

Conceptual Helium Atom Radius

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Abstract: Applying the double-surface concept two possible exact values of the Helium atom radius are predicted, i.e. $r_{He} = 0.309 \times 10^{-10} m$ or $r_{He} = 0.313 \times 10^{-10} m$.

Keywords: effective nuclear charge number, Bohr-like orbit, double-surface, Helium atom radius

1. PREFACE

The subject of interest of this paper is with the help of the Bohr-like orbit on double surface [1] to relate the effective nuclear charge number of the Bohr-like atom to the concerned atom radius. And further, test the concept in the case of the Helium atom.

2. THE BOHR-LIKE ORBIT

On Bohr orbit [2] the electron circulates around a nucleus consisting of only one proton. On the Bohr-like orbit the number of protons in the atom nucleus can amount to more than one. In the case of the Helium atom the mentioned number yields 2.

3. THE ORBIT NATURE

Respecting the double-surface concept [1], [2] the length of the Bohr-like orbit is the average elliptic-hyperbolic length s expressed by the elliptic length n of that orbit as:

$$s(n) = n \left(2 - \frac{1}{\sqrt{1 + \frac{\pi^2}{n^2}}} \right). \quad (1)$$

For lengths given in the units of Compton wavelengths of the electron holds:

$$1 \leq n \in \mathbb{N}. \quad (2)$$

The number of possible orbits n is in principle infinite but their lengths are of irrational discrete values [2].

The shortest length having elliptic value $n = 1$ is the 1st one which satisfying the electron wave nature makes the motion stable. Other lengths surpass this value:

$$s(1) = 1,696685529 \dots \leq s(n), \quad n \in \mathbb{N}. \quad (3)$$

The elliptic length n being the natural number is at the same time the Bohr-like orbit name.

4. THE EFFECTIVE NUCLEAR CHARGE NUMBER

The length of 137th orbit in the ground state of Hydrogen atom (Bohr orbit) equals the inverse fine structure constant α^{-1} [2]. The length of Bohr-like orbits of atoms with atom number $Z > 1$ should equal $\frac{\alpha^{-1}}{Z}$ since Z protons manifest Z – times stronger Coulomb force [3]. But the repulsive interaction committed by neighbouring electrons lessens the attractive effect [3]. So the effective nuclear charge number seen by each electron $Z_{effective}$ is smaller than that one expected from the impact of solely atom number Z :

$$Z_{effective} < Z. \quad (4)$$

Consequently the Bohr-like orbit lengths (with exception of Bohr orbit length itself) differ from the lengths expected by the solely attractive force:

$$s = \frac{\alpha^{-1}}{Z_{effective}} > \frac{\alpha^{-1}}{Z}, \text{ for } Z \neq 1. \tag{5}$$

The orbit length s and effective nuclear charge number $Z_{effective}$ are in inverse proportion. So, since s is down side limited (2) the maximal effective nuclear charge number $Z_{effective}^{max}$ is finite:

$$Z_{effective}^{max} = \frac{\alpha^{-1}}{s(1)} = \frac{\alpha^{-1}}{1,696685529 \dots} = 80,76689. \tag{6}$$

And since s is upside unlimited (2) the minimal effective nuclear charge number $Z_{effective}^{min}$ is zero:

$$Z_{effective}^{min} = \frac{\alpha^{-1}}{s(\infty)} = \frac{\alpha^{-1}}{\infty} = 0. \tag{7}$$

In general $Z_{effective}$ is expressed as (5):

$$Z_{effective} = \frac{\alpha^{-1}}{s(n)}, \quad 1 \leq n \in \mathbb{N}. \tag{8}$$

5. THE ATOM RADIUS

The relation between the orbit length and atom radius depends on geometry of the concerned electron orbit. In the first extreme case the orbit is a circle. Here the atom radius r_{atom} is related to the orbit length s as:

$$r_{atom} = \frac{s}{2\pi}. \tag{9}$$

In the second extreme case the orbit is a flattened ellipse and the atom radius r_{atom} is related to the orbit length s as:

$$r_{atom} = \frac{s}{2}. \tag{10}$$

The atom radius could theoretically in the case of no other limitations occupy a value between the above (9), (10) extreme values:

$$\frac{s}{2\pi} \leq r_{atom} \leq \frac{s}{2}. \tag{11}$$

The number of possible atom radii r_{atom} is in principle infinite but of irrational discrete values (2).

6. THE EXACT ATOM RADIUS PREDICTION

For the circular orbits the next, let us say, experiential orbit length is proposed(9):

$$s_{experiential} = 2\pi r_{atom}. \tag{12}$$

And following the double-surface concept the next, let us say, conceptual one is considered (1):

$$s_{conceptual} = n \left(2 - \frac{1}{\sqrt{1 + \frac{\pi^2}{n^2}}} \right), \quad 1 \leq n \in \mathbb{N}. \tag{13}$$

At the enough small difference between $s_{experiential}$ and $s_{conceptual}$ the exact atom radius value r_{atom} can be predicted. Assuming a circular orbit the above procedure is applicable for the exact Helium atom radius value prediction.

7. THE EXACT HELIUM ATOM RADIUS PREDICTION

The candidates for the conceptual Helium atom orbit length (1)as well as accompanying nuclear charge number (8)are listed in the Table 1.

Table1. Discrete values of atom orbit length s and effective nuclear charge number $Z_{effective}$ from 68th to 137th Bohr-like orbit

n	s	$Z_{effective}$	n	s	$Z_{effective}$	n	s	$Z_{effective}$	n	s	$Z_{effective}$
68	68,07245	2,01309	86	86,05732	1,59238	104	104,0474	1,31705	122	122,0404	1,12287
69	69,07141	1,98398	87	87,05667	1,57410	105	105,0470	1,30452	123	123,0401	1,11375
70	70,07039	1,95569	88	88,05602	1,55624	106	106,0465	1,29223	124	124,0398	1,10477
71	71,06940	1,92820	89	89,05540	1,53877	107	107,0461	1,28016	125	125,0395	1,09594

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72	72,06844	1,90147	90	90,05478	1,52170	108	108,0457	1,26832	126	126,0391	1,08725
73	73,06751	1,87547	91	91,05418	1,50499	109	109,0452	1,25669	127	127,0388	1,07869
74	74,06660	1,85017	92	92,05359	1,48865	110	110,0448	1,24527	128	128,0385	1,07027
75	75,06571	1,82555	93	93,05302	1,47267	111	111,0444	1,23406	129	129,0382	1,06198
76	76,06485	1,80157	94	94,05245	1,45702	112	112,0440	1,22305	130	130,0379	1,05382
77	77,06401	1,77821	95	95,05190	1,44170	113	113,0436	1,21224	131	131,0377	1,04578
78	78,06319	1,75545	96	96,05136	1,42670	114	114,0433	1,20161	132	132,0374	1,03786
79	79,06239	1,73326	97	97,05083	1,41200	115	115,0429	1,19117	133	133,0371	1,03006
80	80,06161	1,71163	98	98,05032	1,39761	116	116,0425	1,18091	134	134,0368	1,02238
81	81,06085	1,69053	99	99,04981	1,38351	117	117,0422	1,17083	135	135,0365	1,01481
82	82,06011	1,66995	100	100,0493	1,36968	118	118,0418	1,16091	136	136,0363	1,00735
83	83,05939	1,64986	101	101,0488	1,35614	119	119,0414	1,15116	137	137,0360	1,00000
84	84,05869	1,63024	102	102,0483	1,34285	120	120,0411	1,14158			
85	85,05800	1,61109	103	103,0479	1,32983	121	121,0408	1,13215			

It is not known to the author that the Helium atom radius has been measured till now. The known value from Chemistry books $r_{He} = 0.31 \times 10^{-10} m$ is smaller than that one valid for the Hydrogen atom radius $r_H = 0.53 \times 10^{-10} m$. [4] The same ratio applies to the both circular orbit lengths (9). The Helium Bohr-like orbit is thus smaller than Hydrogen Bohr one. Then according to the relation (8) the effective nuclear charge number of Helium atom $Z_{effective}^{He}$ should lie between the Hydrogen and Helium atom number Z^H and Z^{He} , respectively:

$$Z^H = 1 < Z_{effective}^{He} < Z^{He} = 2. \quad (14)$$

And consequently the conceptual Helium atom orbit length $s_{conceptual}^{He}$ should lie between 68^{th} and 137^{th} Bohr-like orbit length(8), (1) listed in *Table 1*:

$$68.07245 < s_{conceptual}^{He} < 137.0360. \quad (15)$$

The official Helium atom radius $r_{He} = 0.31 \times 10^{-10} m$ implies the orbit length $s_{He} = 80.27776$ Compton wave lengths of the electron. The given value is not conceptual but lies between the next neighbouring conceptual values listed in *Table 1*:

$$s(80) = 80.06161 \dots < s_{He} = 80.27776 < s(81) = 81.06085 \dots \quad (16)$$

Following the double-surface approach (1), (10) two possible predictions of the exact Helium atom radius value are offered:

$$r_{predicted}^{He} = 0.3092 \times 10^{-10} m \text{ or } 0.3130 \times 10^{-10} m. \quad (17)$$

The second one has the advantage since it is very close to $r_{QM}^{He} = 0.3136 \times 10^{-10} m$ –the Helium atom radius value derived from the effective nuclear charge number $Z_{effective}^{He} = \frac{27}{16}$ (8), (9) given by the variational method of quantum mechanics [5]. Indeed (1), (9), (13):

$$r_{conceptual}^{He} (81) = \frac{s(81)}{2\pi} \times \lambda_e = \frac{81}{2\pi} \left(2 - \frac{1}{\sqrt{1 + \frac{\pi^2}{81^2}}} \right) \times \lambda_e = 0.3130 \times 10^{-10} m \approx r_{QM}^{He} = 0.3136 \times 10^{-10} m. \quad (18)$$

8. CONCLUSION

The given results in the present paper show that the double-surface concept and QM could have the same essence.

DEDICATION

This fragment is dedicated to my dear sister Darinka for her 69^{th} birthday.

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