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Cosmological General Theory of Relativity

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Abstract: *In expanded universe, we found gravity field equation and solution. We found Schwarzschild solution, Kerr-Newman solution in expanded universe. Hence, We found new general relativity theory-Cosmological General Theory of Relativity(CGTR).*

Keywords: *Cosmological General Theory of Relativity; Newtonian Gravity; Schwarzschild solution; Kerr-Newman solution; Robertson-Walker solution*

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1. Introduction

Our article's aim is that we make Cosmological General theory of Relativity (CGRT).

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \quad (1)$$

According to ΛCDM model, our universe's k is zero. In this time, if t_0 is cosmological time,[2]

$$k = 0, t = t_0 >> \Delta t, \ \Delta t \text{ is period of matter's motion} \quad (2)$$

Hence, the proper time is in cosmological time,

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dr^2 + r^2 d\Omega^2]$$

$$= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$

$$= dt^2(1 - \frac{1}{c^2}\Omega^2(t_0)V^2), \quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \quad (3)$$

In this time,

$$d\bar{t} = dt, d\bar{x} = \Omega(t_0)dx, d\bar{y} = \Omega(t_0)dy, d\bar{z} = \Omega(t_0)dz \quad (4)$$

In Cosmological General theory of Relativity(CGTR)'s differential operators are

$$\frac{1}{c}\frac{\partial}{\partial \bar{t}} = \frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial x}\frac{1}{\Omega(t_0)}, \frac{\partial}{\partial \bar{y}} = \frac{\partial}{\partial y}\frac{1}{\Omega(t_0)}, \frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial z}\frac{1}{\Omega(t_0)} \quad (5)$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial \bar{t}^2} - \bar{\nabla}^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x})^2 + (\frac{\partial}{\partial y})^2 + (\frac{\partial}{\partial z})^2\} \quad (6)$$

2. Newtonian Gravity in Expanded Universe

Newton's Gravity is built in static universe. Hence, for making our cosmological theory, we modified Newtonian Gravity in expanded universe.

At first, Newton's gravity acceleration is

$$\bar{\vec{a}} = \vec{a}\Omega(t_0) = -\bar{\vec{\nabla}}\bar{\phi} = -\frac{1}{\Omega^2(t_0)}\vec{\nabla}\phi \quad (7)$$

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = -\frac{GM}{r\Omega(t_0)} \quad (8)$$

or

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = \frac{1}{2}\frac{GM}{R^3\Omega^3(t_0)}r^2\Omega^2(t_0) = \frac{1}{2}\frac{GM}{R^3}r^2\frac{1}{\Omega(t_0)} \quad (9)$$

In this time, if Newton's gravity potential is

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = -\frac{GM}{r\Omega(t_0)} \quad (10)$$

Eq(7) is

$$\bar{\vec{a}} = \vec{a}\Omega(t_0) = -\frac{1}{\Omega^2(t_0)}\vec{\nabla}\phi = -\frac{GM}{r^3}\vec{r}\frac{1}{\Omega^2(t_0)} \quad (11)$$

If Newton's gravity potential is

$$\bar{\phi} = \phi\frac{1}{\Omega(t_0)} = \frac{1}{2}\frac{GM}{R^3}r^2\frac{1}{\Omega(t_0)} \quad (12)$$

Poisson equation is in expanded universe,

$$\bar{\nabla}^2\bar{\phi} = \frac{1}{\Omega^3(t_0)}\nabla^2\phi = 4\pi G\bar{\rho}, \quad \bar{\rho} = \frac{\rho}{\Omega^3(t_0)} \quad (13)$$

Newton force is in expanded universe,

$$\bar{\vec{F}} = m_0\bar{\vec{a}} = m_0\vec{a}\Omega(t_0) = \vec{F}\Omega(t_0) \quad (14)$$

3. Cosmological General Theory of Relativity

Einstein's geodesic equation is in expanded universe,

$$\frac{d^2\bar{x}^\mu}{d\tau^2} + \bar{\Gamma}^\mu_{\alpha\beta}\frac{d\bar{x}^\alpha}{d\tau}\frac{d\bar{x}^\beta}{d\tau} = 0 \quad (15)$$

Schwarzschild solution (vacuum solution) is in expanded universe,

$$ds^2 = -c^2(1-\frac{2GM}{\bar{r}c^2})dt^2 + \frac{d\bar{r}^2}{1-\frac{2GM}{\bar{r}c^2}} + \bar{r}^2d\theta^2 + \bar{r}^2\sin^2\theta d\varphi^2$$

$$= -c^2(1-\frac{2GM}{r\Omega(t_0)c^2})dt^2 + \Omega^2(t_0)[\frac{dr^2}{1-\frac{2GM}{r\Omega(t_0)c^2}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)] \quad (16)$$

Hence, Newtonian approximation is by Eq(11)

$$\bar{a}_r = \frac{d^2 r}{d\tau^2}\Omega(t_0) \approx -\bar{\Gamma}^1_{00}c^2(\frac{dt}{d\tau})^2 \approx \frac{1}{2}c^2\frac{\partial \bar{g}_{00}}{\partial \bar{r}} = \frac{1}{2}\frac{c^2}{\Omega(t_0)}\frac{\partial g_{00}}{\partial r} = -\frac{GM}{r^2}\frac{1}{\Omega^2(t_0)} \tag{17}$$

Hence, the gravity field equation of Einstein in expanded universe,

$$\bar{R}_{\mu\upsilon} - \frac{1}{2}\bar{g}_{\mu\upsilon}\bar{R} = -\frac{8\pi G}{c^4}\bar{T}_{\mu\upsilon} \tag{18}$$

In this time,

$$\bar{T}_{00} = \bar{\rho}c^2 = \frac{\rho}{\Omega^3(t_0)}c^2 = \frac{T_{00}}{\Omega^3(t_0)} \tag{19}$$

Einstein's general solution- Kerr-Newman solution is in expanded universe,[1]

$$ds^2 = \bar{g}_{\mu\upsilon}d\bar{x}^\mu d\bar{x}^\upsilon = -c^2(1-\frac{2c^2GM\bar{r}-kGQ^2}{c^4\bar{\Sigma}})dt^2 - 2(2c^2MG\bar{r}-kGQ^2)\frac{\bar{a}\sin^2\theta}{c^4\bar{\Sigma}}cdtd\varphi$$

$$+\frac{c^4\bar{\Sigma}}{\bar{r}^2 - c^2 2GM\bar{r} + \bar{a}^2 + kGQ^2}d\bar{r}^2 + \bar{\Sigma}d\theta^2$$

$$+\sin^2\theta[\bar{r}^2 + \bar{a}^2 + (2c^2GM\bar{r}-kGQ^2)\frac{\bar{a}^2\sin^2\theta}{c^4\bar{\Sigma}}]d\varphi^2$$

$$\bar{\Sigma} = \bar{r}^2 + \bar{a}^2\cos^2\theta = (r^2 + a^2\Omega^2(t_0)cos^2\theta)\Omega^2(t_0)$$

$$= \Sigma'\Omega^2(t_0)\ , \quad \Sigma' = r^2 + a^2\Omega^2(t_0)cos^2\theta \tag{20}$$

Hence, Kerr-Newman solution is expanded universe,

$$ds^2 = \bar{g}_{\mu\upsilon}d\bar{x}^\mu d\bar{x}^\upsilon = -c^2(1-\frac{2c^2GMr\Omega(t_0) - kGQ^2}{c^4\Sigma'\Omega^2(t_0)})dt^2$$

$$-2(2c^2MGr\Omega(t_0) - kGQ^2)\frac{a\Omega^2(t_0)\sin^2\theta}{c^4\Sigma'\Omega^2(t_0)}cdtd\varphi$$

$$+\Omega^2(t_0)\{\frac{c^4\Sigma'\Omega^2(t_0)}{r^2\Omega^2(t_0) - c^2 2GMr\Omega(t_0) + a^2\Omega^4(t_0) + kGQ^2}dr^2 + \Sigma' d\theta^2$$

$$+\sin^2\theta[r^2 + a^2\Omega^2(t_0) + (2c^2GMr\Omega(t_0) - kGQ^2)\frac{a^2\Omega^4(t_0)\sin^2\theta}{\Omega^4(t_0)c^4\Sigma'}]d\varphi^2\}$$

$$\bar{\Sigma} = \bar{r}^2 + \bar{a}^2\cos^2\theta = (r^2 + a^2\Omega^2(t_0)cos^2\theta)\Omega^2(t_0)$$

$$= \Sigma'\Omega^2(t_0)\ , \quad \Sigma' = r^2 + a^2\Omega^2(t_0)cos^2\theta \tag{21}$$

Robertson-Walker solution is Minkowski space-time by Einstein gravity field equation in CGTR-

Eq(18) in expanded universe.

$$ds^2 = -c^2 dt^2 + [d\bar{r}^2 + \bar{r}^2 d\Omega^2] = -c^2 dt^2 + \Omega^2(t_0)[dr^2 + r^2 d\Omega^2] \quad (22)$$

Eq(22) is equal to Eq(3). Eq(1) is derived by normal Einstein gravity field equation.

According to this theory, the distance traveled by light is changed by cosmological time.

If $x_{t_2 - t_1}$ is the distance traveled by light during the time $t_2 - t_1$ in present cosmological time t_0,

$$\Omega(t_0) x_{t_2 - t_1} = x_{t_2 - t_1} = c(t_2 - t_1), \qquad \Omega(t_0) = 1 \quad (23)$$

If $x_{t_0 - \Delta t + t_2 - t_1}$ is the distance traveled by light in the past universe,

$$\Omega(t_0 - \Delta t) x_{t_0 - \Delta t + t_2 - t_1} = c t_{t_0 - \Delta t + t_2 - t_1} = c(t_2 - t_1) \quad (24)$$

If $x_{t_0 + \Delta t + t_2 - t_1}$ is the distance traveled by light in the future universe,

$$\Omega(t_0 + \Delta t) x_{t_0 + \Delta t + t_2 - t_1} = c t_{t_0 + \Delta t + t_2 - t_1} = c(t_2 - t_1) \quad (25)$$

Universe is expanded. Hence,

$$\Omega(t_0 - \Delta t) < \Omega(t_0) = 1 < \Omega(t_0 + \Delta t) \quad (26)$$

Therefore, the distance traveled by light in the past universe is longer than the distance traveled by light in the future universe.

$$x_{t_0 - \Delta t + t_2 - t_1} > x_{t_2 - t_1} > x_{t_0 + \Delta t + t_2 - t_1} \quad (27)$$

4. Conclusion

We find Cosmological General theory of Relativity. We obtain solution of Einstein gravity field equation in expanded universe..

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