

## Friction on Double-Surface

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**Abstract:** Friction on double surface decreases with the translation time since in that time the moving physical body climbs to higher orbits possessing lower friction coefficient.

**Keywords:** friction coefficient on non-Euclidean and Euclidean sphere, double surface, elliptic, hyperbolic and average elliptic-hyperbolic length

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### 1. PREFACE

Regarding common experience it is easier to keep something in motion across a horizontal surface than to start it in motion from rest [1]. Let us see if the same pattern governs the moving of physical bodies on double surface [2].

### 2. THE APPARENT REST

According to Heracleatean dynamics a physical body possessing the zero kinetic energy insists in the most convenient motion [3]. Let us propose that it spins around itself on double surface measuring the next length values expressed in Compton wavelengths of that body :

The elliptic length on the elliptic side of the sphere

$$n \in \mathbb{N}. \quad (1)$$

The hyperbolic length on the hyperbolic side of the sphere [4]

$$h(n) = n \left( 2 \sqrt{1 + \frac{\pi^2}{n^2}} - 1 \right). \quad (2)$$

Both non-Euclidean sphere orbits lie at the infinite radius far away from the orbit centre [2]:

$$r_{\text{elliptic}} = r_{\text{hyperbolic}} = r_{\text{sphere}} = \infty. \quad (3)$$

This fact allows uniform motion of the physical body on its own orbit.[4]

The average elliptic-hyperbolic orbit is an apparent Euclidean orbit lying finite radius away from the orbit centre:

$$r_{\text{apparent}} \neq \infty. \quad (4)$$

The average orbit length  $s$  is expressed by the elliptic length  $n$  (briefly orbit number) as follows [2]:

$$s(n) = n \left( 2 - \frac{1}{\sqrt{1 + \frac{\pi^2}{n^2}}} \right). \quad (5)$$

The orbit spin projection in the orbit centre is at apparent rest. From this point of view we can say that physical body with zero kinetic energy is there at apparent rest.

### 3. THE GRADUAL TRANSLATION

The translation of physical body is the translation of its orbit centre. The translation event is encouraged by the irrational hyperbolic orbit length  $h$  expressed in Compton wavelengths of that body [4]:

$$h \in \mathcal{R} \setminus \mathbb{Q}. \quad (6)$$

Since the irrational length value does not satisfy wave nature of the physical body.

The orbit centre of physical body can be translated gradually for the average elliptic-hyperbolic length  $s$  mirroring the orbit number  $n$  (5):

$$1,2,3 \dots n \in \mathbb{N}. \tag{7}$$

Since the integer length value satisfies the wave nature of the physical body.

At gradual translation the physical body may jump from  $n^{th}$  to  $(n + 1)^{th}$  orbit. Jumps to non-neighbouring orbits are not allowed.

#### 4. THE TRANSLATION FRICTION

At the translation event the hyperbolic surface slips along the average elliptic-hyperbolic surface. Resulting friction [1] on the contact area depends on the translation friction coefficient, denoted  $\mu$ , being defined as ratio between the hyperbolic length  $h$  of the upper layer and average elliptic-hyperbolic length  $s$  of the lower one:

$$\mu = \frac{h(n)}{s(n)}. \tag{8}$$

The friction force  $F_{friction}$  is in direct proportion to the normal force  $F_{normal}$  (executed perpendicular to the surface) and to the translation friction coefficient  $\mu$  [1]:

$$F_{friction} = F_{normal} \times \mu. \tag{9}$$

At the constant normal force  $F_{normal}$  (no matter how big or small it is) the friction force  $F_{friction}$  depends on solely the translation friction coefficient  $\mu$ .

#### 5. THE TRANSLATION FRICTION COEFFICIENT

The translation friction coefficient  $\mu$  depends on the characteristic orbit number  $n$  (2), (5), (7):

$$\mu(n) = \frac{h(n)}{s(n)} = \frac{n \left( 2\sqrt{1 + \frac{\pi^2}{n^2}} - 1 \right)}{n \left( 2 - \frac{1}{\sqrt{1 + \frac{\pi^2}{n^2}}} \right)}, \quad n \in \mathbb{N}. \tag{10}$$

Some translation friction coefficients are collected in *Table 1*.

**Table1.** Some translation friction coefficients  $\mu$  as a function of the orbit number  $n$ , average elliptic-hyperbolic length  $s$  and hyperbolic length  $h$

n	s	h	$\mu = \frac{h}{s}$
1	1,696685529	5,593816619	3,296908
2	2,925941456	5,448383556	1,862096
3	3,928136632	5,687831582	1,447972
4	4,854243600	6,172434203	1,271554
5	5,766334920	6,810098120	1,181010
6	6,684550414	7,545420540	1,128785
7	7,613682274	8,345306045	1,096093
8	8,553587556	9,189485670	1,074343
9	9,502802744	10,065109955	1,059173
10	10,459717836	10,963740544	1,048187
100	100,049311524	100,098671704	1,000493
$\infty$	$\infty$	$\infty$	1

We see that the translation friction coefficient  $\mu(n) = \frac{h(n)}{s(n)}$  decreases with the increasing orbit number  $n$  tending to reach unit value at Euclidean orbit  $n = \infty$ :

$$3.2969 \dots = \frac{h(1)}{s(1)} > \frac{h(\infty)}{s(\infty)} = 1. \tag{11}$$

And so does the friction (9):

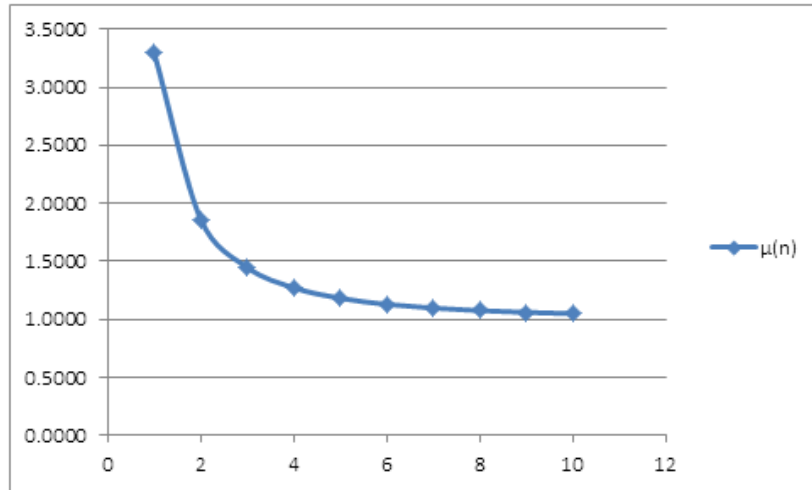
$$\frac{F_{friction}(n = 1)}{F_{friction}(n > 1)} > 1. \tag{12}$$

## 6. THE RELATIVE FRICTION

In a special case comparing the friction values on the non-Euclidean orbit  $n \in \mathbb{N}$  to the friction value on Euclidean orbit  $n = \infty$  the next relative friction is given:

$$\frac{F_{friction}(n)}{F_{friction}(\infty)} = \mu(n). \quad (13)$$

It is represented in *Figure 1*.



**Figure1.** Relative friction  $\mu(n)$  as a ratio of frictions on the non-Euclidean orbit  $n \in \mathbb{N}$  and Euclidean orbit  $n = \infty$  (13)

Maximal relative friction coefficient on double surface  $\mu(1) = 3.3$  is comparable to the maximal ratio between static and kinetic friction in macro world known from physics literature [5]. For instance, the friction of ice on ice and that one in synovial joint of humans is characterised by the same maximal relative friction coefficient:

$$\mu_{relative}^{max} = 3.3. \quad (14)$$

## 7. CONCLUSION

Common experience that it is easier to keep something in motion than to start it in motion from rest seems to originate from - or at least being inspired by - double surface geometry.

## DEDICATION AND TOAST

This fragment is dedicated to my classmates from elementary school in Selnica ob Dravi, 1959-1967.  
Happy 50<sup>th</sup> anniversary!

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## AUTHOR'S BIOGRAPHY



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