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Klein-Gordon Equation and Wave Function for Free Particle in Rindler Space-Time

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Abstract: *Klein-Gordon equation is a relativistic wave equation. It treats spinless particle. The wave function cannot use as a probability amplitude. We made Klein-Gordon equation in Rindler space-time. In this paper, we make free particle's wave function as the solution of Klein-Gordon equation in Rindler space-time.*

Keywords: *Wave Function; Free Particle; Klein-Gordon equation Rindler Space-time;*

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1. Introduction

At first, Klein-Gordon equation is for free particle field ϕ in inertial frame.

$$\frac{m^2c^2}{\hbar^2}\phi+\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}-\nabla^2\phi=0$$

m is free particle's mass (1)

If we write wave function as solution of Klein-Gordon equation for free particle,[3]

$$\phi=A_0\exp i(\omega t-\vec{k}\cdot\vec{x})$$

A_0 is amplitude, ω is angular frequency, $k=|\vec{k}|$ is wave number (2)

Energy and momentum is in inertial frame,[3]

$$E=\hbar\omega,\vec{p}=\hbar\vec{k} \tag{3}$$

Hence, energy-momentum relation is[3]

$$E^2=\hbar^2\omega^2=p^2c^2+m^2c^4=\hbar^2k^2c^2+m^2c^4 \tag{4}$$

Or angular frequency- wave number relation is

$$\frac{\omega^2}{c^2}=k^2+\frac{m^2c^2}{\hbar^2} \tag{5}$$

2. Klein-Gordon Equation and Wave Function for Free Particle Field in Rindler-Space-Time

Rindler coordinates are

$$ct=(\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0}{c}\xi^0)\quad,\quad x=(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0}{c}\xi^0)-\frac{c^2}{a_0}$$

$$y=\xi^2,z=\xi^3 \tag{6}$$

We know Klein-Gordon equation in Rindler space-time.

Klein-Gordon equation is for free particle field ϕ_ξ in Rindler space-time,[1]

$$\frac{m^2c^2}{\hbar^2}\phi_\xi+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2\phi_\xi}{(\partial\xi^0)^2}-\nabla_\xi{}^2\phi_\xi-\frac{\partial\phi_\xi}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}=0 \tag{7}$$

m is free particle's mass

For we write wave function as solution of Klein-Gordon equation for free particle in Rindler space-time, if we insert Eq(6) in Eq(2),[2,3]

$$\phi=A_0\exp i(\omega t-\vec{k}\cdot\vec{x})$$

$$=\phi_\xi=A_0\exp i[(\frac{c^2}{a_0}+\xi^1)\{\frac{\omega}{c}\sinh(\frac{a_0\xi^0}{c})-k_1\cosh(\frac{a_0\xi^0}{c})\}+k_1\frac{c^2}{a_0}-k_2\xi^2-k_3\xi^3] \tag{8}$$

Eq(8) is the solution's function of the wave equation, Eq(7) in Rindler space-time.

In this point, energy-momentum transformation is[2]

$$E=\hbar\omega=E_\xi\cosh(\frac{a_0\xi^0}{c})+p_{\xi^1}c\sinh(\frac{a_0\xi^0}{c})$$

$$=\hbar\omega_\xi\cosh(\frac{a_0\xi^0}{c})+\hbar k_{\xi^1}c\sinh(\frac{a_0\xi^0}{c}) \tag{9}$$

$$p_x=\hbar k_1=\frac{E_\xi}{c}\sinh(\frac{a_0\xi^0}{c})+p_{\xi^1}\cosh(\frac{a_0\xi^0}{c})=\hbar\frac{\omega_\xi}{c}\sinh(\frac{a_0\xi^0}{c})+\hbar k_{\xi^1}\cosh(\frac{a_0\xi^0}{c}) \tag{10}$$

$$p_y=\hbar k_2=p_{\xi^2}=\hbar k_{\xi^2},p_z=\hbar k_3=p_{\xi^3}=\hbar k_{\xi^3} \tag{11}$$

In this time, we suppose $E_\xi=\hbar\omega_\xi,\vec{p}_\xi=\hbar\vec{k}_\xi$ .In this careful point is we know $\omega,\vec{k}$ are constant.

But, in Eq(9),E(10), ω_ξ,k_{ξ^1} are variable functions with ξ^0. Hence, $\omega_\xi=\omega_\xi(\xi^0),k_{\xi^1}=k_{\xi^1}(\xi^0)$

don't have to use in Eq(7),Eq(8). Energy-momentum relation is Rindler space-time,

$$E_\xi{}^2-p_\xi{}^2c^2=\hbar^2\omega_\xi{}^2-\hbar^2k_\xi{}^2c^2=m^2c^4=E^2-p^2c^2=\hbar^2\omega^2-\hbar^2k^2c^2 \tag{12}$$

3. CONCLUSION

We found the wave function of Klein-Gordon's free particle in Rindler space-time. The wave function cannot use as a probability amplitude. In this paper, the particle has to do a spinless particle.

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