Relation of Accelerations in Two Inertial Frames in Special Relativity Theory

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Abstract: *In special relativity theory, we discover the relation of inertial frames' accelerations. In this theory, we can understand general state of the relation of inertial frames' accelerations*

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1. INTRODUCTION

In special relativity theory, we discover the relation of inertial frames' accelerations.

At first, 4-dimension-Lorentz differential transformation is

$$d\vec{x} = d\vec{x}' + \gamma \vec{v}_0 dt' - (1-\gamma)\frac{\vec{v}_0 \cdot d\vec{x}}{v_0^2}\vec{v}_0 \tag{1}$$

$$dt = \gamma(dt' + \frac{\vec{v}_0}{c^2} \cdot d\vec{x}') , \gamma = 1/\sqrt{1-\frac{v_0^2}{c^2}} \tag{2}$$

If we rewrite Eq(1)-(2) in the other way,

$$\frac{d\vec{x}}{d\tau} = \frac{d\vec{x}'}{d\tau} + \gamma\vec{v}_0\frac{dt'}{d\tau} - (1-\gamma)\frac{\vec{v}_0}{v_0^2} \cdot \frac{d\vec{x}'}{d\tau}\vec{v}_0 \tag{3}$$

$$dt = \gamma(dt' + \frac{\vec{v}_0}{c^2} \cdot d\vec{x}) = \gamma dt'(1+\frac{\vec{v}_0}{c^2} \cdot \vec{u}), \vec{V} = \frac{d\vec{x}}{dt}, \vec{u} = \frac{d\vec{x}'}{dt'} \tag{4}$$

2. THE RELATION OF INERTIAL FRAMES' ACCELERATIONS

For calculating the acceleration,

$$\vec{a} = \frac{d}{dt}(\frac{d\vec{x}}{d\tau}) = \frac{1}{\gamma(1+\frac{\vec{v}_0 \cdot \vec{u}}{c^2})}[\frac{d}{dt'}(\frac{d\vec{x}'}{d\tau}) + \gamma\vec{v}_0\frac{d}{dt'}(\frac{dt'}{d\tau}) - (1-\gamma)\frac{\vec{v}_0}{v_0^2} \cdot \frac{d}{dt'}(\frac{d\vec{x}'}{d\tau})\vec{v}_0]$$

$$= \frac{1}{\gamma(1+\frac{\vec{v}_0 \cdot \vec{u}}{c^2})}[\vec{a}' + \gamma\vec{v}_0\frac{d}{dt'}(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}) - (1-\gamma)\frac{\vec{v}_0 \cdot \vec{a}'}{v_0^2}\vec{v}_0] \tag{5}$$

In this time, the constant acceleration a_0' is

$$a_0' = \frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \rightarrow u = \frac{a_0't'}{\sqrt{1+\frac{a_0'^2 t'^2}{c^2}}} \tag{6}$$

Hence,

$$\frac{d}{dt'}\left(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\right) = \frac{d}{dt'}\left(\sqrt{1+\frac{a_0{'}^2 t{'}^2}{c^2}}\right) = \frac{a_0{'}}{c^2}\frac{a_0{'}t'}{\sqrt{1+\frac{a_0{'}^2 t{'}^2}{c^2}}} = \frac{a_0{'}}{c^2}u \tag{7}$$

$$\vec{v}_0 \cdot \vec{u} = v_0 u \cos\theta{'}, \quad \vec{v}_0 \cdot \vec{a}_0{'} = v_0 a_0{'}\cos\theta{'} \tag{8}$$

Therefore, Eq(5) is

$$\vec{a} = \frac{1}{\gamma\left(1+\frac{\vec{v}_0 \cdot \vec{u}}{c^2}\right)}\left[\vec{a}_0 + \gamma\vec{v}_0\frac{a_0{'}}{c^2}u - (1-\gamma)\frac{\vec{v}_0 \cdot \vec{a}_0{'}}{v_0^2}\vec{v}_0\right]$$

$$= \frac{1}{\gamma\left(1+\frac{v_0 u \cos\theta{'}}{c^2}\right)}\left[\vec{a}_0 + \gamma\vec{v}_0\frac{a_0{'}}{c^2}u - (1-\gamma)\frac{v_0 a_0{'}\cos\theta{'}}{v_0^2}\vec{v}_0\right] \tag{9}$$

If we calculate Eq(9),

$$a^2 = \vec{a}\cdot\vec{a} = \frac{1}{\gamma^2\left(1+\frac{v_0 u \cos\theta{'}}{c^2}\right)^2}\left[a_0{'}^2 + \gamma^2 v_0^2 \frac{a_0{'}^2}{c^4}u^2 + (1-\gamma)^2 a_0{'}^2 \cos^2\theta{'}\right.$$

$$\left. + 2a_0{'}^2 \gamma v_0 \cos\theta{'}\frac{u}{c^2} - 2\gamma v_0 \frac{a_0{'}^2}{c^2}u(1-\gamma)\cos\theta{'} - 2(1-\gamma)a_0{'}^2 \cos^2\theta{'}\right]$$

$$= \frac{1}{\left(1+\frac{v_0 u}{c^2}\cos\theta{'}\right)^2}a_0{'}^2\left[\frac{1}{\gamma^2} + \left(\frac{v_0^2 u^2}{c^4} + 2\frac{v_0 u}{c^2}\cos\theta{'} + \frac{v_0^2}{c^2}\cos^2\theta{'}\right)\right] \tag{10}$$

Finally, the relation of inertial frames' accelerations is

$$a = \frac{\sqrt{(1-v_0^2/c^2) + \left(\frac{v_0^2 u^2}{c^4} + 2\frac{v_0 u}{c^2}\cos\theta{'} + \frac{v_0^2}{c^2}\cos^2\theta{'}\right)}}{\left(1+\frac{v_0 u}{c^2}\cos\theta{'}\right)}a_0{'} \tag{11}$$

In special case, if $\theta{'} = 0$ in Eq(11), [1]

$$a = a_0{'} = \frac{d}{dt'}\left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = a_0 = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right), \quad \vec{V} = \frac{d\vec{x}}{dt}, \vec{u} = \frac{d\vec{x}{'}}{dt'} \tag{12}$$

Eq(11) is the general state of Eq(12).

According to the relativity principle, if we use $-v_0, V, \theta$ instead of $v_0, u, \theta{'}$ in Eq(11), we can obtain the next relation of inertial frames' accelerations.

$$a{'} = \frac{\sqrt{(1-v_0^2/c^2) + \left(\frac{v_0^2 V^2}{c^4} - 2\frac{v_0 V}{c^2}\cos\theta + \frac{v_0^2}{c^2}\cos^2\theta\right)}}{\left(1-\frac{v_0 V}{c^2}\cos\theta\right)}a_0 \tag{13}$$

In this time, the constant acceleration a_0 is

$$a_0 = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right), \ \vec{v}_0 \cdot \vec{V} = v_0 V \cos\theta \ , \ \ \vec{v}_0 \cdot \vec{a}_0 = v_0 a_0 \cos\theta \tag{14}$$

Additional special case, if $\theta = 0$ in Eq(13), [1]

$$a = a_0 = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right) = a_0' = \frac{d}{dt'}\left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right), \ \vec{V} = \frac{d\vec{x}}{dt}, \vec{u} = \frac{d\vec{x}'}{dt'} \tag{15}$$

3. CONCLUSION

We can understand general state of the relation of inertial frames' accelerations in this theory.

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