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Dirac Equation in Cosmological Inertial Frame

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Abstract: *Dirac equation is a one order-wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Dirac Equation from wave function, Type A in cosmological inertial frame. The Dirac equation satisfy Klein-Gordon equation in cosmological inertial frame.*

Keywords: *Gordon Equation; Cosmological Inertial Frame;Dirac equation;*

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1. Introduction

Dirac equation is in special relativity theory,[8]

$$(i\hbar\gamma^{\mu}\partial_{\mu} - mcI)\psi = 0 \quad ,$$

I is 4×4 unit matrix ,

$$\gamma^0 = \begin{pmatrix} I' & 0 \\ 0 & -I' \end{pmatrix},\ \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},\ \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

I' is 2×2 unit matrix, σ^i is Pauli's matrix. (1)

2. Dirac Equation from Wave Function-Type A in Cosmological Inertial Frame

Dirac equation is the wave equation. Therefore, Dirac equation is in cosmological inertial frame,[2]

Wave function Type A:

$$r \to r\sqrt{\Omega(t_0)} \quad , \quad t \to \frac{t}{\sqrt{\Omega(t_0)}},$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of universe in the cosmological time t_0 .

$$(i\hbar\sqrt{\Omega(t_0)}\gamma^0\partial_0 + i\hbar\frac{1}{\sqrt{\Omega(t_0)}}\gamma^i\partial_i - mcI)\phi = 0 \tag{2}$$

If $\bar{\partial}_{\mu}$ is

$$\bar{\partial}_{\mu} = (\sqrt{\Omega(t_0)}\partial_0, \frac{1}{\sqrt{\Omega(t_0)}}\partial_i) \tag{3}$$

Dirac equation is in cosmological inertial frame,

$$(i\hbar\gamma^{\mu}\bar{\partial}_{\mu} - mcI)\phi = 0 \tag{4}$$

Eq(4) multiply $i\hbar\gamma^{\upsilon}\bar{\partial}_{\upsilon}$, hence

$$(-\hbar^2(\gamma^\mu\bar{\partial}_\mu)(\gamma^\upsilon\bar{\partial}_\upsilon)-i\hbar(\gamma^\upsilon\bar{\partial}_\upsilon)mcI)\phi=0 \tag{5}$$

In this time,

$$i\hbar\gamma^\upsilon\bar{\partial}_\upsilon\phi=mcI\phi \tag{6}$$

Hence, Eq(5) is

$$(-\hbar^2\gamma^\mu\gamma^\upsilon\bar{\partial}_\mu\bar{\partial}_\upsilon-m^2c^2I)\phi=0 \tag{7}$$

In this time, matrix γ^μ is

$$\frac{1}{2}(\gamma^\mu\gamma^\upsilon+\gamma^\upsilon\gamma^\mu)=\frac{1}{2}\{\gamma^\mu,\gamma^\upsilon\}=\eta^{\mu\upsilon}I \tag{8}$$

Therefore,[1],[3]

$$\frac{1}{2}(\gamma^\mu\gamma^\upsilon+\gamma^\upsilon\gamma^\mu)\bar{\partial}_\mu\bar{\partial}_\upsilon\phi+\frac{m^2c^2}{\hbar^2}I\phi$$

$$=(\eta^{\mu\upsilon}\bar{\partial}_\mu\bar{\partial}_\upsilon+\frac{m^2c^2}{\hbar^2})I\phi=0 \tag{9}$$

Eq(9) is the matrix equation of Klein-Gordon.

Dirac spinor ϕ is $\phi=(\phi_1,\phi_2,\phi_3,\phi_4)$. ϕ 's hermitian conjugate $\phi^+=(\phi^*_1,\phi^*_2,\phi^*_3,\phi^*_4)$.

Hence, ϕ 's adjoint spinor $\bar{\phi}$ is

$$\bar{\phi}=\phi^+\gamma^0,\quad \bar{\phi}(i\gamma^\mu\overleftarrow{\bar{\partial}}_\mu+mcI)=0 \tag{10}$$

Hence, positive probability density j^0 is

$$j^0=\bar{\phi}\gamma^0\phi=\phi^+\phi=|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2 \tag{11}$$

3. Conclusion

We found Dirac equation from Wave Function-Type A in cosmological special theory of relativity. The wave function uses as a probability amplitude.

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