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Methods of the Nonlinear Tensor Function Theory in the Models of Two-Phase Flow through Anisotropic Porous Media

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Abstract: *A new presentation of tensors of relative phase permeabilities for anisotropic porous media is considered.The relation between the tensors of absolute and phase permeabilities is postulated to be represented by a tensor of 4th rank. It is shown that in this case the relative phase permeabilities depend not only on the saturation, but also on the ratios of main values of an absolute permeability tensor. The obtained relations for phase permeabilities are used for analyzing the results of numerical modeling of two-phase flow in the orthotropic porous medium. The versions of complex studies for obtaining phase permeabilities in anisotropic porous media from the results of laboratory measurements are discussed.*

Keywords: *limit groups of symmetry, tensor relations between absolute and phase permeabilities, theoretical presentation and experimental determination of relative phase permeabilities, two-phase flow through anisotropic porous media*

1. Introduction

The problems of applicability and generalization of classical models of two-phase filtration, which use tensors of phase permeabilities and the notion of relative phase permeability, became recently a subject of many studies, in which the modern possibilities of numerical modeling, the methods of non-equilibrium thermodynamics, of the percolation theory and averaging are applied [1]-[5]. However, as a rule, the isotropic media are considered in these studies, and in those rare cases, when anisotropic porous media are investigated, the problems of interpretation of results and construction of generalized links in basic relations arise. Besides, the methods of the theory of nonlinear tensor functions, developed in works by L.I. Sedov and his school's pupils [6]-[8], allow to study the structure of links between tensors of absolute and phase permeabilities for all crystallophysic and limiting groups of symmetry, to reveal possible effects and to obtain in the explicit form the general relations for functions of relative phase permeabilities, which play extremely important part in the solution of applied problems of hydrocarbon raw material mining [9], [10].

2. Tensor Relations in Two-Phase Flow Theory through Anisotropic Porous Media

In the phenomenological theory of two-phase flows of immiscible fluids (oil and water or water and gas, for example), the Darsy law is supposed to be fulfilled for each of phases:

$$w_i^\alpha = -\frac{k_{ij}^\alpha}{\mu^\alpha}\nabla_j p, \qquad \alpha = 1,2 \ , \tag{1}$$

where $w_i^\alpha -$ are filtration velocity vector components, k_{ij}^α - is the symmetric tensor of phase permeability coefficients, $\mu_\alpha -$ is the viscosity of liquid, p – is the pressure. Here and hereafter the Latin indices denote components of vectors and tensors, and Greek ones, as a rule , the α - number of a phase or function; the repeating Latin indices denote summation, the Greek ones mean "no summation", and for convenience of designation the only Cartesian coordinate system is used here.

Thus, in the filtration theory along with the tensor of absolute permeability k_{ij}, which specifies material properties of a medium in the Darsy law for the filtration of one homogeneous liquid, relations (1) introduce some additional material characteristics k_{ij}^{α}, which specify filtration properties in a combined flow of two homogeneous immiscible liquids. Note that in the crystallophysics [11] the vectors and tensors, which specify the material properties, are called material ones, and vectors and tensors, which specify the influence on the medium and the response to it, are called field ones. Hence, the porous medium for one-phase and two-phase flows is characterized by various material tensors. However, for isotropic porous media was found experimentally, that there exists some relation between k_{ij}^{α} and k_{ij} tensors:

$$k^{\alpha} = f^{\alpha}(s)k \text{ or } k^{\alpha}\delta_{ij} = f^{\alpha}(s)k\delta_{ij}, \tag{2}$$

where δ_{ij} is the Kroneker delta; k and k^{α} are coefficients of absolute and phase permeabilities, respectively; $f^{\alpha}(s)$ - are coefficients of proportionality, which are represented as functions of parameter s - the medium saturation with one of phases (water, as usual). Since the phase permeability depends on an empty space geometry, which determines the absolute permeability, and on the saturation, and then relations (2) actually postulate the possibility of the "separation of variables". Function $f^{\alpha}(s)$ is called the relative phase permeability, since it follows from (2), that $f^{\alpha}(s)=k^{\alpha}/k$.

The generalization of relation (2) to the general case, when filtration properties of a porous medium may be anisotropic, allows for various versions of constructing the relations between k_{ij}^{α} and k_{ij}. Initially, when the relative phase permeabilities have been considered to be universal functions of saturation, the relation of the following type was suggested [12]:

$$k_{ij}^{\alpha} = f^{\alpha}(s)k_{ij}, \tag{3}$$

in which it was supposed that the symmetry of tensors k_{ij}^{α} and k_{ij} is similar and can be arbitrary, i.e. the tensors can be anisotropic with arbitrary type of symmetry. But, as the results of numerical experiment [1] and of construction of an effective filtration law for a stratified medium [4] have shown, equalities (3) in the case of anisotropic media k_{ij}^{α} and k_{ij} are not satisfied.

Other versions of construction of relations between k_{ij}^{α} and k_{ij} for anisotropic media were considered in [13]-[15]. However, in these cases tensors k_{ij}^{α} and k_{ij} were supposed to be coaxial and having similar symmetry. The main result of these studies was reduced to the introduction of own function of relative phase permeability, which depends on the saturation only, along each main direction of tensors. Thus, equalities (3) were proposed to be replaced by equalities, which have the following form in the principal coordinate system:

$$k_{\beta}^{\alpha} = f_{\beta}^{\alpha}(s)k_{\beta}, \beta = 1,\quad 2,\quad 3 \tag{4}$$

where k_{β} and k_{β}^{α} are main values of tensors of absolute and phase permeability, respectively, $f_{\beta}^{\alpha}(s)$ are "main" values of relative phase permeabilities.

In spite of the fact, that equalities (4) generalize relations (3), the strict limitations, imposed on the symmetry of tensors, do not allow to consider and take into account possible effects of a combined flow of two liquids. As known [16], the difference in the wettability of a porous medium with liquids results in the situation, when a more wetting liquid fills-in small pores and channels while leaving larger pores and channels for filtration with another, less wetting liquid. On the other hand, as it was shown in [17], the tensor of absolute permeability coefficients can be presented in the form of a composition of tensors of shape and transparency coefficients, and the effective conductivity of porous channels of various diameters may occur to be the same for filtration of one homogeneous liquid. Therefore, the symmetry of tensor k_{ij} may occur to be higher, than that of tensor k_{ij}^{α}. For example, in the flow of one homogeneous liquid the isotropic filtration properties may correspond to an empty space with a rhombic symmetry (which is usually called orthotropy in the elasticity theory), whereas in the filtration of two immiscible liquids the tensor k_{ij}^{α} with a rhombic symmetry may correspond to the mentioned empty space due to the influence of wettability. The other combinations are also possible, but the symmetry of k_{ij} is always higher or equal to the symmetry of k_{ij}^{α}. The accounting of possible difference in the symmetry of tensors k_{ij} and k_{ij}^{α}, as well as the most general analysis of the relation between absolute and phase permeabilities, can obviuosly be performed similarly to the method used in the determination of an elastic body and viscous fluid [6]. The existence of relation between k_{ij} and k_{ij}^{α} actually implies that there exists a functional tensor dependence of general form, which is specified by equalities:

$$k_{ij}^{\alpha} = F_{ij}^{\alpha}(k_{lm}, T^{\alpha}, \chi^{\alpha}), \tag{5}$$

where $F_{ij}^{\alpha} -$ is a tensor function which depends on the absolute permeability tensor k_{lm}, on tensors T^{α}, which determine and specify the symmetry of an empty space [8], and on the parameters of physical-chemical nature χ^{α}. The difference of equalities (5) from classical ones, by means of which the elastic bodies and viscous fluids are determined, consists in the fact, that relations (5) associate the material, rather than field characteristics between each other. Considering functions F_{ij}^{α} to be smooth enough and expanding them into a series to an accuracy of linear terms, we obtain from relation (5):

$$k_{ij}^{\alpha} = F_{ijkl}^{\alpha} k_{kl}, \tag{6}$$

where tensor F_{ijkl}^{α} may be called a tensor of relative phase permeabilities. This tensor associates the quantities of the same dimensionality and, therefore, its invariant coefficients are dimensionless, and the internal symmetry (assuming the relation to be potential one) is the same as that of the tensor of elasticity coefficients in Hook's law. Relations (6) are a natural generalization of relation (2), which is presented as a particular case for isotropic k_{ij} and k_{ij}^{α}. The situation similar to that under consideration (6) has already been encountered in the filtration theory [18]. The tensor of the 4th rank - the tensor of dispersion coefficients of a porous medium - was introduced for specifying the tensor of filtration-convective diffusion.

Using the results, obtained by L.I. Sedov and his school, one can analyze relation (6) for all types of anisotropy and, besides, construct the nonlinear generalizations of the relation as well. However, at present there are no experimental data for constructing these nonlinear generalizations. The linear relation (6) for symmetry groups, which are most frequently used in applied calculations - transversally-isotropic and orthotropic ones, has the following form:

$$\begin{gathered} k_{ij}^{\alpha} = \varphi_1^{\alpha} k_1 \gamma_{ij} + \varphi_3^{\alpha} k_3 B_{ij}, \varphi_1^{\alpha} = F_1^{\alpha} + F_2^{\alpha}(1-\theta_{13})/\theta_{13}, \\ \gamma_3^{\alpha} = F_3^{\alpha}\theta_{13} + F_4^{\alpha}(1-\theta_{13}), \end{gathered} \tag{7}$$

$$\begin{gathered} k_{ij}^{\beta} = \varphi_{1\beta} k_1 a_i a_j + \varphi_2^{\beta} k_2 c_i c_j + \varphi_3^{\beta} k_3 b_i b_j, \\ \varphi_1^{\beta} = F_1^{\beta} + F_2^{\beta}\theta_{21} + F_3^{\beta}\theta_{31}, \varphi_2^{\beta} = F_2^{\beta}\theta_{12} + F_4^{\beta} + F_5^{\beta}\theta_{32}, \quad \varphi_3^{\beta} = F_3^{\beta}\theta_{13} + F_5^{\beta}\theta_{23} + F_6^{\beta}, \end{gathered} \tag{8}$$

where $a = e1$, $c = e2$, $b = e3$ are the orths of a crystallophisic basis; $B_{ij} = b_i b_j, \gamma_{ij} = \delta_{ij} - B_{ij}, \varphi_{\beta}^{\alpha} k_{\beta}$ – are the main values of a tensor of phase permeabilities presented in the form of a product of main values of an absolute permeability tensor k_{β} by "main" values of relative phase permeabilities $\varphi_{\beta}^{\alpha}; F_{\beta}^{\alpha}$ are presented in the form of linear combinations of invariant coefficients of basis tensors specifying and determining the given classes of symmetry, $\theta_{\alpha\beta} = k_\alpha / k_\beta$.

As follows from relations (7) and (8), functions φ^α_β, which depend not only on saturation, but on anisotropy parameters $\theta_{\alpha\beta}$ (presented as ratios of main values of a tensor of absolute permeability) correspond to classical presentation of relative phase permeabilities.

3. Theoretical Representation of Relative Phase Permeabilities as Compared with Numerical Experiment

At present, there are no laboratory experimental data required for analysis and verification of the obtained presentation of functions of relative phase permeabilities in anisotropic porous media; but there are the results of numerical modeling of a two-phase flow in the orthotropic porous medium [1]. Of course, a single numerical modeling result is still insufficient for a comprehensive analysis of the relation between absolute and phase permeabilities and for establishing a reliable form of functions of relative phase permeabilities. The absence of results of laboratory investigations is mainly explained by minimization of measurements on a core-sample material. As a rule, the medium is a priori supposed to be either isotropic or, at least, transversally isotropic. In this case the measurements do not allow to obtain reliable characteristics of porous medium's permeability and, the more so as, to determine the relative phase permeabilities. By this reason, below we shall consider possible versions of complex investigations for obtaining the relative phase permeabilities in anisotropic porous media from laboratory measurement results, and also we shall use numerical modeling results [1] for obtaining a possible explicit form of functions φ^β_α for an orthotropic porous medium. We shall begin with the latter action.

As a result of numerical modeling, the relative phase permeabilities for an orthotropic porous medium with a prominent anisotropy were obtained in [1]. The ratios of main values of an absolute permeability tensor were equal to: k_y/k_x = 0,207 and k_z/k_x = 0,030. The relative phase permeabilities were calculated for water only; they are shown by dark circles, small rhombuses and squares along axis z, y and x, accordingly in Fig.1. To approximate the results we shall analyze the structure of obtained expressions. The general form of obtained relations in (8) is presented by expressions:

$$\varphi^1_\alpha = F_\gamma + F_\chi \theta_{\nu\alpha} + F_\tau \theta_{\mu\alpha}, \tag{9}$$

which satisfy the conditions: for $s = 0$, $\varphi^1_\alpha = 0$ and for $s = 1$, $\varphi^1_\alpha = 1$. Besides, the plots are such, that the minimum values of relative phase permeabilities correspond to the maximum value of the absolute permeability, and vice versa. Supposing all F_γ functions to be of the same order, expressions (9) can be transformed into the following approximate equalities:

$$\varphi^1_\alpha = \frac{I_1(k)}{k_\alpha} \cdot F_\gamma \cdot s^\varepsilon . \tag{10}$$

Multiplier s^ε guarantees fulfillment of condition $\varphi^1_\alpha = 0$ for s = 0 and corresponds to a generally accepted approximation of relative phase permeabilities for isotropic porous media [9, 10]. Multiplier $I_1(k)/k_\alpha$, where $I_1(k)$ is the first invariant of a tensor of absolute permeability, qualitatively confirms the numerical modeling results: a larger value of the relative phase permeability corresponds to a lower value of the permeability. Further, in constructing the approximation of function F_γ one should take into account the condition $\varphi^1_\alpha = 1$ for s = 1, as well as the important feature of the behavior of functions for s = 1: the second derivative of functions of relative phase permeabilities for directions y and z is negative and that for direction x is positive. Besides, as it was already noted, in the approximation of relative phase permeabilities for water the multiplier s^ε satisfies both "boundary conditions" even in the case of nonzero limiting saturations s_* for water and s*– for oil (gas):

$\varphi^1 = (s - s_*)^{\varepsilon}/(1 - s_*)$. Therefore, the multiplier $I_1(k)F_\gamma/k_\alpha$ may be supposed to be caused by anisotropy and be equal to unity for isotropic media. One of approximation versions, that satisfies the listed conditions, is presented in the form:

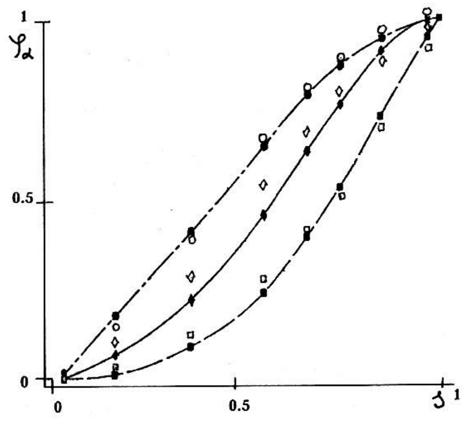
$$\varphi^1_\alpha = \left[1 + \left(\frac{I_1(k)}{3k_\alpha} - 1\right)(1 - s)\right] s^{\varepsilon} , \qquad (11)$$

where ε is the only parameter to be determined. It is clear that the requirement of equality of ε values for all main directions is a strong limitation, since according to relations (8), each of φ^1_α quantities is determined by its own set of functions F_γ. Nevertheless, it is also clear that the exponent ε must be expressed in terms of invariant quantities, which specify and determine filtration-capacity parameters of a medium, and in transition from anisotropy to the isotropic case its value must correspond to well-known approximations [9], [10]. However, the data presented in [1] are insufficient for performing such an analysis.

The numerical analysis of expressions (11), according to the data of [1], has shown that the assumption on the equality of the order of functions F_γ is not fulfilled. Indeed, the orthotropic porous medium modelled in [1] possesses prominent anisotropy: the permeability in the direction of axis x is more than 30 times greater than that in the direction of axis z. Therefore, it is logical to suppose that function F_3, which represents a multiplier at $\theta_{31} = 0.030$ in the presentation of φ^1_1 and at $\theta_{13} = 33.3$ – in the presentation of φ^1_3, is considerably lower than remaining ones and can be neglected. Under this supposition relation (11) for the numerical data of [1] becomes:

$$\begin{aligned} &\varphi^1_1(s) = (0{,}4 + 0{,}6s)s^{\varepsilon}, \varphi^1_2(s) = (2 - s)s^{\varepsilon}, \\ &\varphi^1_3(s) = (2{,}6 - 1{,}6s)s^{\varepsilon}, \end{aligned} \qquad (12)$$

and well approximates the numerical modeling results for ε = 1,6. The values of relative phase permeabilities, calculated by formulae (12) for ε = 1.6, are presented by corresponding light marks in Fig.1.



It is clear that relations (12) represent only one of possible versions of approximation, and new laboratory experimental investigations are necessary for further studies and for determination of an explicit form of φ^β_α . Below we shall outline the possible strategy of such investigations.

4. EXPERIMENTAL DETERMINATION OF THE MATERIAL TENSOR FUNCTIONS

To determine phase permeabilities and, then, to calculate the relative phase permeabilities, it is necessary, first of all, to determine the components of a tensor of absolute permeabilities. Available techniques, in which the measurements are carried out on core samples in the arbitrary direction in the stratification plane, do not guarantee in any case, that the axis of symmetry of a cylindric specimen is directed along the main direction of a tensor of absolute permeability, or, taking into account the possible change of a symmetry group in transition from k_{ij} to k_{ij}^{α}, it is directed along the main direction of a tensor of phase permeabilities. Therefore, the permeability values obtained in similar experiments can be effective, rather than main values of a tensor of absolute permeability; and further investigations of two-phase flows will not provide experimental data which could be properly interpreted, since the replacement of main values of permeability by effective ones can lead to essential errors [19]. Hence, it is necessary, at first, to perform a series of measurements on differently oriented cores and on cores with the same orientation, but with various length, in order to determine the directions of main axes of an absolute permeability tensor. In establishing main directions of a tensor of permeability versions are possible, depending on a porous medium structure. We consider the case, when one of main directions is known. Usually, this direction is chosen to be that perpendicular to the stratification plane, especially in the case of prominent stratification.

We introduce the laboratory coordinate system: axis z is directed perpendicular to the stratification plane; axes x and y, if there are no prominent structure or any other method of orientation, can be directed in the plane perpendicular to axis z, mutually perpendicular in arbitrary directions. Further, for performing measurements one should cut off two cores of different length in the direction of main axis z. The second core of smaller (or greater) length is required for checking the measurement of a permeability: if the axis of symmetry of a core coincides with the main direction of tensor k_{ij}, then the measured value does not depend on a sample length. Three cores are sufficient for determining three components k_{11}, k_{12}, and k_{22} of the permeability tensor. One of these cores is cut off parallel to axis x, the second one - parallel to axis y, and third one - along the bisecting line of right angle 0xy. For a greater accuracy and reliability of determination of tensor components the fourth core can be produced by shortening one of samples oriented along one of the coordinate axes. The measurements carried out during the filtration of one homogeneous liquid will be sufficient for determination of main values of tensor k_{ij} and orientation of main directions in the stratification plane [20].

Further measurements of components of a tensor of phase permeability coefficients can be performed both on old cores and on old and new ones, which were cut off after determination of main directions of tensors k_{ij} and k_{ij}^{α}. Indeed, the measurements, carried out on old cores, under assumption that axis z is perpendicular to the stratification plane and remains to be main one for a tensor of phase permeabilities as well, will allow to determine main values and main directions of tensor k_{ij}^{α}. For anisotropic media the coincidence of main directions of tensors k_{ij} and k_{ij}^{α} also in the stratification plane is most probable. If this assumption is valid, it is desirable to perform a greater number of measurements for checking the validity of a tensor character of relation (1). By this reason one can cut off two more cores along the main directions of permeability tensors in the stratification plane. Then the measurements on these new core samples will allow to determine main values of tensors of coefficients of phase permeabilities and the "main" values of relative phase permeabilities. The measurements on old core samples for a two-phase flow together with the measurements on new samples are interrelated by the tensor transformation law and, therefore, the analytical relations exist between them [21], [22]. The coincidence of experimental results with analytical values will be an evidence of the validity of equalities (1). Besides, the use of the results, obtained both for one- and for two-phase filtration flows, will allow to substantiate the relation (6) and to provide an extra data for determining an explicit form of "main" values of relative phase permeabilities.

The measurements and the technique of performing experimental investigations for two-phase flows in anisotropic media on arbitrarily oriented core samples can, probably, be remained the same; but the interpretation of measurement results and the number of measurements are changed [21].

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