the "universal cognitions *a priori*" (A300) or "a cognition from a principle" (B357), he or she belongs among the category, which signifies appearances themselves which are to disappear in nullity in space-time, 5) "the original unity of consciousness" in regard to the "universal cognitions *a priori*" or "a cognition from a principle" signifies the fundamental trait of humanity, indicating that the category – *Homo sapiens* – is distinct from others. We think that since nullity in space-time – death itself – is taken from experience (by deduction), the "universal proposition" (A300) of nullity in space-time – space-time itself – serves "as the major premise in a syllogism" (A300), enabling humans to attain to "a cognition from a principle." When we say that something is "taken from experience (by deduction)," it is meant to be "taken from experience (by the 'metaphysical deduction' (B159) in conjunction with the 'transcendental deduction' (B159)." Since "the major premise always gives a concept such that everything subsumed under its condition can be cognized from it according to a principle" (B357), "that cognition in which I cognize the particular in the universal through concepts" (B357) – the category – signifies nothing other than "universal propositions *a priori*" (B357) or "principles" (B357). We say that the categories, which appear to "serve only for the possibility of empirical cognition" (B147), would last forever in virtue of "synthetic *a priori* principles" (B305) – the "transcendental truth" (A146) or the "transcendental ideality of appearances" (A506/B534) – irrespective of the existence of humans on this planet.

Being keenly conscious of the fact that 1) when a human, which appears in the world of sense, is to necessarily disappear in nullity in space-time, it stands under the law of nature – death itself – 2) since a human is to encounter death itself either in "experience itself" (A123) or in "possible experience" (A489/B517), "experience itself" or the "possible experience" is homogeneous with the "necessity," which "is nothing other than the existence that is given by possibility itself" (B111) – "real possibility" (B302) – 3) since "the necessity" determines the category – a human or *Homo sapiens* – this "provides us with the rule and is the source of truth" (B375), we have already said, "As for category Kant says, 'Categories are concepts that prescribe laws *a priori* to appearances, thus to nature as the sum total of all appearances (*natura materialiter spectata*), and, since they are not derived from nature and do not follow it as their pattern…' (B163). Contrary to Kant, we say that 1) nature prescribes laws *a priori* to appearance, 2) the law of nature, as the sum total of all appearances, prescribes concepts as categories, 3) thus, categories are concepts that prescribe laws *a priori* to appearances" (YAMAMOTO 2017a: 19-37). What does this mean? It means that categories, which are taken from experience (by deduction), thereby being "derived from nature," and which "follow it as their pattern" (B163), "determine *a priori* the combination of the manifold of nature" (B163). We have to take note of the fact that there is one category, which is "derived from nature," being taken from experience (by deduction), but which does not "follow it as their pattern," determining "*a priori* the combination of the manifold of nature." What is this category, which does not follow it as the pattern? It is the "pure concept of the understanding" (A143), i.e., death itself – space-time itself – for which there are two patterns for a human to follow; "experience itself" (A123), and "possible experience" (A489/B517) in virtue of "real possibility" (B302). In addition, humans have myriads of categories, which are taken from experience (by deduction), thereby being "derived from nature," and

which "follow it as their pattern." What does "follow it as their pattern" mean? It means "general logic" (A72) or "the categorical syllogisms, whose major premise, as a principle, states the relation of a predicate to a subject" (A406-B433). What does the "major premise, as a principle," (A406) state? Since "the law of nature is that humans appear in a world of sense and then disappear" (YAMAMOTO 2016: 87-100), a human, which stands under the category, can state "the relation of a predicate to a subject," i.e., that nullity in space-time – space-time itself – permeates subjects which "follow it as their pattern." In other words, categories, which follow the law of nature as their pattern, are to determine themselves *a priori*. In regard to this issue, we have already said, "'Every syllogism is a form of derivation of a cognition from a principle' (B357). When 'a cognition from a principle' belongs among the categories, it corresponds to the 'categorical syllogisms' (A406). Therefore, 'a cognition from a principle' – nullity in space-time – works as the major premise in categorical syllogisms, stating 'the relation of a predicate to a subject' (B433), i.e., that nullity in space-time – space-time itself – permeates a subject. Since the major premise – nullity in space-time – 'always gives a concept such that everything subsumed under its condition can be cognized from it according to a principle' (B357), humans' cognition can ground in nullity in space-time, indicating that it has 'the reality (i.e., objective validity)' (A28) of space and time, which is necessary and universal" (YAMAMOTO 2017d: 19-29).

The discourse above indicates that categories are the causality of "the manifold of nature" (B163). Though the law of nature which edicts that a thing that appears is to disappear in nullity in space-time seems to be distinct from "the law of nature that everything that happens has a cause" (A542/B570), actually, there is no difference between the two on account of the fact that "everything that happens has a cause" in virtue of categories, which let loose the "temporal sequence" (B163): that a thing that appears in filled space-elapsing time is to disappear in nullity in space-time. When Kant makes a discourse in regard to categories, saying, "We cannot think any object except through categories; we cannot cognize any object that is thought except through intuitions that correspond to those concepts. Now all our intuitions are sensible, and this cognition, so far as its object is given, is empirical. Empirical cognition, however, is experience. Consequently no *a priori* cognition is possible for us except solely of objects of possible experience" (B165-B166), we should like to put it in another way, "We can think any object through categories; we can cognize any object that is thought through intuitions that correspond to those concepts. Now all our intuitions are sensible, and this cognition, so far as its object is given, is empirical. Empirical cognition, however, is experience. Consequently *a priori* cognition is possible for us solely of objects of possible experience." What does "objects of possible experience" mean in this context? It is nothing but the category, i.e., nullity in space-time, or categories, i.e., filled space-elapsing time, which are grounded in "synthetic *a priori* cognition" (A14/B28). When we acknowledge, in this regard, that the "categories are concepts that prescribe laws *a priori* to appearances" (B163), it becomes possible for us to reach the abyss of causality through transcendental analytic. We have already explicated in regard to the issue of causality, as follows: "Then, following the scheme in the 'Table of Categories' (B106), which ordains that 'allness (totality) is nothing other than plurality considered as a unity, limitation is nothing other than reality combined with negation, community is the causality of a substance in the reciprocal determination of others, finally necessity is nothing other than the existence that is given by possibility itself' (B111), we have to deal with the issue of community (reciprocity) in 'allness (totality),' i.e., 'plurality considered as a unity' since 'the schema of community (reciprocity), or of the reciprocal causality of substances with regard to their accidents, is the simultaneity of the determinations of the one with those of the other, in accordance with a general rule' (A144-B184)…1) since there are no parts of space in space itself – nullity in space – space, as space itself, is not 'subordinated to one another' and are not 'coordinated with one another' (B439): 2) therefore, space, as space itself, does not constitute a series: 3) since there are filled spaces as manifold parts of space itself, the manifold parts of space itself are subordinated to one another or are coordinated with one another: 4) therefore, a filled space – a manifold part – can be the condition of the possibility of another part, and filled space, like elapsing time, does in itself constitute a series: 5) the synthesis of the manifold part of space itself (synthesis of filled space or of a filled space and empty space) is 'successive, and thus occurs in time and contains a series' (B439): 6) since the synthesis takes place in the manifold of sensibility, succession, subordination and coordination pertain to filled space-elapsing time: 7) since the world-whole consists of all appearances – filled space-elapsing time and nullity in space-time – succession,

subordination and coordination which take place in filled space-elapsing time would affect the world-whole. Following what Kant says (B112), contrarily, we have to say, 'Now a similar connection is thought of in an entirety of things, since one is subordinated, as effect, under another, as the cause of its existence, or coordinated with the other simultaneously and reciprocally as cause with regard to its determination.' What does this mean? It means that 'the members of the division exclude each other and yet are connected in one sphere, so in the latter case the parts are represented as ones to which existence (as substances) pertains to each exclusively of the others, and which are yet connected in one whole' (B113). We think that 'the members of the division' signifies categories, through which it would become possible for us to cognize 'whatever objects may come before our senses,…as far as the laws of their combination are concerned' (B159)" (YAMAMOTO 2017b: 72-81). "This metaphysical causality must be the ground on which mathematical axioms rest. We feel that mathematical axioms, in commensurate with metaphysical axioms, would be the clue to solve the conundrum in regard to Hilbert's mathematical problems posed in 1900 (HILBERT 1902: 437-479)" (YAMAMOTO 2017d: 19-29).

According to Kant, "when we consider nature, experience provides us with the rule and is the source of truth" (B375). When Kant adds, in regard to nature, the rule and experience, saying, "The genus is representation in general (*repraesentatio*). Under it stands the representation with consciousness (*perceptio*). A perception that refers to the subject as a modification of its state is a sensation (*sensatio*); an objective perception is a cognition (*cognitio*). The latter is either an intuition or a concept (*intuitus vel conceptus*). The former is immediately related to the object and is singular; the latter is mediate, by means of a mark, which can be common to several things. A concept is either an empirical or a pure concept, and the pure concept, insofar as it has its origin solely in the understanding (not in a pure image of sensibility), is called *notio*. A concept made up of notions, which goes beyond the possibility of experience, is an idea or a concept of reason" (A320-B377), we, in half agreement and in half disagreement with Kant, should like to put it in another way, saying, "a perception – 'the representation with consciousness' – is immediately related to the object itself, i.e., death itself through a cognition – 'an objective perception' – on account of the fact that cognizing 'the actuality of things' is to have a 'connection with some actual perception in accordance with the analogies of experience, which exhibit all real connection in an experience' (A225), and is not singular. Consciousness, which is to be immediately marked by death itself, could perceive a modification of its state through sensation and intuition, which accompany another's experience of death itself and cognize it as nullity in space-time, which is understood to be common to all consciousnesses. While nullity in space-time is cognized through sensibility, thereby being empirical, death itself is a pure concept, which is perceived from nullity in space-time and the synthesis of apprehension. The pure concept, insofar as it has its origin in the understanding and in a pure image of sensibility, does not go beyond the possibility of experience. This 'transcendental schema' (A138/B177) – 'pure *a priori* concepts' (A95) – is regarded as an idea or a concept of reason – the category. The 'transcendental schema' – the category – manifests only in humans, yielding *Homo sapiens*." Thus, a consciousness – the "manifold in a given intuition" (B143) – "necessarily stands under categories" (B143). Since the pure concept, insofar as it has its origin in the understanding and in a pure image of sensibility is to be called *notio*, we think that "the pure concept" or "*notio*" is equivalent to the "transcendental schema," which belongs to "pure *a priori* concepts" – "*a priori* conditions for a possible experience" (A95). Furthermore, in the same line of thoughts, we say, following what Kant says regarding the conditions, the conditioned and the unconditioned (B379), that "since the unconditioned (nullity in space-time, i.e., space-time itself) alone makes possible the totality of conditions (the world-whole or allness, i.e., totality), and conversely the totality of conditions (the world-whole or allness, i.e., totality) is always itself unconditioned (nullity in space-time, i.e., space-time itself), a pure concept of reason in general (the world-whole or allness, i.e., totality) can be explained through the concept of the unconditioned (nullity in space-time, i.e., space-time itself), insofar as it contains a ground of synthesis (death itself, i.e., "transcendental schema") for what is conditioned (*Homo sapiens*)." Therefore, we should like to say that 1) a human or *Homo sapiens* is homogeneous with the "transcendental concept of reason" (B379), namely the concept of the totality of conditions (the world-whole or allness, i.e., totality), and 2) the "transcendental schema" (A138/B177), which belongs among the "pure *a priori* concepts" (A95), is homogeneous with "pure concepts of reason or transcendental ideas" (B378). What does the enigmatic saying mean?

It means that the number in regard to the category – *Homo sapiens* – is to be commensurate with the number, $i^2 = -1$. This issue will be discussed later.

Furthermore, we would add, following what Kant refers to in regard to time (A138/B177-B178-A139), saying, "a human, i.e., a filled space-elapsing time, as the condition of the manifold of inner sense, and 'outer sense' (B37), thus of the connection of all representations, contains an *a priori* manifold in pure intuition – nullity in space-time, i.e., space-time itself. Now this transcendental space-time-determination – space-time itself – is homogeneous with the category (which constitutes its unity) insofar as it is universal and rests on a rule *a priori*. And it is at the same time homogeneous with the appearance – a human – insofar as space-time itself can be thought, 'by means of a mark' (B377), to be contained in every empirical representation of the manifold. Hence an application of the category to the appearance – every empirical representation of the manifold – becomes possible by means of the transcendental space-time-determination which, as 'the schema of the pure concept of the understanding' (B224), or as 'the schemata of the concepts of pure understanding' (B185-A146), mediates the subsumption of every empirical representation of the manifold under the category, i.e., the 'pure concept of the understanding' (A143) or 'a concept of reason' (B377)." In addition, when Kant makes a discourse in regard to the pure concepts of the understanding (A137/B176-A138/B177), we should like to say, in an opposite manner, that "the pure concept of the understanding, i.e., death itself, in comparison with empirical (indeed in general sensible) intuitions, are homogeneous, and can be encountered in any intuition. Now the subsumption of any intuition including empirical (indeed in general sensible) intuitions under the pure concept of the understanding, thus the application of the category to the empirical intuitions is possible, since one can say that the category, e.g., causality, is contained in the empirical (indeed in general sensible) intuitions and could be perceived through the concept of death. This question, so natural and important, is the cause which makes a transcendental doctrine of the power of judgment unnecessary, in order, namely, to show the possibility of applying the pure concept of the understanding to the empirical (indeed in general sensible) intuitions." "Experience provides us with the rule and is the source of truth" (B375) in regard to *Homo sapiens* and "appearances in general" (A138/B177), which edicts: 1) "categories – parts of filled space-elapsing time – exclude each other and yet are connected in one sphere, so in the latter case categories – parts of filled space-elapsing time – are represented as ones to which existence (as substances) pertains to each exclusively of the others, and which are yet connected in one whole," 2) categories are to disappear in nullity in space-time – space-time itself. Therefore, when Kant refers to Plato and Platonism (B374-A318-B375), we say, following him, "Plato was right to see clear proofs of an origin in ideas not only where human reason shows causality, and where ideas become causes (of action and their objects) in regard to the 'whole object of experience' (A694/B722). A plant, an animal, the regular arrangement of the world's structure (presumably thus also the whole order of nature) – these show clearly that they are possible according to ideas; although no individual creature other than humans, under the individual conditions of its existence, is congruent with the idea of what is most perfect of its species, these ideas – the original causes of things which are discovered – are in the highest understanding not individual, unalterable, thoroughly determined, and the whole of its combination in the totality of a world is fully adequate to its idea." What does the idea mean? From our viewpoint, it is nothing but the category, i.e., *phaenomena* of death itself, "to the extent that as objects they are thought in accordance with the unity of categories" (B305-A249). Therefore, we say, in an opposite manner to what Kant says (A181-B224), "Appearances themselves (e.g., humans) must be subsumed under the category, but only under the schemata of disappearance of 'the things themselves, which appear' (B324). Since the objects to which these principles are to be related are things in themselves, it is possible to cognize anything about them synthetically *a priori*. Now it is nothing but appearances themselves whose complete cognition, to which in the end all *a priori* principles must come down, is the 'possibility of experience' (A156) or 'possible experience' (A489/B517). Consequently these principles can have as their goal the conditions of the unity of empirical cognition in the synthesis of the appearances. However, since these conditions of the unity are to be thought only in the schema of nullity in space-time, the category contains the function, unrestricted by any sensible condition, of their unity, as of a synthesis in general." We think that "the function, unrestricted by any sensible condition, of their unity, as of a synthesis in general" (B224) pertains to space-time itself. Therefore, following this discourse, we say that 1) the schema of nullity in space-time – "the objects of nature itself" (A114) – is equivalent to "the original causes of things"

(A318), i.e., space-time itself, 2) the categories – the "object of all possible experience" (A114) or the conditions of the unity, which are "thought only in the schema of the pure concept of the understanding" (B224) – are the causality, 3) space-time itself – the category – and filled space-elapsing time – the categories – signify "principles" – the cause and the causality. Since the issue of Cantor's transfinite numbers and "the cardinal number of the continuum" or Riemann zeta function and prime numbers cannot be separated from the issue of categories (YAMAMOTO 2017d: 19-29), we try to solve the conundrums in regard to these issues by means of transcendental analytic, which stands for the "metaphysical axioms."

2. CONTINUUM HYPOTHESIS, REAL NUMBERS AND METAPHYSICAL AXIOMS

Among the twenty-three mathematical problems posed in 1900, Hilbert raised "Cantor's problem of the cardinal number of the continuum" as the first that was expected to be solved in the 20th century (HILBERT 1902: 437-479). In regard to the issue of continuum, we have made a preliminary discourse as follows: "Here, Hilbert first defines Cantor's problem as follows: 'Two systems, i.e., two assemblages of ordinary real numbers or points, are said to be (according to Cantor) equivalent or of equal *cardinal number*, if they can be brought into a relation to one another such that to every number of the one assemblage corresponds one and only one definite number of the other' (HILBERT 1902: 437-479). Even if Cantor asserts that 'every system of infinitely many real numbers, i.e., every assemblage of numbers (or points), is either equivalent to the assemblage of natural integers, 1, 2, 3…or to the assemblage of all real numbers and therefore to the continuum, that is, to the points of a line; *as regards equivalence there are, therefore, only two assemblages of numbers, the countable assemblage and continuum*' (HILBERT 1902: 437-479), the most serious problem arises in terms of the theorem that 'the continuum has the next cardinal number beyond that of the countable assemblage' (HILBERT 1902: 437-479). In other words, the issue of what the cardinal number signifies inevitably comes forwards. In order to clarify it, 'a new bridge between the countable assemblage and the continuum' (HILBERT 1902: 437-479) is absolutely necessary. However, it would never come out from mathematical axioms. From where would it come out? It would come out only through metaphysical axioms. Kant elaborates on it as follows: 'since every reality has its degree that can decrease to nothing (emptiness) through infinite steps while the extensive magnitude of the appearance remains unaltered, it must yield infinitely different degrees with which space or time is filled, and the intensive magnitude in different appearances can be smaller or greater even though the extensive magnitude of the intuition remains identical' (B214-A173). We think that what Kant refers to here indicates that 0, $i^2 = -1$, and 1 would signify cardinality. Since the numbers 0, $i^2 = -1$, and 1 can be regarded as a 'real number,' metaphysical axioms dictate that; 1) 0 signifies nullity in space-time, 2) number 1 signifies the consummation of filled space-elapsing time, and 3) $i^2 = -1$ signifies every reality, which 'has its degree that can decrease to nothing (emptiness) through infinite steps' (B214-A173). We have to think that $i^2 = -1$ is the bridge between the countable assemblage and continuum" (YAMAMOTO 2017c: 57-70).

When we say that 0, $i^2 = -1$ and 1 signify the cardinality, it is also meant to be indicative of the change of "the accidents" (A184) in the world of sense in virtue of "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$." We think that the change of the accidents, namely "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" signifies the synthetic *a priori* propositions (YAMAMOTO 2017d: 19-29) on account of the fact that 1) these propositions are "valid only in relation to possible experience" (B228), and 2) they are to be proved successively through "our apprehension of the manifold of appearance" (A182). While "such a proof can never be conducted dogmatically, i.e., from concepts" (A184-B228), humans are to apprehend the manifold of appearance – nullity in space-time – upon encountering the disappearance of the manifold, namely death itself in regard to *Homo sapiens*. Since death itself is "necessity," among *Homo sapiens*, which "is nothing other than the existence that is given by possibility itself" (B111), i.e., "real possibility" (B302), the "synthetic *a priori* propositions" (B205), i.e., "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" can be proved through a deduction of the possibility of "possible experience," i.e., a deduction of the possibility of death itself, thereby standing, "as it deserves to, at the head of the pure and completely *a priori* laws of nature" (A184). We think that the metaphysical axioms, i.e., "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$," which stands for the "transcendental deduction" (B159) in which the "possibility as *a priori* cognitions of objects of an intuition in general was exhibited" (B159), and the "metaphysical deduction" in which "the origin of the *a priori*

categories in general was established through their complete coincidence with the universal logical functions of thinking" (B159), signifies "the three primitive ideas in Peano's arithmetic…0, number, successor,…" (RUSSELL 1971: 1-10). Furthermore, we have to say that it belongs among "universal cognitions *a priori*" (A300) or "a cognition from a principle" (B357). We are in agreement with Russell, who says, "It is a principle, in all formal reasoning, to generalize to the utmost, since we thereby secure that a given process of deduction shall have more widely applicable results;…And in thus generalizing we have, in effect, created a set of new deductive systems, in which traditional arithmetic is at once dissolved and enlarged;…" (RUSSELL 1971: 194-206). What does "a set of new deductive systems" mean? It means the discovery of the so-called imaginary number $i^2 = -1$ – "laws of the laws of nature" (FREGE 1980: 99-119). Once having found that the cardinality, i.e., "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" signifies not only "necessity" but universality, we have to seek another cardinality – "system of infinitely many real numbers, i.e., every assemblage of numbers (or points)" (HILBERT 1902: 437-479) – which is "equivalent to the assemblage of natural integers, 1, 2, 3…" (HILBERT 1902: 437-479). Another cardinality is deemed to be "equivalent or of equal *cardinal number*, if they can be brought into a relation to one another such that to every number of the one assemblage corresponds one and only one definite number of the other" (HILBERT 1902: 437-479). Since the "equal cardinal number," if thought by means of "transfinite induction" (RUSSELL 1971: 117-130), corresponds to $e^{i\pi} + 1 = 0$, we have to say that the cardinality, "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" is equivalent to "$0 \rightarrow 1 \rightarrow e^{i\pi} = -1 \rightarrow 0$" or "$0 \rightarrow e^{i\pi} = -1 \rightarrow 0$." This issue will be discussed later.

In regard to the cause and the causality, when Kant says that "the schema of the cause and of the causality of a thing in general is the real upon which, whenever it is posited, something else always follows. It therefore consists in the succession of the manifold insofar as it is subject to a rule" (A144), we entirely agree with him. Seeing that the cause and the causality is to be commensurate with a rule to which the succession of the manifold is subject, we think that there is no causality other than categories, which let loose the "temporal sequence" (A553/B581): that a thing that appears in filled space-elapsing time is to disappear in nullity in space-time. Furthermore, we have to think that the schema of the cause and of the causality of a thing – "the real" (A144) – is tantamount to the "temporal sequence" in such a way that a thing that appears in filled space-elapsing time is to disappear in nullity in space-time. In regard to the issue of "the real" and human's cognizing it, Kant says, "It is possible experience alone that can give our concepts reality; without it, every concept is only an idea, without truth and reference to an object. Hence the possible empirical concept was the standard by which it had to be judged whether the idea is a mere idea and a thought-entity or instead encounters its object within the world" (A489/B517). We are absolutely requested to search for "possible experience" as "the real" which is to assign an "object within the world" to the "possible empirical concept." It must be thought of as the "truth and reference to an object" on the grounds that a human, endowed with "three subjective sources of cognition: sense, imagination, and apperception" (A115), is to necessarily encounter its object within the world or is determined to necessarily encounter its object in "possible experience." The only thing that meets all these demands: "possible experience as the real," "truth and reference to an object," "necessarily encounter its object within the world," "determined to necessarily encounter its object in possible experience," "possible empirical concept" is death itself, namely the disappearance of a human that appears (YAMAMOTO 2016: 87-100). It signifies "*a priori* principles of the possibility of experience" (B294) and "all synthetic *a priori* propositions" (B294), determining their "possibility itself" (B294) as the real. Death itself is the "necessity" – "nothing other than the existence that is given by possibility itself" (B111) – in regard to *Homo sapiens*. We say that since "the limitation of a judgment" (A27/B43) is added to "the concept of the subject, then the judgment is unconditionally valid" (A27/B43). When we acknowledge that "the real" is to pertain to "all reality in perception" (B214) accompanied with "its degree that can decrease to nothing (emptiness) through infinite steps" (B214) and the "temporal sequence in such a way that a thing that appears in filled space-elapsing time is to disappear in nullity in space-time," the numbers; 0, $i^2 = -1$, and 1, which coincide with "the real" in regard to the categories, are regarded to be "real numbers" (YAMAMOTO 2017b: 72-81). We, who stand under metaphysical axioms, would say that the cardinality, i.e., "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" signifies the "nondenumerability of the real numbers" (DAUBEN 1990: 47-76) or "the transfinite cardinal numbers" (DAUBEN 1990: 149-168), suggesting that the cardinal numbers; 0, $i^2 = -1$, and 1 would be "the smallest of infinite

cardinals" (RUSSELL 1971: 77-88) – "Aleph" or $\aleph_0$.

3. Numbers, Magnitudes, One Homogeneous Unit and Reality

Contrary to the cardinal numbers; 0, $i^2 = -1$, and 1, the natural or real numbers in mathematics are regarded as the "imaginary numbers" (YAMAMOTO 2017b: 72-81, YAMAMOTO 2017c: 57-70, YAMAMOTO 2017d: 19-29). Why? We have to listen to how Kant elaborates on numbers in the *Critique of Pure Reason*, which says, "The pure image of all magnitudes (*quantorum*) for outer sense is space; for all objects of the senses in general, it is time. The pure schema of magnitude (*quantitatis*), however, as a concept of the understanding, is number, which is a representation that summarizes the successive addition of one (homogeneous) unit to another. Thus number is nothing other than the unity of the synthesis of the manifold of a homogeneous intuition in general, because I generate time itself in the apprehension of the intuition" (B182-A143). Thus, Kant asserts that number in mathematics is "a concept of the understanding" in virtue of the "pure schema of magnitude (*quantitatis*)," which signifies "the unity of the synthesis of the manifold of a homogeneous intuition" – "a representation that summarizes the successive addition of one (homogeneous) unit to another." What does "the manifold of a homogeneous intuition" mean? We will clarify it here. It is impossible for a human to cognize "the manifold of a homogeneous intuition" upon encountering a thing which appears in filled space-elapsing time – "the manifold that is given in a sensible intuition" (B143). On the contrary, the "decomposition" (A525/B553) of the unity of the synthesis of "the manifold that is given in a sensible intuition" is to be encountered in all sensible intuitions in such a way that it is either "experience itself," or "real possibility" (B302). Here, "real possibility" is meant to be the "necessity," i.e., "the existence that is given by possibility itself" (B111). The "decomposition" of "the manifold that is given in a sensible intuition" corresponds to nullity in space-time – "the manifold of pure intuition" (B104-A79) – in regard to *Homo sapiens*. We have to take note of the fact that 1) "the manifold of pure intuition," i.e., nullity in space-time, which is to be homogeneous, constitutes a "true correlate" (A30) with "empirical (indeed in general sensible) intuitions" (A137/B176) which appear to be un-homogeneous, 2) the former signifies "*quantum continuum*" (A527/B555), while the latter signifies "*quantum discretum*" (A526/B554), 3) death itself, namely the disappearance of a human that appears, is to be called *phaenomena*, "to the extent that as objects they are thought in accordance with the unity of the categories" (B305-A249), 4) the "pure concept of the understanding" (A143) in regard to death itself rests on the "possibility of experience" (A156) or "possible experience" (A489/B517), while the "pure concepts of the understanding" (B166) other than death itself rest on "possibility itself" (B111) or "possibility *a priori*" (A222), 5) since death itself permeates "empirical (indeed in general sensible) intuitions," the "pure concept of the understanding" – the category – is to arise through "the logical functions for judgment" (B143), i.e., the "very functions for judging, insofar as the manifold of a given intuition is determined with regard to them" (B143). From which it follows that we can say that, in regard to "objects of experience" (B258), the categories other than death itself could arise through "the logical functions for judgment," i.e., the "very functions for judging, insofar as the manifold of a given intuition is determined" with regard to the "possibility itself" (B111) or "possibility *a priori*" (A222) – the intensive magnitude in appearance or "the intensive magnitude in different appearances" (A173). Humans are to perceive "the manifold of a homogeneous intuition" only upon encountering nullity in space-time – death itself – in possible experience, insofar as this "possible experience" is "the real." How does that "possible experience is the real" can happen among them? When the "possibility of experience" or "possible experience" is enhanced to "real possibility" (B302), i.e., "necessity" – "the existence that is given by possibility itself" (B111) – by means of "a mathematical synthesis" (B221), the "possibility of experience" or "possible experience" becomes correspondent to "experience itself" (A222) in the "converse domain" (RUSSELL 1971: 52-62) or "an analogy of experience" (A180) in "the domain." Here, we define "the domain" as "the physical world" and the "converse domain" as "Platonic mathematical world" (PENROSE 2007: 7-24, 1010-1047). In contrast to "the manifold of a homogeneous intuition," "all objects of perception" (B208), i.e., "objects of experience" are ascribed to an intensive magnitude, i.e., a degree of influence on sense" (B208) – "sensation in itself" (B208). When "sensation in itself" corresponds to the intensive magnitude in appearance or "the intensive magnitude in different appearances" (A173) – "possibility itself" or "possibility *a priori*" – "all objects of perception," i.e., "objects of experience" could arise through "the logical functions for judgment," i.e., the "very functions for judging, insofar as

"sensation in itself" is determined with regard to the "possibility itself" or "possibility *a priori*." Therefore, we would say that "all objects of perception," i.e., "objects of experience" are called *phaenomena*, "to the extent that as objects they are thought in accordance with the unity of the categories."

When Kant makes a discourse in regard to "the schema of each category (A145), saying "the schema of each category contains and makes representable: in the case of magnitude, the generation (synthesis) of time itself, in the successive apprehension of an object" (A145), we think that the enigmatic remark indicates that 1) the "pure schema of magnitude (*quantitatis*)" (B182) – number – is commensurate with "the manifold for an empirical intuition" (B144) – a filled space-elapsing time – on account of the fact that filled space, i.e., "the pure image of all magnitudes (*quantorum*) for outer sense" (B182) cannot be separated from elapsing time, i.e., the pure image of all magnitudes (*quantorum*) "for all objects of the senses in general" (B182), 2) concurrently the "pure schema of magnitude (*quantitatis*)" – number – is to be commensurate with "a manifold in itself" (A99) – nullity in space-time – on account of the fact that space itself, i.e., the "pure image of all magnitudes (*quantorum*) for outer sense" cannot be separated from time itself, i.e., the pure image of all magnitudes (*quantorum*) "for all objects of the senses in general." Why is "the manifold for an empirical intuition" or "a manifold in itself" supposed to be commensurate with the "pure schema of magnitude" – "the pure image of all magnitudes"? Kant explains it, saying, "Every space intuited within its boundaries is such a whole, whose parts in every decomposition are in turn spaces, and it is therefore divisible to infinity. From this there also follows quite naturally the second application, to an external appearance enclosed within its boundaries (a body). Its division is grounded on the divisibility of space, which constitutes the possibility of the body as an extended whole. The latter is thus divisible to infinity, without, however, therefore consisting of infinitely many parts. To be sure, it appears that since a body has to be represented as substance in space, it is to be distinguished from a space as far as the law of the divisibility of space is concerned;…" (A524/B552-A525/B553). From our point of view, the enigmatic sayings should indicate four things; that 1) "an external appearance enclosed within its boundaries (a body)" – *quantum discretum* – is not distinguished from space-time itself – *quantum continuum* – "as far as the law of the divisibility of space is concerned," 2) while "an external appearance enclosed within its boundaries (a body)" – *quantum discretum* – appears to be divisible to infinity, the end product of the division is without consisting of infinitely many parts, indicating that it signifies *quantum continuum*, 3) thus, "an external appearance enclosed within its boundaries (a body)" – *quantum discretum* – is to signify *quantum continuum*, 4) since "every space intuited within its boundaries" merely appears to signify *quantum discretum*, it become possible for us to assign "every space intuited within its boundaries" to "pure image." From this it follows that we can say that "number – "a concept of the understanding" – is to come, in virtue of the "pure schema of magnitude (*quantitatis*)," from the synthesis of the "pure image of all magnitudes (*quantorum*) for outer sense" (B182), i.e., filled space in conjunction with the synthesis of the pure image of all magnitudes (*quantorum*) "for all objects of the senses in general" (B182), i.e., elapsing time, while number is also to come, in virtue of the "pure schema of magnitude (*quantitatis*)," from the "pure image of all magnitudes (*quantorum*) for outer sense" (B182), i.e., space itself and the pure image of all magnitudes (*quantorum*) "for all objects of the senses in general" (B182), i.e., time itself. Seeing that number holds biphasic properties in regard to the "pure schema of magnitude (*quantitatis*)," we say that it signifies either "the unity of the synthesis of the manifold of a homogeneous intuition" (A143) – *quantum discretum* – or "the manifold of a homogeneous intuition" – *quantum continuum*. Since both "the unity of the synthesis" and "the manifold of a homogeneous intuition" arise through cognizing space-time itself – death itself – in "the apprehension of the intuition" (A143), they signify "one consciousness" (A103), or "one experience" (A108) or "one reality" (B329). Now, it has been found that the "unity of the manifold in a subject" (A116) is nothing but "synthetic *a priori* unity" (A217). Therefore we can say that "a principle of the synthetic unity of the manifold in all possible intuition" (A117) which "pure apperception, i.e.., the thoroughgoing identity of oneself in all possible representations" (A116) yields – *quantum discretum* – is correspondent to "the transcendental principle of the unity of all the manifold of our representations" (A116) – *quantum continuum*. Here, we dare to say that 1) "one consciousness," or "one experience," or "one reality" is homogeneous with "one representation" (A99), 2) since "the unity of the synthesis of the manifold of a homogeneous intuition in general" (A143) arises as a result of cognizing space-time itself – death itself – in "the

apprehension of the intuition" as "a representation," "one representation" is commensurate with "the representation I think" (B132) – a human or humans – which is supposed to be "a new bridge between the countable assemblage and the continuum" (HILBERT 1902: 437-479), namely $i^2 = -1$, 3) this representation "summarizes the successive addition of one (homogeneous) unit to another," thereby producing the natural numbers in mathematics, which "are thought in accordance with the unity of the categories" (B305-A149). When the "pure schema of magnitude (*quantitatis*), i.e., "0" or "1," is assigned to "a representation" for "the successive addition of one (homogeneous) unit to another" (B182), "the representation I think" performs it. Since "one (homogeneous) unit" signifies "allness (totality)" or "a unity" or "a whole" (A524/B552), which is correspondent to "the pure image of all magnitudes (*quantorum*)," which can be thought to signify either "amount" (B215) of magnitude in filled space-elapsing time – the intensive magnitude – or "volume" (B215) of magnitude in nullity in space-time – the extensive magnitude – we have to say that "a representation" is counting "how-many-times" (B300) a part of "the intensive magnitude" has disappeared in the domain, which corresponds to how-many-times "the extensive magnitude" has appeared in the converse domain when it is summarizing the successive addition of one (homogeneous) unit to another.

In regard to the issue of number, when Kant says, "The infinite division indicates only the appearance as *quantum continuum*, and is inseparable from the filling of space; for the ground of its infinite divisibility lies precisely in that. But as soon as something is assumed as a *quantum discretum*, the multiplicity of units in it is determined; hence it is always equal to a number" (A527/B555), we have already said that "Is the former number – 'a concept of the understanding' – which is supposed to be equivalent to 'the pure schema of magnitude (*quantitatis*)' different from the latter, which is supposed to be equivalent to 'the multiplicity of units' in something assumed as a *quantum discretum*? No, they are not different, but the same. The former number – 'a representation that summarizes the successive addition of one (homogeneous) unit to another' – signifies *quantum continuum* in terms of nullity in space-time, while the latter number, which signifies *quantum continuum*, appears to be *quantum discretum* under the assumption that a thing signifies *quantum discretum*. In Kant's metaphysics, number – *quantum continuum* – is homogeneous with 'the appearance as *quantum continuum*' (A527/B555). However, when it is under the aegis of Kant's assumption, number is transformed into one in which the multiplicity of units is determined" (YAMAMOTO 2017a: 19-37). Here we should like to add, saying that 1) "Kant's assumption" is nothing but mathematicians' assumption, 2) Plato's "Allegory of the Cave" (The Republic 514-516[2]) has been raised for the purpose of coping with the issue of number, 3) number – "pure schema of magnitude (*quantitatis*)" – holds biphasic properties on account of the fact that "the intensive magnitude in different appearances can be smaller or greater even though the extensive magnitude of the intuition remains identical" (A173), indicating that the intensive magnitude arises, as *quantum discretum*, through the apprehension of the magnitude in regard to "sensation in itself," in which "the empirical consciousness can grow in a certain time from nothing = 0 to its given measure" (B208), i.e., from the "mere perception (sensation and thus reality)" (B212) of filled space-elapsing time while the extensive magnitude arises, as *quantum continuum*, through "a synthesis of the generation of the magnitude of a sensation" (B208), which begins in "the pure intuition = o" and can expand to "any arbitrary magnitude" (B208), i.e., from the synthesis of the "pure image of all magnitudes (*quantorum*)" in virtue of space-time itself. From this, it follows that it becomes possible to say that the intensive magnitude – *quantum discretum* – is "already intuited as aggregates (multitudes of antecedently given parts)" (B204) while the extensive magnitude – *quantum continuum* – is "represented and apprehended by us as extensive" (B204). We would say that the intensive magnitude in the domain, which is "already intuited as aggregates (multitudes of antecedently given parts)," signifies, in the domain, nothing but "the manifold that is given in a sensible intuition (B143) – "alterability" (B213) or "the limits of created beings, i.e., negations" (B329) – while the extensive magnitude in the domain, which is "represented and apprehended by us as extensive" (B204), signifies, in the domain, "the manifold of sensible representation (intuition)" (A129) or "the manifold of given representations" (B143) – the categories. Here, the categories are to be homogeneous with "aggregates (multitudes of antecedently given parts)." Furthermore, we say that the intensive magnitude in the domain, which is "already intuited as aggregates (multitudes of

[2] 'The Republic 514-516' designates Stephanus pagination 514-516 of *The Republic* (Plato, *The Republic Books VI-X*, Harvard University Press, 1935).

antecedently given parts)," can be thought to signify, in the converse domain, nothing but "the manifold of sensible representation (intuition)" or "the manifold of given representations" – "continuous magnitudes" (A171) – while the extensive magnitude in the domain – "continuous magnitudes" – can be thought to signify, in the converse domain, "the manifold that is given in a sensible intuition (B143) – "alterability" or "the limits of created beings, i.e., negations," i.e., the intensive magnitude. On the contrary, the intensive magnitude in the converse domain – "alterability" or "the limits of created beings, i.e., negations" – can be "represented and apprehended by us" as "continuous magnitudes" in the domain while the extensive magnitude in the converse domain can be "already intuited as aggregates (multitudes of antecedently given parts)" in the domain. In other words, the intensive magnitude in the converse domain – "alterability" or "the limits of created beings, i.e., negations," i.e., "$0 \rightarrow 1 \rightarrow e^{i\pi} = -1 \rightarrow 0$" or "$0 \rightarrow e^{i\pi} = -1 \rightarrow 0$" – is to be "represented and apprehended by us as extensive" in the domain, while the intensive magnitude in the domain – "alterability" or "the limits of created beings, i.e., negations," i.e., "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" – is to be "represented and apprehended by us as extensive" in the converse domain, indicating a one-one relation between "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" in the domain and "$0 \rightarrow 1 \rightarrow e^{i\pi} = -1 \rightarrow 0$" or "$0 \rightarrow e^{i\pi} = -1 \rightarrow 0$" in the converse domain. Furthermore, we should like to say that number – the "pure schema of magnitude" (B182) – is to arise from the "synthetic *a priori* cognition" (A14/B28), implying that number is the "transcendental product of the imagination" (A142), which stands under "the schematism of the understanding through the transcendental synthesis of imagination" (B185). Therefore, number is to signify the category – "a rule of unity according to concepts in general, which the category expresses" (A142). Here, the transcendental synthesis of imagination is equivalent to the synthesis of the extensive magnitude in the domain and the intensive magnitude in the converse domain. We have to take note of the fact that there are two modes of converse relations here; "intensive magnitude vs. extensive magnitude" and "intuition vs. representation-apprehension." That is, if put in another way, "intensive magnitude (intuition) vs. extensive magnitude (representation-apprehension)." There must be a bridge between the two modes of converse relations, which is to reside in the field between the domain and the converse domain. What does this mean? It means that the bridge is $i^2 = -1$, which signifies the "representation I think," i.e., a human – the category. Of course, a human is endowed with intuition and "perception (sensation and thus reality)" (B212), which operate together at the same point in the same instance. This is "a system interconnected in accordance with necessary laws" (A645/B673). If intuition and "perception (sensation and thus reality)," which operate together at the same point in the same instance, i.e., in the domain and in the converse domain, cease to operate, then "the synthesis of the manifold of appearance is interrupted" (B212). Even if the transcendental synthesis of imagination ceases to operate in conjunction with the decomposition of intuition and "perception (sensation and thus reality)," "the repetition of an ever-ceasing synthesis" (B212) – the extensive magnitude in the converse domain and the intensive magnitude in the domain – would never cease to operate, generating "an aggregate of many appearances" (B212) – the categories. Therefore, we say that "in such a way there arise exactly as many pure concepts of the understanding, which apply to objects of intuition in general *a priori*" (B105). Furthermore, we should like to say that this is the "transcendental logic" (B104), which "teaches how to bring under concepts" not only "the representations but the pure synthesis of representations" (B104) since "our transcendental logic, which 'has a manifold of sensibility that lies before it *a priori*' (B102), i.e., disappearance (death), is considered to 'provide the pure concepts of the understanding with a matter' (A77)" (YAMAMOTO 2016: 87-100). Here, "a matter" is commensurate with space-time itself.

While a "reality, in contrast to negation" (B300) – a human other than me – can be thought to signify the intensive magnitude, i.e., *quantum discretum*, a "reality combined with negation" (B111) – the disappearance (death itself) in regard to a human other than me – can be thought to signify nullity in space-time – "pure image of all magnitudes (*quantorum*)." We think that when a human, in primordial times, found, through cognizing nullity in space-time upon encountering disappearance of that which looks like oneself, that a "reality, in contrast to negation" (B300) decreases to nothing (emptiness) through the successive or abrupt subtraction of the "reality, in contrast to negation," while the intensive magnitude is to "decrease to nothing (emptiness) through infinite steps" (A173), or is to come to "the extensive magnitude" through abrupt subtraction of the intensive magnitude, then "one (homogeneous) unit" – number "1" – or "the concept of a number (which belongs to the category of

allness)" (B111) arose by means of the "transcendental function of the imagination" (A123) – "productive imagination" (A123). We have already put it in another way, saying that "any imagination, even if it is spontaneous, should not be productive. Otherwise, it would cause a disaster, in which sheer illusions will be thought to be productive. Only when an imagination arises, spontaneously, in conjunction with what happens in accordance with the law of nature, it is to be productive. When such *phaenomena* that what appears never fails to disappear, is repeatedly witnessed by 'the standing and lasting I (of pure apperception)' (A123), it could precipitate a productive imagination in it, thereby enabling it to arise as 'the representation I think' (B132)" (YAMAMOTO 2016: 87-100). Therefore, the number "1" – "a representation that summarizes the successive addition of one (homogeneous) unit to another" (B182) – is regarded as the "possible empirical concept" (A489/B517). We also think that "the category of allness" (B111) signifies nothing but "one (homogeneous) unit" (B182) – number "1" – which could signify either the extensive magnitude ("unbounded reality" (B322), i.e., empty space-nullified time) arising from space-time itself, i.e., "the matter of all possibility" (B322) or the intensive magnitude (bounded reality, i.e., a filled space-elapsing time) arising from the synthesis of filled space-elapsing time, "under the condition that it is possible for 'the standing and lasting I (of pure apperception)' (A123) or 'consciousness of itself (apperception)' (B68) to conjure up *a priori* the limitation for space-time itself" (YAMAMOTO 2017d: 19-29). Furthermore, we should like to put it in another way, saying that the number "1," as "the concept of a number (which belongs to the category of allness)," is thought to signify *quantum continuum*, which is commensurate with the extensive magnitude ("unbounded reality," i.e., empty space-nullified time) or *quantum discretum*, which is commensurate with the intensive magnitude (bounded reality, i.e., a filled space-elapsing time). From this, it follows that when the number "1" – "one (homogeneous) unit" – is enhanced to the "pure schema of magnitude (*quantitatis*)" (B182) – "a representation that summarizes the successive addition of one (homogeneous) unit to another" (B182) – it becomes possible to say "how many units are posited in it" (B300) on account of the fact that "a representation" – the "pure schema of magnitude (*quantitatis*)" – is "the determination of a thing through which it can be thought how many units are posited in it" (B300). In other words, "how many units are posited in it" is commensurate with "how-many-times" (B300) a representation "summarizes the successive addition of one (homogeneous) unit to another." Here, we have to stress that insofar as the "pure schema of magnitude (*quantitatis*)" remains an epistemological naught, "one (homogeneous) unit" or "the successive addition of one (homogeneous) unit to another" does not become comprehensible. We have already indicated above that "one (homogeneous) unit" – the number "1" – is correspondent to "the pure image of all magnitudes (*quantorum*)," i.e., either a pure image of a human in filled space-elapsing time – *quantum discretum* – or a pure image of a deceased in nullity in space-time – *quantum continuum.* Therefore, we say that 1) since the product of "the successive addition of one (homogeneous) unit to another" is equivalent to nullity in space-time – "one complete whole" (A676/B704), i.e., *quantum continuum* – the product of the successive addition of "one complete whole" to space-time itself is the same as the number "1," 2) since number "1" is commensurate with "a representation that summarizes the successive addition of one (homogeneous) unit to another" (B182), number "2" is also a representation that summarizes the successive addition of one (homogeneous) unit to another, and so on. We can repeat it infinitely since the product of "the successive addition of one (homogeneous) unit to another" or the successive addition of "one complete whole" to another is equivalent to nullity in space-time, i.e., space-time itself – the number "1." "Thus number is nothing other than the unity of the synthesis of the manifold of a homogeneous intuition in general" (B182-A143). Since a number as a unity is "allness (totality)" – "one (homogeneous) unit" (B182) or "one complete whole" in virtue of "pure schema of magnitude (*quantitatis*)" (B182) – number, i.e., "a representation that summarizes the successive addition of one (homogeneous) unit to another" is thought to signify the category as nullity in space-time or the category as a filled space-elapsing time. We should like to put it in another way, saying that number – the unity of the synthesis of the manifold of a homogeneous intuition in general" – signifies "pure *a priori* concepts" (A95) or "the transcendental schema" (A138/B177) – "*notio*" (B377) – which does not go "beyond the possibility of experience" (B377). Therefore, we say that number also signifies the "possible empirical concept." If nullity in space-time, as the "possible experience" of death itself, is presented to a human – to "the representation I think" (B132) – several times in the domain with the same or different "inner determinations of a *substantia phaenomenon*"

(B321), it is always the same since "it counts as an object of pure understanding, not many but only one thing (*numerica identitas*)" (B319) – "pure *a priori* concepts" or "the transcendental schema." If an object is presented to a human – to "the representation I think" – many times with the same or different "inner determinations of a *substantia phaenomenon*," it is not the same since it counts, as "objects of sensibility" (B320) or "objects given to the mere understanding"(A500), not one thing (*numerica identitas*)" but "an aggregate of many appearances" (B212). Since an object is "the appearance itself" (B53) – a filled space-elapsing time – "the issue is not the comparison of concepts, but rather, however identical everything may be in regard to that, the difference of the places of these appearances at different times is still an adequate ground for the numerical difference of the object (of the senses) itself" (B319). Therefore, we should like to say that 1) since an object – "one thing (*numerica identitas*)" – signifies the difference of the places of the appearance at the same time, it signifies "an aggregate of many appearances," i.e., an aggregate of many "one thing (*numerica identitas*)," 2) since an object – "one thing (*numerica identitas*)" – signifies the difference in the time of the appearance at the same place, it signifies an appearance – "a series" (B389) of "one thing (*numerica identitas*)" – 3) since an object – "one thing (*numerica identitas*)" – signifies the difference in the time of the appearance at different places, it signifies an aggregate of many "one thing (*numerica identitas*)," and a series of "one thing (*numerica identitas*)," 4) since an object – "one thing (*numerica identitas*)" – signifies no difference in the time of the appearance at the same place, it is commensurate with "one thing (*numerica identitas*)" – the category – 5) since "an object of pure understanding" (B319) – "one thing (*numerica identitas*)" – signifies nullity in space-time, it belongs among the category – "one thing (*numerica identitas*)." Since an object – "one thing (*numerica identitas*)" – in the domain signifies "the manifold that is given in a sensible intuition (B143) – "alterability" or "the limits of created beings, i.e., negations" – in the domain, the cardinality in the domain is to be "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$." Furthermore, since "an object of pure understanding" in the domain – nullity in space-time or "continuous magnitude" – is to be thought to signify "the manifold that is given in a sensible intuition" – "alterability" or "the limits of created beings, i.e., negations" – in the converse domain, the cardinality in the converse domain is to be "$0 \rightarrow 1 \rightarrow e^{i\pi} = -1 \rightarrow 0$" or "$0 \rightarrow e^{i\pi} = -1 \rightarrow 0$." We have to take note of the fact that 1) the numbers e and π are to be composed of natural "number (which belongs to the category of allness)" (B111) by means of "the function of the categorical judgment" (B128) – numerical formulas – 2) the cardinality in the domain; "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" or that in the converse domain; "$0 \rightarrow 1 \rightarrow e^{i\pi} = -1 \rightarrow 0$" or "$0 \rightarrow e^{i\pi} = -1 \rightarrow 0$," is thought to signify "the schema of necessity," i.e., "the existence of an object at all times" (A145), 3) $i^2 = -1$ signifies "the concepts that give this pure synthesis unity, and that consist solely in the representation of this necessary synthetic unity" (A79). In regard to $i^2 = -1$, we have already said, "The manifold of pure intuition, i.e., death, is 'given to us *a priori* for the cognition of all objects' according to the law of nature. When Kant says that 'empirical intuition is possible only through the pure intuition (of space and time)' (B206), whose attributes are supposed to be identical with 'geometry' (B206), we think that since the 'pure intuition (of space and time)' is already abstracted from all 'forms of sensible intuition' on account of the analogy with geometry, the pure intuition of space and time is tantamount to the intuition of empty space-nullified time. If 'the synthesis of this manifold of pure intuition' is possible 'by means of the imagination' (A79), this synthesis would be achieved through the synthesis of the manifold of space and time which are abstracted from all forms of sensible intuition, in other words, empty space-nullified time. We think that this is the 'pure synthesis of representations,' which is to take place with the unity necessary for pure synthesis, which consists 'solely in the representation of this necessary synthetic unity' (A79)" (YAMAMOTO 2016: 87-100). Here, "the representation of this necessary synthetic unity" (A79) is commensurate with "$i^2 = -1$."

In regard to the issue of "numerical difference of the object (of the senses) itself" (B319), we have already said, "Instead of Kant, we have to say, 'a number – *quantum continuum* – is homogeneous with the appearance itself, and appears to be homogeneous with the multiplicity, insofar as thing in itself – space-time itself – appears as *quantum discretum*.' We comprehend, through empirical intuition and synthesis – 'the apprehension of the intuition' (A143) – that upon the entire cessation of movement of the standing and lasting I, number and magnitude emerges as *quantum continuum*, while during its movement, number and magnitude appear as *quantum discretum*. Therefore, it can be said

that number and magnitude enables the standing and lasting I of pure apperception to generate time itself in *quantum continuum* and determines 'elapsing time' as *quantum discretum*. In other words, time itself – 'the absolute whole of magnitude (the world-whole)' (A483) – emerges from number and magnitude as *quantum continuum* which is homogeneous with number and magnitude of 'the standing and lasting Is,' which existed or exist or will exist while a time – 'every determinate magnitude of time' (A32-B48) – emerges from number and magnitude as *quantum discretum*, which is homogeneous with number and magnitude of the standing and lasting Is, which exist. In view of this, our discourse can be said to agree with Kant's metaphysics, which says, 'Every intuition contains a manifold in itself, which however would not be represented as such if the mind did not distinguish the time in the succession of impressions on one another; for as contained in one moment no representation can ever be anything other than absolute unity' (A99). Our metaphysics, in conformity with Kant's, says that 1) one moment in which appearance disappears signifies the representation of 'a manifold in itself,' which is equivalent to 'absolute unity' – nullity in time: 2) intuition is a manifold in itself – absolute unity – insofar as the mind distinguishes nullity in time in one moment 'in the succession of impressions on one another'" (YAMAMOTO 2017a: 19-37). What does "the entire cessation of movement of the standing and lasting I" mean? It means "the entire cessation of movement of the standing and lasting I" other than me, i.e., another I's death itself. In this regard, we can say that 1) the number "1" signifies another I's death itself while the number "0" signifies "possible experience" or the "possibility of experience" of death itself for myself, 2) "a new bridge between the countable assemblage and the continuum," namely $i^2 = -1$ is the true bridge between "1" and "0," i.e., the "necessity," among *Homo sapiens*, which "is nothing other than the existence that is given by possibility itself" – "real possibility" – 3) therefore, once the cardinal numbers; 0, $i^2 = -1$, and 1 are discovered, the cardinality, i.e., "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" could be reenacted forever insofar as categories appear in "the absolute whole of magnitude (the world-whole)" (A483).

In relation to this issue, we have already said, "Accordingly, Kant says, 'All appearances whatsoever are accordingly continuous magnitudes, either in their intuition, as extensive magnitudes, or in their mere perception (sensation and thus reality), as intensive ones. If the synthesis of the manifold of appearance is interrupted, then it is an aggregate of many appearances, and not really appearance as a quantum, which is not generated through the mere continuation of productive synthesis of a certain kind, but through the repetition of an ever-ceasing synthesis' (B212). This remark seems to indicate that 1) all appearances themselves which have continuous magnitudes, pertain to sensibility and signify reality, 2) appearance as a quantum is generated through "the mere continuation of productive synthesis of a certain kind," 3) an aggregate of many appearances is not a quantum since it is generated through the repetition of an ever-ceasing synthesis, 4) magnitudes are to be made into a quantum by means of mere continuation of productive synthesis of a certain kind – a device with which one 'abstracts from everything empirical in the appearances' (A96)" (YAMAMOTO 2017a: 19-37). This remark, in conjunction with the discourse so far, exposes "the transcendental ground of this unity, undoubtedly lie too deeply hidden for us, who know even ourselves only through inner sense, thus as appearance, to be able to use such an unsuitable tool of investigation to find out anything except always more appearances, even though we would gladly investigate their non-sensible cause" (B334). What does it mean? What does "the mystery of the origin of our sensibility" (B334) mean? It means that 1) a living thing, which found "more appearances," i.e., death itself, has come out as a human, 2) when it cognized "continuous magnitudes," "the mere continuation of productive synthesis" began, identifying itself as "a representation," 3) the identity – "unity" or "one representation" – has separated it from the living things other than the unity, i.e., animals which cannot go beyond "the repetition of an ever-ceasing synthesis."

4. Notions, Categories, Numbers, and Numerical Formulas

When "Leibniz took the appearances for things in themselves," "his principle of non-discernibility (*principium identitatis indiscernibilium*)" (B320) could be disputed. Though "they are objects of sensibility, and the understanding with regard to them is not of pure but of empirical use" (B320), "multiplicity and numerical difference" are not already given by filled space-elapsing time "as the condition of outer appearances" (B320). Multiplicity and numerical difference could arise, as the "propositions of numerical relation" (B205), by means of "the function that corresponds to inner sense

(to a receptivity)" (B185). It is clear that, first, Leibniz and mathematicians need "the self-evident propositions of numerical relation" (B205) – the product of "the function that corresponds to inner sense." What are the self-evident propositions of numerical relation? They are the cardinality, i.e., "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0" in the domain or the cardinality, i.e., "0 → 1 → $e^{i\pi}$ = -1 → 0" or "0 → $e^{i\pi}$ = -1 → 0" in the converse domain, which is commensurate with "pure *a priori* concepts" (A95) or "the transcendental schema" (A138/B177). We think that "*intelligibilia*, i.e., objects of the pure understanding" (B320) do not go beyond the possibility of experience on account of the fact that they are the product of "the function that corresponds to inner sense." Therefore, we should say, in opposition to what Kant refers to (B320), that "parts of filled space-elapsing time, even though they appear to be completely dissimilar and unequal to another, are nevertheless inside the condition of outer appearances, and are on that account the same parts as that which is added to them in order to constitute larger parts of filled space-elapsing time, and this must therefore hold of everything that exists simultaneously in the various positions in filled space-elapsing time, no matter how dissimilar and unequal they appear to be." How and where does the synthesis of parts of filled space-elapsing time in order to constitute larger parts of filled space-elapsing time take place? It must take place in the converse domain by means of "the function that corresponds to inner sense," implying that the larger parts of filled space-elapsing time correspond to "a transcendental product of the imagination, which concerns the determination of the inner sense in general" (A142), in accordance with "filled" space and "elapsing" time "in regard to all representations, insofar as these are to be connected together *a priori* in one concept in accord with the unity of apperception" (A142). Kant's discourse in such a way that 1) "the schema of a pure concept of the understanding" (A142) is "the pure synthesis, in accord with a rule of unity" (A142), 2) "the schema of a pure concept of the understanding" – "a transcendental product of the imagination" (A142) – come out by means of "pure synthesis" as "*intelligibilia*, i.e., objects of the pure understanding" (B320), is tautology. When Kant tries to produce "a rule of unity according to concepts in general, which the category expresses" "merely regulatively" (A180), with the aid of "a pure imagination" (A124), it is wrong. We have already criticized this way of thinking, saying that "any imagination, even if it is spontaneous, should not be productive. Otherwise, it would cause a disaster, in which sheer illusions will be thought to be productive. Only when an imagination arises, spontaneously, in conjunction with what happens in accordance with the law of nature, it is to be productive." (YAMAMOTO 2016: 87-100). Therefore, "the schemata of the concepts of pure understanding" (B185-A146) must be "the true and sole conditions" (A146) for providing "the function that corresponds to inner sense" with a relation to objects. We, while agreeing with Kant who suggests that "the schema of a pure concept of the understanding" (A142) is "the pure synthesis," have to think that "a rule of unity" for "the pure synthesis" must be "nullity in space-time," i.e., space-time itself in virtue of the "possible empirical concept."

The product of "the function that corresponds to inner sense" is called "a concept of reason" (B377), which does not go beyond the possibility of experience, since we think that "a concept of reason" has "its origin in the understanding and in a pure image of sensibility." We say that the number "0" or "1" belongs among "a concept of reason" in virtue of the product of "the function that corresponds to inner sense." Since the product of "the function that corresponds to inner sense" belongs among "the possible empirical concept" (A489/B517), "the schema of a pure concept of the understanding" (A142) – the schema of the possible empirical concept – can be thought to correspond to "the pure synthesis." Seeing that "the pure synthesis" is "in accord with a rule of unity according to concepts in general, which the category expresses" (A142), it becomes possible to say that "the pure synthesis" and the product of "the function that corresponds to inner sense" – "the possible empirical concept" (A489/B517) – also belong among the categories. Since "a transcendental product of the imagination" – number – is thought to correspond to the product of "the transcendental synthesis of imagination" (B185) – the categories – we think that number, as the product of "the transcendental synthesis of imagination," signifies three things; 1) "the natural numbers in mathematics," 2) the cardinality in the converse domain, i.e., "0 → 1 → $e^{i\pi}$ = -1 → 0" or "0 → $e^{i\pi}$ = -1 → 0," 3) a one-one relation between "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0" in the domain and "0 → 1 → $e^{i\pi}$ = -1 → 0" or "0 → $e^{i\pi}$ = -1 → 0" in the converse domain. Since the cardinality, which signifies "the manifold that is given in a sensible intuition (B143), i.e., "alterability" or "the limits of created beings, i.e., negations," is correspondent with "the multiplicative axiom" (RUSSELL 1971: 117-130), they pertain to the

causality of "the manifold of nature" (B163). We think that this enables us to comprehend Russell's enigmatic remarks, which say, "For purposes of defining multiplication, it is the selectors we require rather than the selections. Thus we define: 'The product of the numbers of the members of a class of classes *k*' is the number of selectors from *k*" (RUSSELL 1971: 117-130). "The product of the numbers of the members of a class of classes *k*" is equivalent to "$e^{i\pi} = -1$" – the categories – while "the number of the selectors from *k*" is nothing but "$i^2 = -1$" – the category. In contrast with "the function that corresponds to inner sense," "a function of the power of judgment" (B304) pertains to the "concept of reason" which could go beyond the possibility of experience if "it has its origin solely in the understanding (not in a pure image of sensibility)" (B377). We think that "the natural numbers in mathematics" belong among "a concept of reason" in virtue of the product of "a function of the power of judgment" in conjunction with "the function that corresponds to inner sense." If the natural numbers in mathematics – the imaginary numbers in our metaphysics or "inductive numbers" in Russell's mathematical philosophy (RUSSELL 1971: 20-28) – can be "thought in accordance with the unity of the categories" (B305-A249), they are to be enhanced to the real numbers. Then, the "transcendental product of the imagination" (A142) is to be "in accord with a rule of unity according to concepts in general, which the category expresses" (A142), making it possible for us to say that the real numbers such as the natural numbers, i.e., "inductive numbers" and $i^2 = -1$ pertain to the causality of "the manifold of nature" (B163) – the categories.

We have said above that "appearances themselves (e.g., humans) must be subsumed under the category, but only under the schemata of disappearance of 'the things themselves, which appear' (B324)." From this, it follows that the real numbers, which signify the real, i.e., "appearances themselves," are to be subsumed under the category, i.e., under the schemata of disappearance of "the things themselves, which appear" – the schema of nullity in space-time, i.e., space-time itself. What are the real numbers that are subsumed under the category? They are nothing but the cardinality; "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$." The cardinality can "have as their goal nothing but the conditions of the unity of empirical cognition in the synthesis of the appearances" (A181-B224) since "every reality has its degree that can decrease to nothing (emptiness) through infinite steps" (B214-A173). There are two modes of reality in regard to appearances themselves: "realities in appearance (*realitas phaenomenon*)" (A265) which "can certainly be in opposition with each other" (B321), and "reality" which is "represented only through the pure understanding (*realitas noumenon*)" (B320). The former, united in the same subject, "can partly or wholly destroy the consequence of the other" (B321), while as for the latter, since "no opposition between realities can be thought, i.e., a relation such that when they are bound together in one subject they cancel out their consequences, as in 3 – 3 = 0" (B320-A265). Therefore, it is not farfetched to assume that the cardinal numbers; 0, $i^2 = -1$, and 1 pertain to the "realities in appearance (*realitas phaenomenon*)" (A265), while the natural numbers pertain to "the pure understanding (*realitas noumenon*)" (B320). "Since every reality has its degree that can decrease to nothing (emptiness) through infinite steps while the extensive magnitude of the appearance remains unaltered" (B214-A173), the natural numbers can be thought to pertain to "the extensive magnitude of the appearance." Furthermore, since reality "must yield infinitely different degrees with which space or time is filled, and the intensive magnitude in different appearances can be smaller or greater even though the extensive magnitude of the intuition remains identical' (A173), the cardinality; "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" can be thought to pertain to "the intensive magnitude in different appearances" or the intensive magnitude in appearance.

When Kant makes a discourse in regard to the natural numbers and numerical formulas (B205), we have to say, in an opposite manner, that "the propositions of numerical relation in regard to 'the synthesis of that which is homogeneous (of units)' – are analytic and general, and for that reason can be called axioms, and could be named numerical formulas. That 7 + 5 = 12 is an analytic proposition. For we do not think the number 12 either in the representation of 7 or in that of 5 or in the representation of the combination of the two (that we ought not to think this in the addition of the two is here at issue; in the case of an analytic proposition the question is whether we actually think the predicate in the representation of the subject)." Furthermore, we have to say, in opposition to what Kant says (B205-A165), that "since it is analytic, it is not a singular proposition. Insofar as it is the synthesis of that which is homogeneous (of units) that is at issue here, the synthesis can take place in a single way. Consequently, the subsequent use of the pure concept of the understanding, i.e., "synthetic

a priori propositions" – the use of axioms – is general among humans." Why is "the synthesis of that which is homogeneous (of units)" at issue here? It is because Kant believes that "that equals added to or subtracted from equals give an equal are analytic propositions, since I am immediately conscious of the identity of one generation of a magnitude with the other" (A164-B205). We think that this remark indicates three things; 1) "concerning magnitude (*quantitas*), i.e., the answer to the question 'How big is something?'" (B204), there must be axioms in the proper sense, since these propositions are analytic and "immediately certain (*indemonstrabilia*)" (A164), 2) in other words, the self-evident propositions – "that equals added to or subtracted from equals give an equal" in regard to "the synthesis of that which is homogeneous (of units)" – are the "synthetic *a priori* propositions" (B294), 3) the "synthesis of things not homogeneous" (A530/B558) can be thought to correspond to "the synthesis of that which is homogeneous (of units)" when the "synthesis" is commensurate with "the identity of one generation of a magnitude with the other" (B205). We have to say that the axioms – synthetic *a priori* propositions – are equivalent to "the identity of one generation of a magnitude with the other" (B205) or "the conditions of the unity of empirical cognition in the synthesis of the appearances" (A181-B224). In other words, the axioms signify "the dynamical synthesis, in causal connection as well as in the connection of the necessity with the contingent" (A530/B558). Since "these conditions are thought only in the schema of the pure concept of the understanding" (B224), the "conditions of the unity of empirical cognition," i.e., "the identity of one generation of a magnitude with the other" – a human – is thought to contain the function, restricted or unrestricted by a "sensible condition, of their unity, as of a synthesis in general" (B224). From this it follows that it becomes possible for us to say that Euclid's "Common Notions" (EUCLID 2002:1-36) "cannot be called axioms, but could rather be named numerical formulas" (B205) on account of the fact that they seem to have grounded on "the synthesis of that which is homogeneous (of units)" carried out by the "unity of empirical cognition," which "contains the function, unrestricted by any sensible condition, of their unity, as of a synthesis in general" (B224). However, if Euclid's "Common Notions" (EUCLID 2002:1-36) were grounded on "the synthesis of that which is homogeneous (of units)" carried out by the "conditions of the unity of empirical cognition," which is restricted by a "sensible condition, of their unity, as of a synthesis in general" (B224), they can be called axioms.

Here we have to take note of the fact that there are two modes of "Common Notions" (YAMAMOTO 2017c: 57-70).

The first edicts:

1. Since there is nullity in space-time which is equal to nullity in space-time in the world-whole, a thing – nullity in space-time – which is equal to the same thing, is equal to one another,
2. Since there is nullity in space-time which is equal to nullity in space-time in the world-whole, if equal – nullity in space-time – is added to equal, i.e., nullity in space-time, the wholes are equal.
3. Since there is nullity in space-time which is equal to nullity in space-time in the world-whole, if equal – nullity in space-time – is subtracted from equal, i.e., nullity in space-time, the remainders are equal.
4. Since there is nullity in space-time which coincides with one another in the world-whole, things – nullity in space-time – are equal to one another.
5. Since the whole consists of filled space-elapsing time and nullity in space-time, and nullity in space-time permeates filled space-elapsing time, the whole – nullity in space-time – cannot be greater than the part – nullity in space-time (YAMAMOTO 2017c: 57-70).

The second edicts:

1. Since there is no part of filled space-elapsing time which is equal to the same part of filled space-elapsing time in the world-whole, things – parts of filled space-elapsing time – which are not equal to the same thing, are not equal to one another.
2. Since there is no part of filled space-elapsing time which is equal to another part of filled space-elapsing time in the world-whole, it is impossible for equal to be added to equal.
3. Since there is no part of filled space-elapsing time which is equal to another part of filled space-elapsing time in the world-whole, it is impossible for equal to be subtracted from equal.

4. Since there are no parts of filled space-elapsing time which coincide with one another, things – parts of filled space-elapsing time – are not equal to one another (YAMAMOTO 2017c: 57-70).
5. If the whole is assumed to consist only of filled space-elapsing time, the whole is greater than the part of filled space-elapsing time.

It seems that the first "Common Notions" are compatible with Euclid's "Common Notions" (EUCLID 2002:1-36) in the converse domain, while the second "Common Notions" are with that in the domain. At this stage of our discourse, we cannot fail to remember Plato's "Allegory of the Cave" (The Republic 514-516). When the relations between the Platonic mathematical world, the physical world, and the mental world are assumed, it was said that there were three profound mysteries between them (PENROSE 2007: 7-24, 1010-1047). From our viewpoint, the most profound mystery is; what kind of cardinal numbers are used in the Platonic mathematical world and in the physical world. The relation between the Platonic mathematical world and the mental world is not a mystery on account of the fact that since the advent of the former is subordinated to the latter, it necessarily stands under the latter, implying that the cardinal numbers used in the Platonic mathematical world are the same as those in the mental world. We think that if the Platonic mathematical world manifests as "the transcendental ideality of appearances" (A506/B534) or "synthetic *a priori* principles" (B305), i.e., "transcendental truth," it would last forever irrespective of the existence of humans on this planet (YAMAMOTO 2017c: 57-70, YAMAMOTO 2017d: 19-29). We, humans, are requested to attain to "synthetic *a priori* principles," i.e., "transcendental truth," through the "transcendental principle of the mathematics of appearances" (B206). How to do it? We have already said, "Since 'nowhere beyond the field of possible experience can there be any synthetic *a priori* principles' (A248-B305), 'all of our cognitions' (A146), which 'lie in the entirety of all possible experience' (A146), can be enhanced to 'synthetic *a priori* principles' (B305), i.e., 'transcendental truth, which precedes all empirical truth and makes it possible' (A146)" (YAMAMOTO 2017c: 57-70). Since "all of our cognitions, which can be enhanced to 'synthetic *a priori* principles,' belong among categories" (YAMAMOTO 2017d: 19-29), the category, or categories are the crucial elements through which the Platonic mathematical world would manifest as the converse domain with regard to the physical world, i.e., the domain. If "there is one-one relation whose domain is the one class and whose converse domain is the other" (RUSSELL 1971: 52-62), in other words, if there is one-one relation between the Platonic mathematical world and the physical world, we could know what kind of class the Platonic mathematical world is endowed with, since we have already clarified that the class in the physical world is the cardinality as "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$." It is not far-fetched to assume that the world inside the cave, as narrated in the "Allegory of the Cave," pertains to the Platonic mathematical world or the mental world, though the Platonic mathematical world does not completely correspond to the mental world, while the world outside the cave signifies the physical world. From the discourse we have made so far, it follows that it is possible to think that "the class" (RUSSELL 1971: 117-130) outside the cave is to pertain to the cardinality: "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$" while that inside the cave is to pertain to the natural numbers and numerical formulas – Euclid's "Common Notions" or Peano axioms – on account of the fact that "the shadows of the artificial objects" (The Republic 514-516), which the prisoners see inside the cave, are to be thought (by the people outside the cave) to arise as a result of "logical reflection" (B318) or "a mere comparison (*comparatio*)" (B318) with the physical world.

5. Plato's Allegoery of the Cave, Reality, And Numbers

Plato's Allegory of the Cave would show a way through which we can attain to the fundamental concept in regard to mathematics, and that the elucidation of the meaning of this enigmatic, bizarre allegory would enable us to comprehend the foundations of numbers and arithmetic. Plato narrates the allegory of the cave in *The Republic*, as follows; "Picture men dwelling in a sort of subterranean cavern with a long entrance open to the light on its entire width. Conceive them as having their legs and necks fettered from childhood, so that they remain in the same spot, able to look forward only, and prevented by the fetters from turning their heads. Picture further the light from a fire burning higher up and at a distance behind them, and between the fire and the prisoners and above them a road along which a low wall has been built, as the exhibitors of puppet-shows have partitions before the men themselves, above which they show the puppets…See also, then, men carrying past the wall

implements of all kinds that rise above the wall, and human images and shapes of animals as well, wrought in stone and wood and every material, some of these bearers presumably speaking and others silent…Then in every way such prisoners would deem reality to be nothing else than the shadows of the artificial objects…When one was freed from his fetters and compelled to stand up suddenly and turn his head around and walk and to lift up his eyes to the light,…at first he would most easily discern the shadows and, after that, the likenesses or reflections in water of men and other things, and later, the things themselves,…he would be able to look upon the sun itself and see its true nature, but by reflections in water or phantasms of it in an alien setting,…" (The Republic 514-516). Since the artificial objects in this allegory are thought to parallel "the objects of experience" (A158) outside the cave – "the likenesses or reflections in water of men and other things" (The Republic 514-516), it is possible for us to think that the objects of experience outside the cave pertain to the "realities in appearance (*realitas phaenomenon*)" (A265), while "the shadows of the artificial objects" pertain to the reality as "*realitas noumenon*" (B320), which is presented "through the pure understanding" (B320) of "the shadows." Here, we have to take note of the fact that the objects of experience outside the cave – "the likenesses or reflections in water of men and other things" – are thought to signify "the things themselves" – "appearance themselves." In regard to appearances themselves, we have already made the following discourse: "we think that 1) since all appearances themselves, which signify 'filled space-elapsing time' or 'empty space-nullified time,' are homogeneous with space-time itself, they are quantum and have continuous magnitude, 2) therefore, an aggregate of many appearances themselves, which is already quantum, is generated 'through the repetition of an ever-ceasing synthesis' of the manifold of appearance which is not interrupted" (YAMAMOTO 2017a: 19-37). Here, we assume that the prisoners are mathematicians, and, if freed from their fetter, they have to work as mathematicians. What kind of numbers do they use outside the cave? Since "appearances themselves are nothing but sensible representations" (A104), the cardinal numbers, which the prisoners-mathematicians use outside the cave, are requested to be commensurate with the cardinality: "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$." What kind of cardinal number do the prisoners-mathematicians use inside the cave? Though the shadows of the artificial objects do not necessarily signify "the objects of experience" outside the cave, the cardinal number inside the cave must pertain to the reality as "*realitas noumenon*," which is assumed to be presented "through the pure understanding" of the "realities in appearance (*realitas phaenomenon*)." In other words, it is assumed to be "a reflexive cardinal number," which has the "property of reflexiveness" (RUSSELL 1971: 77-88).

Furthermore, we have to take note of the fact that 1) "the shadows of the artificial objects," which appear to be homogeneous inside the cave, can be thought to signify a "true correlate" (A30) with "the objects of experience" outside the cave, which are to be entirely un-homogeneous outsides the cave, 2) "the shadows of the artificial objects" can be thought to pertain to "*quantum continuum*" while "the objects of experience" pertain to "*quantum discretum*," 3) while "the shadows of the artificial objects" are derived from the artificial objects themselves, i.e., the objects of experience outside the cave, it can be thought that they arise as a result of "logical reflection" or "a mere comparison" with the objects of experience outside the cave, 4) from this it follows that the prisoners' "shadows of the artificial objects" are thought to rest on the "possibility of experience" or "possible experience," 5) in contrast, the prisoners' "realities in appearance (*realitas phaenomenon*)" or realities in the objects of experience are to stand under "possibility itself" or "possibility *a priori*," 6) the prisoners would call the "shadows of the artificial objects" *phaenomena*, "to the extent that as objects they are thought in accordance with the unity of the categories," i.e., "the shadows," 7) furthermore, since the prisoners have no doubts in regard to the "shadows of the artificial objects," they call the shadows *phaenomena*, "to the extent that as objects they are thought in accordance with the unity of the categories," i.e., "*intelligibilia*, i.e., objects of the pure understanding (*realitas noumenon*)," though the shadows are not given to the prisoners through "the reception of representations (the receptivity of impressions)" (A50/B74). Here, we have to stress that "the shadows of the artificial objects" signify notion or "*notio*" (B377) insofar as it has its origin in the understanding and in a pure image of sensibility. The prisoners can call "the shadows of the artificial objects" *phaenomena*, provided they have no doubts in regard to "the pure understanding (*realitas noumenon*)" (B320). In this regard, we say that the prisoners' belief in the shadows of the artificial objects is tantamount to believing that it is possible to "say anything synthetically about things in themselves through the pure understanding (which is

nevertheless impossible)" (A276). The prisoners' way of thinking is wrong. Since whether or not the prisoner is "freed from his fetters and compelled to stand up suddenly and turn his head around and walk and to lift up his eyes to the light" is contingent, this is impossible. Furthermore, even if "the shadows of the artificial objects" have their origin in the understanding and in a pure image of sensibility, they are not to arise through "transcendental reflection" (B317). Therefore, we have to say that "inductive numbers" inside the cave, which have their origin in a pure image of sensibility, are "the mere category" (B151) while numerical formula can induce more and more numbers, i.e., "inductive numbers" (RUSSELL 1971: 29-41), we dare to say, infinitely. The prisoners-mathematicians inside the cave would use "inductive numbers" for summarizing "the successive addition of one (homogeneous) unit to another," which can go "beyond the possibility of experience" (B377). However, "inductive numbers" and numerical formulas in virtue of "the pure understanding (*realitas noumenon*)" have a boundary. They must stand under "a reflexive cardinal number," which has the "property of reflexiveness" (RUSSELL 1971: 77-88).

The prisoners-mathematicians' "concept made up of notions" (B377), i.e., the concept made up of "inductive numbers" with the contrivance of numerical formulas stands under their "function." The function, which resides in "the mental world" of the prisoners-mathematicians, could go "beyond the possibility of experience," though their "inductive numbers" and numerical formulas in virtue of "the pure understanding (*realitas noumenon*)" must have a boundary, i.e., "a reflexive cardinal number." At the same time, the pure concept (*notio*) could not go "beyond the possibility of experience" outside the cave, since it has its origin both in the understanding and in a pure image of sensibility in conjunction with the law of nature. When "a concept made up of notions" and the pure concept (*notio*) cannot go "beyond the possibility of experience," they signify nothing but the categories – the "pure *a priori* concepts" (A95) or "the transcendental schema" (A138/B177). As indicated above, the "pure *a priori* concepts" or "the transcendental schema" outside the cave signify the cardinal numbers; 0, i^2 = -1, and 1. In view of the fact that "a concept made up of notions" outside the cave cannot go "beyond the possibility of experience," and that the prisoners' "concept made up of notions" inside the cave must be limited by "a boundary for given concepts" (B310) – "a reflexive cardinal number" – it becomes possible for us to say that there is one-one relation between "a concept made up of notions" outside the cave and that inside the cave. "A concept made up of notions" – the cardinality as "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0" – cannot go "beyond the possibility of experience" on account of the fact that the cardinal numbers; 0, i^2 = -1, and 1 merely signify "possible experience." We think that the prisoners' "concept made up of notions" is to be equivalent to "a reflexive cardinal number," i.e., "0 → 1 → $e^{i\pi}$ = -1 → 0" or "0 → $e^{i\pi}$ = -1 → 0," which is limited by "a boundary concept" (A255-B311) – "i^2 = -1." This indicates one-one relation between the cardinality in the domain: "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0" and "a reflexive cardinal number" in the converse domain: "0 → 1 → $e^{i\pi}$ = -1 → 0" or "0 → $e^{i\pi}$ = -1 → 0." In view of the fact that the cardinal numbers in the domain; 0, i^2 = -1, and 1 are "the smallest of infinite cardinals" (RUSSELL 1971: 77-88), namely "Aleph" or $\aleph_0$., we would say that "reflexive cardinal numbers" in the converse domain; 0, $e^{i\pi}$ = -1 and 1 correspond to $\aleph_0$. Here, we have to take note of the fact that the cardinal numbers in the domain; 0, i^2 = -1, and 1 signify the "classes and cardinals which are neither reflexive nor inductive" (RUSSELL 1971: 77-88).

In contrast to the prisoners' "shadows of the artificial objects," *phaenomena* or the objects of experience outside the cave are to signify "quantum" (B212). Since "quantum" – "a continuous magnitude" (B212) – is "generated through the mere continuation of productive synthesis of a certain kind" (B212), we think that quantum is to be commensurate with the number i^2 = -1, which signifies "the representation I think" – the category. Therefore, we say that the category generates categories – quantum. In other words, categories are the causality of "the manifold of nature" (B163). If the "mere continuation of productive synthesis of a certain kind" (B212) is interrupted, "an aggregate of many appearances" is to be generated "through the repetition of an ever-ceasing synthesis" (B212). Who carries out the "productive synthesis of a certain kind" continuously or "the repetition of an ever-ceasing synthesis"? The former is to be carried out by the people outside the cave, while the latter is to be carried out by the prisoners-mathematicians inside the cave. The prisoners-mathematicians have "inductive numbers" and numerical formulas in order to deal with "an aggregate of many appearances," which is generated "through the repetition of an ever-ceasing synthesis." On the other hand, the people outside the cave have the cardinality: "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0" in order to deal with "quantum," which is generated "through the mere continuation of productive

synthesis of a certain kind." Does "quantum" need to be generated "through the mere continuation of productive synthesis of a certain kind" continuously? No, it need not be generated any more once "quantum" is "added to the concept, which expresses merely its possibility" (A599/B627), i.e., to the objects of experience in the domain. Here, it is clear that "the actual" must contain something "more than the merely possible" (A599/B627). Is there "the actual" for the prisoners-mathematicians? Yes, there is. What is it? It is "the shadows of the artificial objects." However, "the actual" evaporates when they are freed from their fetters. In contrast to the prisoners-mathematicians, we have "the actual," which is "more than the merely possible," i.e., the schemata of disappearance of "the things themselves, which appear" (B324). Therefore, we can say that "quantum" – categories or *phaenomena* – is to be there forever irrespective of the existence of the people outside the cave or the prisoners inside the cave. We should like to put it in another way, saying that once "$i^2 = -1$" is added to appearances themselves – the "sensible representations" (A104) – which are subject to the "law of reproduction" (A100), they are to transmute into the categories, which are to be in "the absolute whole of magnitude (the world-whole)" (A483) forever irrespective of the existence of humans on this planet, as "the actual," containing something "more than the merely possible." This is our "transcendental place" (B324) and "the transcendental topic, a doctrine that would thoroughly protect against false pretenses of the pure understanding and illusions arising therefrom by always distinguishing to which cognitive power the concepts properly belong" (B324).

Furthermore, we should like to say in another way, what we have already said above in distinction from what Kant says (A181-B224), that "in the 'Allegory of the Cave,' appearances themselves – the objects of experience outside the cave — are thought to be subsumed under the categories under the schemata of 'the shadows,' i.e., 'logical reflection' (B318). Even if the objects of experience outside the cave to which the shadows are related are things in themselves, it would be impossible for the prisoners to cognize anything about them synthetically *a priori*. Now the prisoners' complete cognition of the objects of experience outside the cave, to which in the end all *a priori* principles must come down, rests on the possibility of being freed from their fetters. Consequently these principles cannot have as their goal the conditions of the unity of empirical cognition in the synthesis of the appearances, since the conditions must be thought only in 'the identity of one generation of a magnitude with the other' (B205), namely in the schema of nullity in space-time." We add, saying, "The prisoners-mathematicians would use 'inductive numbers' and numerical formulas insofar as they are subject to the conditions, restricted by 'the shadows of the artificial objects,' while their function might go beyond the possibility of experience." We think that "inductive numbers" – the natural numbers in mathematics – and numerical formulas must be restricted by a sensible condition, of their unity, as of a synthesis in general. However, in this allegory, since the prisoners "deem reality to be nothing else than the shadows of the artificial objects" (The Republic 514-516), they are ordained to think that the shadows themselves – the shadows of the artificial objects or the shadows of the objects of experience outside the cave – signifies *phaenomena*, "to the extent that as objects they are thought in accordance with the unity of the categories" (B305-A249). We cannot think that "inductive numbers," which the prisoners-mathematicians use inside the cave, are the "transcendental product of the imagination, which concerns the determination of the inner sense" (A142) – "the mathematical categories" (B110). This is the Plato's schemata of "the shadows" – a product of "a mere function of thinking" (A253). We have to take note of the fact that the categories which the prisoners-mathematicians think in terms of *phaenomena* belong among "merely sophistical (dialectical) concepts" (A644/B672) or "the mere category" (B151). While people outside the cave would think that the shadows of the artificial objects arise as a result of "logical refection," i.e., "a mere comparison" (B318) on the part of the prisoners, the prisoners-mathematicians would think likewise, if freed, that the shadows of the artificial objects arise as a result of "logical refection," i.e., "a mere comparison" (B318). Here, we have to seek an answer to the question; what "a concept made up of notions" (B377) means. We, as the people outside the cave who are destined to disappear, think that "a concept made up of notions" – "a concept of reason" (B377) – is to be "synthetic *a priori* cognitions of things" (A247) or "synthetic *a priori* principles" (B305), which signify categories or *phaenomena*, leading us to "truth and reference to an object" (A489/B517), which teaches that "a concept made up of notions," even if containing the function, unrestricted by a sensible condition, cannot go beyond the possibility of experience. On the contrary, Kant thinks that "a concept made up of notions" belongs to the "transcendental ideas" (B368) or "concepts of pure reason" (B368), which are grounded in the

"the transcendental dialectic" (B366) – Kant's *notio*. Kant's *notio* – his "thing in itself" – is meant to signify the "transcendental dialectic" as the ground of categories or *phaenomena*. This is the problem of metaphysics in the *Critique of Pure Reason*.

The prisoners-mathematicians inside the cave can be thought to pertain to the Platonic mathematical world and the mental world, while the prisoners-mathematicians who are freed from their fetters are to pertain to the physical world. The numbers which the prisoners-mathematicians inside the cave use seems to be the same as those which the prisoners-mathematicians who are freed from their fetters use. Is it possible for the prisoners-mathematicians who are freed from their fetters to use "inductive numbers" and numerical formulas outside the cave? "Inductive numbers" can be functional insofar as the prisoners-mathematicians believe that the objects of experience outside the cave – "objects of possible experience" (B166) – signify the likenesses of the shadows of the artificial objects, or "an analogy of experience" (A180). Here, we have take note of the fact that the "possibility of experience" or "possible experience" does not stand under the "necessity," which "is nothing other than the existence that is given by possibility itself" (B111) – "real possibility" (B302) – but stands under the possibility of being freed from their fetters. Since "prisoners would deem reality to be nothing else than the shadows of the artificial objects" (The Republic 514-516), namely the shadows of the objects of experience outside the cave, the prisoners-mathematicians can use, inside the cave, "inductive numbers" without harboring any doubts in regard to the "arithmetical truths" (FREGE 1980: 1-5). However, if the prisoners-mathematicians, freed from their fetters, find that "inductive numbers" – "a representation that summarizes the successive addition of one (homogeneous) unit to another" (B182) in terms of counting the shadows appear to have little to do with the objects of experience outside the cave – reality – a serious doubt about "inductive numbers" could arise among them. Frege explicates the difficulty in regard to "inductive numbers" and the "arithmetical truths" with which the prisoners-mathematicians, freed from their fetter, have to cope, saying "If we try to produce the number by putting together different distinct objects, the result is an agglomeration in which the objects contained remain still in possession of precisely those properties which serve to distinguish them from one another; and that is not the number. But if we try to do it in the other way, by putting together identicals, the result runs perpetually together into one and we never reach a plurality. If we use 1 to stand for each of the objects to be numbered, we make the mistake of assigning the same symbol to different things. But if we provide the 1 with differentiating strokes, it becomes unusable for arithmetic" (FREGE 1980: 39-67). Thus, mathematicians feel concerns about if it is appropriate to use "inductive numbers" for counting the objects of experience outside the cave, i.e., "an agglomeration in which the objects contained remain still in possession of precisely those properties which serve to distinguish them from one another." How to solve this conundrum is a crucial issue for mathematicians. If it cannot be solved, it shakes the foundation of mathematics. The clue to the solution resides in "categories" which "are concepts that prescribe laws *a priori* to appearances, thus to nature as the sum total of all appearances" (B163) and their relation to prime numbers in conjunction with the metaphysical axioms. We think that it would provide us with "the rule" (B375), "arithmetical truths" and "reference to an object" (A489/B517). This issue will be further discussed in our following discourse.

6. EULER'S IDENTITY, INDUCTIVE NUMBERS AND COMPLEX NUMBERS

As indicated above, the numbers, which the prisoners-mathematicians inside the cave use, are relevant to counting "the shadows of the artificial objects," while the numbers, which the prisoners-mathematicians who are freed from their fetters use, must be relevant to the objects of experience outside the cave. Since the numbers, which the prisoners-mathematicians inside the cave use, are "inductive numbers" – the natural numbers – which belong among "a concept of the understanding" (B182) or "the pure schema of magnitude (*quantitatis*)" (B182), they are, as "*intelligibilia*, i.e., objects of the pure understanding" (B320), subject to Peano axioms or Euclid's Common Notions. In contrast, since the numbers, which the prisoners-mathematicians who are freed from their fetters use, must pertain to "realities in appearance (*realitas phaenomenon*)" (A265), being subject to the metaphysical axioms, they are to be the cardinal numbers; 0, $i^2 = -1$, and 1 (YAMAMOTO 2017c: 57-70). Therefore, we entirely agree with Russell when he says, "This point, that '0' and 'numbers' and 'successor' cannot be defined by means of Peano's five axioms, but must be independently understood, is important. We want our numbers not merely to verify mathematical formulae, but to

apply in the right way to common objects" (RUSSELL 1971: 1-10). Seeing that the cardinal numbers; 0, i^2 = -1, and 1 signify "the three primitive ideas in Peano's arithmetic…0, number, successor" (RUSSELL 1971: 1-10), we think that the cardinality: "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0" indicates that "1) 0 is to signify nullity in space-time – space-time itself – while number to signify infinite steps of alteration of filled space-elapsing time or its consummation or nullity in space-time; 2) succession is to signify the alteration from nullity in space-time to a part of filled space-elapsing time or from a part of filled space-elapsing time to empty space-nullified time, i.e., nullity in space-time, suggesting that 1) i^2 = -1 is to signify infinite steps of alteration of space-time itself – quantum – between filled-elapsing and empty-nullified, 2) 0 = nullity in space-time – space-time itself – is to permeate filled space-elapsing time, 3) i^2 = -1 is to come across its cessation any time before the consummation of itself as number 1" (YAMAMOTO 2017c: 57-70).

If we clarify one-one correlation between "inductive numbers" – the natural numbers in mathematics – and the cardinality: "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0" by means of "producing a chain of deductions with no link missing, such that no step in it is taken which does not conform to some one of a small number of principles of inference recognized as purely logical" (FREGE 1980: 99-119), the conundrum in regard to numbers and numerical formulas, which Frege indicates (FREGE 1980: 39-67), will be solved. According to Russell, "the key to our problem lies in *mathematical inductio*n. It will be remembered that, in Chapter 1., this was the fifth of the five primitive propositions which we laid down about the natural numbers. It stated that any property which belongs to 0, and to the successor of any number which has the property, belongs to all natural numbers. This was then presented as a principle, but we shall now adopt it as a definition" (RUSSELL 1971: 20-28). On the contrary, we, thinking that the key to the problem of mathematics lies in "a deduction of the pure concepts of the understanding" (B144), have searched for "variables," being taken from experience (by deduction), which enable us to make a "universal proposition" (A300). We have clarified that a "universal proposition" which can "serve as the major premise in a syllogism" (A300) is nothing but nullity in space-time, which is the product of the transcendental deduction in conjunction with the metaphysical deduction in which "the origin of the *a priori* categories in general was established through their complete coincidence with the universal logical functions of thinking" (YAMAMOTO 2016: 87-100, YAMAMOTO 2017c: 57-70, YAMAMOTO 2017d: 19-29). As a result of this exploration, the cardinal numbers; 0, i^2 = -1, and 1, and metaphysical axioms have manifested (YAMAMOTO 2017c: 57-70, YAMAMOTO 2017d: 19-29). The objects of experience outside the cave – appearances themselves – are regarded as signifying the categories, through which we can think of any object. Since "the number of individuals in the universe is not finite" (RUSSELL 1971: 20-28), the "number" and "successor" pertinent to the categories must be, as we have shown above, the cardinality in the domain, "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0," and a reflexive cardinal number in the converse domain, "0 → 1 → $e^{i\pi}$ = -1 → 0" or "0 → $e^{i\pi}$ = -1 → 0." Russell explicates the direction for us to go, saying, "Of the usual formal laws of addition, multiplication, and exponentiation, all are obeyed by transfinite cardinals, but only some are obeyed by transfinite ordinals, and those that are obeyed by them are obeyed by all relation-numbers" (RUSSELL 1971: 89-96), suggesting that "the commutative law," "the associative law," "the distributive law," and "the laws of exponentiation" would operate in "the domain" and in "the converse domain." What does it mean? It means that 1) "transfinite cardinals" – "i^2 = -1" – obey "the commutative law," "the associative law," "the distributive law," and "the laws of exponentiation," 2) "all relation-numbers" – "$e^{i\pi}$ = -1 = i^2" – obey "the commutative law," "the associative law," "the distributive law," and "the laws of exponentiation." In view of the fact that 1) Euler's number *e* and the transcendental, irrational number π are composed of "inductive numbers" with the contrivance of numerical formulas, 2) "i^2 + 1 = 0" and so-called Euler's identity, i.e., "$e^{i\pi}$ + 1 = 0" has "a one-one relation whose domain is the one class and whose converse domain is the other" (RUSSELL 1971: 52-62), we say that on the ground of this one-one relation, "inductive numbers," i.e., the natural numbers in mathematics, which are to be the elements of *e* and π, can be thought of as the real numbers. When we say one-one relation, we have in mind Russell's "definition of similarity" (RUSSELL 1971: 11-19)," which says, "One class is said to be 'similar' to another when there is a one-one relation of which the one class is the domain, while the other is the converse domain" (RUSSELL 1971: 11-19). Since "two relations have the same structure when they have likeness, i.e., when they have the same relation-number" (RUSSELL 1971: 52-62), and "a relation number is a class of relations consisting of all those relations that are similar to

one member of the class" (RUSSELL 1971: 52-62), we think that "all relation-numbers" – all real numbers such as e, π, $i^2 = -1$ in the converse domain – have a structural "likeness" to those in the domain, indicating that the complex numbers in the converse domain, consisting of "inductive numbers" – the natural numbers – and $i^2 = -1$, could be the real numbers. When all complex numbers in the converse domain are found to have the property relevant to the real numbers, we are approaching the solution of the conundrums in regard to mathematics and physics.

However, when Russell feels concerned in regard to "the multiplicative axiom" (RUSSELL 1971: 117-130), saying, "The problem of multiplication when the number of factors may be infinite arises in this way: Suppose we have a class *k* consisting of classes; suppose the number of terms in each of these classes is given. How shall we define the product of all these numbers?...It is the case when *k* is infinite, even though its members may be finite, that we have to find a way of dealing with…" (RUSSELL 1971: 117-130), we have to tackle this issue. We think that, here, Russell is dealing with the issue of mathematics in the converse domain – in the Platonic mathematical world. From our viewpoint, what he refers to indicates three things: 1) "a class *k* consisting of classes" is meant to be "a reflexive cardinal number" – "$e^{i\pi} = -1$" – in the converse domain, 2) the classes which constitute a class *k* are meant to be "inductive numbers" – the natural numbers – 3) "the product of all these numbers" is meant to be e, π or "a reflexive cardinal number," i.e., "$e^{i\pi} = -1$." From this it follows that since "*k* is infinite, even though its members may be finite," "a reflexive cardinal number," i.e., "$e^{i\pi} = -1$" in the converse domain, i.e., in "the Platonic mathematical world" signifies categories in the domain, i.e., in "the physical world." This is the product of the "transfinite induction" (RUSSELL 1971: 117-130), which could occur in "the mental world." The most important things are how the "transfinite induction" occurs in the mental world and what implications this has in regard to "the possibility of things in the world of sense" (A677/B705) and "the possibility of a world-whole itself" (A677/B705). Seeing that these issues are the problem of metaphysics in the *Critique of Pure Reason*, we have already clarified them (YAMAMOTO 2016: 87-100, YAMAMOTO 2017a: 19-37, YAMAMOTO 2017b: 72-81, YAMAMOTO 2017c: 57-70), indicating that the "transfinite induction" could occur in virtue of the "transcendental deduction" (B159) and the "metaphysical deduction" (B159) in conjunction with the law of nature. The law of nature, i.e., the schemata of disappearance of "the things themselves, which appear" (B324) manifests in humans in virtue of "$i^2 + 1 = 0$" or "$e^{i\pi} + 1 = 0$." In this regard, the product of the "transfinite induction" is nothing but "the transfinite cardinal numbers" (DAUBEN 1990: 149-168); 0, $i^2 = -1$, and 1, and "a reflexive cardinal number" (RUSSELL 1971: 77-88), i.e., "$e^{i\pi} = -1$," which are to rest on "death itself which is taken from experience (by deduction)," i.e., nullity in space-time. Therefore, when Russell refers to "selection" and "selectors" in regard to the issue of defining "the product of all these numbers" (RUSSELL 1971: 117-130) or "the product of the numbers of the separate constituencies" (RUSSELL 1971: 117-130), saying, "The class of all 'selections' from *k* we shall call the 'multiplicative class' of *k*. The number of terms in the multiplicative class of *k*, i.e., the number of possible selections from *k*, is defined as the product of the numbers of the members of *k*. This definition is equally applicable whether *k* is finite or infinite…For purposes of defining multiplication, it is the selectors we require rather than the selections. Thus we define: 'The product of the numbers of the members of a class of classes *k*' is the number of selectors from *k*" (RUSSELL 1971: 117-130), we, in entire agreement with Russell, should like to elaborate on it, saying that 1) "the class of all 'selections' from *k*" – "the 'multiplicative class' of *k*" – is meant to be the cardinality in the domain; "$0 \rightarrow 1 \rightarrow i^2 = -1 \rightarrow 0$" or "$0 \rightarrow i^2 = -1 \rightarrow 0$," 2) "the product of the numbers of the members of *k*" – "the product of the numbers of the members of a class of classes *k*" – is commensurate with "$e^{i\pi} = -1$," 3) "the number of terms in the multiplicative class of *k*" – "the number of selectors from *k*" – is nothing but "$i^2 = -1$." Therefore, we are surprised and annoyed to hear from Russell, who says, "we cannot prove that a product is only zero when one of its factors is zero" (RUSSELL 1971: 117-130). It is said that, because of this conundrum, Zermelo had assumed that "if *a* be any class, and *k* all sub-classes of *a* with the exception of the null-class, then there is at least one selector from *k*" and regarded "the axiom as an unquestionable truth" (RUSSELL 1971: 117-130). We think that "Zermelo's axiom" is not to be an assumption but "an unquestionable truth."

This issue seems to be related to the "Cantor's problem of the cardinal number of the continuum" (HILBERT 1902: 437-479). When Russell attempts to "explain the theory of transfinite or infinite cardinal numbers" (RUSSELL 1971: 77-88), we would say, as explicated above, that the "transfinite or infinite cardinal numbers" are correspondent with the cardinality in the domain; "$0 \rightarrow 1 \rightarrow i^2 = -1$

→ 0" or "0 → i^2 = -1 → 0," or the cardinality in the converse domain; "0 → 1 → $e^{i\pi}$ = -1 → 0" or "0 → $e^{i\pi}$ = -1 → 0." Since the infinite or transfinite cardinal numbers can be thought to signify the "infinite collections in the world" (RUSSELL 1971: 77-88), we say that the "axiom of infinity" (RUSSELL 1971: 77-88) is not the assumption but the "unquestionable truth." When Russell says, "A series is called 'Dedekindian' when every section has a boundary, upper or lower as the case may be. We have seen that the series of ratios in order of magnitude is not Dedekindian" (RUSSELL 1971: 63-76), the enigmatic remarks seem to indicate two things: 1) Dedekindian series is homogeneous with "$e^{i\pi}$ = -1" or "i^2 = -1," which has the upper boundary 1 and the lower boundary 0, 2) "the series of ratios in order of magnitude" is meant to be nothing but the transfinite cardinality: "0 → 1 → i^2 = -1 → 0" or "0 → i^2 = -1 → 0" or the reflexive cardinality: "0 → 1 → $e^{i\pi}$ = -1 → 0" or "0 → $e^{i\pi}$ = -1 → 0." Furthermore, when Russell makes remarks in regard to "Cantorian continuity," saying, "A series is 'continuous' when (1) it is Dedekindian, (2) it contains a median class having $\aleph_0$ terms. To avoid confusion, we shall speak of this kind as 'Cantorian continuity.'…All series having Cantorian continuity are similar, but not all series having Dedekindian continuity" (RUSSELL 1971: 97-106), we think, in agreement with him, that the enigmatic remarks indicate that "Dedekindian continuity" has the property of "Cantorian continuity" on account of the fact that "it contains a median class having $\aleph_0$ terms, i.e., $e^{i\pi} = -1 = i^2$." These findings show us a new path through which we can proceed. Now, it becomes possible for us to say that 1) though "Dedekindian series," i.e., $e^{i\pi}$ = -1 is composed of "inductive numbers" – the natural numbers – with the contrivance of numerical formulas, and "the number of possible selections from *k*," i.e., "i^2 = -1," Dedekindian series or "Dedekindian continuity," i.e., "$e^{i\pi}$ = -1" contains "the Dedekindian gap" (RUSSELL 1971: 63-76), 2) the gap is to be filled by a "Dedekind cut" (RUSSELL 1971: 63-76) – by "transfinite induction" – 3) the Dedekindian series or "Dedekindian continuity" can be thought to signify categories insofar as the gap is filled by means of "a median class having $\aleph_0$ terms, i.e., $e^{i\pi} = -1 = i^2$." From this it follows that 1) the Dedekindian series or "Dedekindian continuity" is commensurate with the products made from the complex numbers with the contrivance of numerical formulas, 2) the products, which are supposed to correspond to "the *abelian* extension of a field *k*" (WEIL 2005: 334-341), i.e., "abelian groups," are correspondent with categories – "all series having Cantorian continuity" (RUSSELL 1971: 97-106) – where the "Dedekindian gap" has been filled through "transfinite induction" – a "Dedekind cut." Therefore, we entire agree with what Russell refers to in regard to "the Cantorian infinite" (RUSSELL 1971: 63-76), which says, "zero and infinity, alone among ratios, are not one-one. Zero is one-many, and infinity is many-one" (RUSSELL 1971: 63-76). It is indeed such a penetrating insight that Russell expresses his elated feeling, saying, "The Cantorian infinite, on the other hand, is of the greatest and most fundamental importance; the understanding of it opens the way to whole new realms of mathematics and philosophy" (RUSSELL 1971: 63-76). We, having a similar feeling, will tread on the path which "the Cantorian infinite" opens, in an attempt to solve another conundrum of mathematics in the following discourse.

REFERENCES

Dauben, Joseph Warren, *Georg Cantor: His Mathematics and Philosophy of the Infinite*, Princeton University Press, 1990.

Einstein, Albert, Podolsky, Boris, Rosen, Nathan, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," Physical Review 47: 777-780, 1935.

Euclid, *Euclid's Elements*, Green Lion Press, 2002.

Frege, Gottlob, *The Foundations of Arithmetic*, Northwestern University Press, 1980.

Hilbert, David, "Mathematical Problems," Bulletin of the American Mathematical Society 8: 437-479, 1902.

Kant, Immanuel, *Critique of Pure Reason*, Cambridge University Press, 1999.

Penrose, Roger, *The Road to Reality: a Complete Guide to the Laws of the Universe*, Vintage Books, 2007.

Plato, *The Republic Books VI-X*, Harvard University Press, 1935.

Russell, Bertrand, *Introduction to Mathematical Philosophy*, Simon & Schuster, 1971.

Weil, André, "A 1940 Letter of André Weil on Analogy in Mathematics," Notices of the American Mathematical Society 52: 334-341, 2005.

Yamamoto, Kazuhiko, "The Pure Concepts of the Understanding and Synthetic *A Priori* Cognition: the Problem of Metaphysics in the *Critique of Pure Reason* and a Solution," Proceedings of the European Conference on Ethics, Religion & Philosophy 2016, pp 87-100, 2016.

Yamamoto, Kazuhiko, "The Transcendental Aesthetic and Absolute Totality of Conditions: The Problem of Metaphysics in the *Critique of Pure Reason* and a Solution," International Journal of Humanities Social Sciences and Education 4 (2): 19-37, 2017a.

Yamamoto, Kazuhiko, "Absolute Totality, Causality, and Quantum: The Problem of Metaphysics in the *Critique of Pure Reason*," International Journal of Humanities Social Sciences and Education 4 (4): 72-81, 2017b.

Yamamoto, Kazuhiko, "Numbers, Geometry, and Mathematical Axioms: The Problem of Metaphysics in the *Critique of Pure Reason*," International Journal of Humanities Social Sciences and Education 4 (6): 57-70, 2017c.

Yamamoto, Kazuhiko, "Categories, Causality, and Numbers: The Problem of Metaphysics in the *Critique of Pure Reason*," International Journal of Humanities Social Sciences and Education 4 (8): 19-29, 2017d.

Citation: *Kazuhiko Yamamoto. "The Real, the Infinite, Categories, and Cardinal Numbers: The Problem of Metaphysics in the Critique of Pure Reason" International Journal of Humanities Social Sciences and Education (IJHSSE), vol 5, no. 4, 2018, pp. 20-45. doi: http://dx.doi.org/10. 20 431/ 2349-0381.0504003.*