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Families of Rational Solutions to the KPI Equation of Order 7 Depending on 12 Parameters

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Abstract: *We construct in this paper, rational solutions as a quotient of two determinants of order 2N = 14 and we obtain what we call solutions of order N = 7 to the Kadomtsev-Petviashvili equation (KPI) as a quotient of 2 polynomials of degree 112 in x, y and t depending on 12 parameters. The maximum of modulus of these solutions at order 7 is equal to $2(2N + 1)^2 = 450$. We make the study of the patterns of their modulus in the plane (x, y) and their evolution according to time and parameters a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5, b_6. When all these parameters grow, triangle and ring structures are obtained.*

Keywords: *KPI equation; Fredholm determinants; Wronskians; rogue waves; lumps.*

PACS numbers: *33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd*

1. Introduction

We consider the Kadomtsev-Petviashvili equation (KPI), first proposed in 1970 [1] in the following normalization

$$(4u_t - 6uu_x + u_{xxx})_x - 3u_{yy} = 0. \tag{1}$$

As usual, subscripts x, y and t denote partial derivatives.

The first rational solutions were constructed in 1977 by Manakov, Zakharov,

Bordag and Matveev [2]. Other more general rational solutions of the KPI equation were found by Krichever in 1978 [3, 4], Satsuma and Ablowitz in 1979 [5], Matveev in 1979 [6], in particuler among many works on this subject.

We construct rational solutions of order N depending on 2N − 2 parameters which can be written as a ratio of two polynomials in x, y and t of degree 2N (N + 1).

The maximum of the modulus of these solutions at order N is equal to $2(2N + 1)^2$. Here we construct the explicit rational solutions of order 7, depending on 12 real parameters, and the representations of their modulus in the plane of the coordinates (x, y) according to the real parameters a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, a_5, b_5, a_6, b_6 and time t. When the parameters grow, we obtain N (N +1)/2 peaks in particular structures, such as triangles, rings, or concentric rings.

2. Rational Solutions to KPI Equation of Order N Depending on 2N − 2 Parameters

The rational solutions to the KPI equation are given by the following result [7, 8]:

Theorem 2.1 The function v defined by

$$v(x,y,t) = -2\frac{|\det((n_{jk})_{j,k\in[1,2N]})|^2}{\det((d_{jk})_{j,k\in[1,2N]})^2} \tag{2}$$

is a rational solution of the KPI equation (1), where

$$
\begin{array}{l}
n_{j1} = \varphi_{j,1}(x,y,t,0),\ 1 \le j \le 2N \quad n_{jk} = \frac{\partial^{2k-2}\varphi_{j,1}}{\partial \epsilon^{2k-2}}(x,y,t,0),\\
n_{jN+1} = \varphi_{j,N+1}(x,y,t,0),\ 1 \le j \le 2N \quad n_{jN+k} = \frac{\partial^{2k-2}\varphi_{j,N+1}}{\partial \epsilon^{2k-2}}(x,y,t,0),\\
d_{j1} = \psi_{j,1}(x,y,t,0),\ 1 \le j \le 2N \quad d_{jk} = \frac{\partial^{2k-2}\psi_{j,1}}{\partial \epsilon^{2k-2}}(x,y,t,0),\\
d_{jN+1} = \psi_{j,N+1}(x,y,t,0),\ 1 \le j \le 2N \quad d_{jN+k} = \frac{\partial^{2k-2}\psi_{j,N+1}}{\partial \epsilon^{2k-2}}(x,y,t,0),\\
2 \le k \le N,\ 1 \le j \le 2N
\end{array}
\tag{3}
$$

The functions φ and ψ are defined in (4), (5), (6), (7)

$$
\begin{array}{l}
\varphi_{4j+1,k} = \gamma_k^{4j-1}\sin X_k, \quad \varphi_{4j+2,k} = \gamma_k^{4j}\cos X_k,\\
\varphi_{4j+3,k} = -\gamma_k^{4j+1}\sin X_k, \quad \varphi_{4j+4,k} = -\gamma_k^{4j+2}\cos X_k,
\end{array}
\tag{4}
$$

For $\le k \le N$, and

$$
\begin{array}{l}
\varphi_{4j+1,N+k} = \gamma_k^{2N-4j-2}\cos X_{N+k}, \quad \varphi_{4j+2,N+k} = -\gamma_k^{2N-4j-3}\sin X_{N+k},\\
\varphi_{4j+3,N+k} = -\gamma_k^{2N-4j-4}\cos X_{N+k}, \quad \varphi_{4j+4,N+k} = \gamma_k^{2N-4j-5}\sin X_{N+k},
\end{array}
\tag{5}
$$

For $1 \le k \le N$.

The functions $\psi_{j,k}$ for $1 \le j \le 2N$, $1 \le k \le 2N$ are defined in the same way, the term X_k is only replaced by Y_k

$$
X_\nu = \frac{\kappa_\nu x}{2} + i\delta_\nu y - i\frac{x_{3,\nu}}{2} - i\frac{\tau_\nu}{2}t - i\frac{e_\nu}{2},
$$

$$
Y_\nu = \frac{\kappa_\nu x}{2} + i\delta_\nu y - i\frac{x_{1,\nu}}{2} - i\frac{\tau_\nu}{2}t - i\frac{e_\nu}{2},
$$

For $1 \le \nu \le 2N$ with k_ν, δ_ν, $x_{t,\nu}$, defined in (9) and parameters e_ν defined by (10)

$$
\begin{array}{l}
\psi_{4j+1,k} = \gamma_k^{4j-1}\sin Y_k, \quad \psi_{4j+2,k} = \gamma_k^{4j}\cos Y_k,\\
\psi_{4j+3,k} = -\gamma_k^{4j+1}\sin Y_k, \quad \psi_{4j+4,k} = -\gamma_k^{4j+2}\cos Y_k,
\end{array}
\tag{6}
$$

For $1 \le k \le N$, and

$$
\begin{array}{l}
\psi_{4j+1,N+k} = \gamma_k^{2N-4j-2}\cos Y_{N+k}, \quad \psi_{4j+2,N+k} = -\gamma_k^{2N-4j-3}\sin Y_{N+k},\\
\psi_{4j+3,N+k} = -\gamma_k^{2N-4j-4}\cos Y_{N+k}, \quad \psi_{4j+4,N+k} = \gamma_k^{2N-4j-5}\sin Y_{N+k}.
\end{array}
\tag{7}
$$

Real numbers λ_j are such that $-1 < \lambda_\nu < 1$, $\nu = 1, \ldots, 2N$ depending on a parameter ọ which will be intended to tend towards 0; they can be written as

$$
\lambda_j = 1 - 2\epsilon^2 j^2, \quad \lambda_{N+j} = -\lambda_j, \quad 1 \le j \le N.
\tag{8}
$$

The terms κ_ν, δ_ν, γ_ν, τ_ν and $x_{r,\nu}$ are functions of λ_ν, $1 \le \nu \le 2N$; they are defined by the formulas:

$$
\begin{array}{l}
\kappa_j = 2\sqrt{1-\lambda_j^2}, \quad \delta_j = \kappa_j\lambda_j, \quad \gamma_j = \sqrt{\frac{1-\lambda_j}{1+\lambda_j}},\\
x_{r,j} = (r-1)\ln\frac{\gamma_j - i}{\gamma_j + i}, \quad r = 1,3, \quad \tau_j = -12i\lambda_j^2\sqrt{1-\lambda_j^2} - 4i(1-\lambda_j^2)\sqrt{1-\lambda_j^2},\\
\kappa_{N+j} = \kappa_j, \quad \delta_{N+j} = -\delta_j, \quad \gamma_{N+j} = \gamma_j^{-1},\\
x_{r,N+j} = -x_{r,j},, \quad \tau_{N+j} = \tau_j \quad j = 1, \ldots, N.
\end{array}
\tag{9}
$$

e_ν, $1 \le \nu \le 2N$ are defined in the following way:

$$
\begin{array}{l}
e_j = 2i\left(\sum_{k=1}^{1/2M-1} a_k (je)^{2k-1} - i\sum_{k=1}^{1/2M-1} b_K (je)^{2k-1}\right),\\
e_{N+j} = 2i\left(\sum_{k=1}^{1/2M-1} a_k (je)^{2k-1} + i\sum_{k=1}^{1/2M-1} b_k (je)^{2k-1}\right), \quad 1 \le j \le N,\\
a_k, b_k \in \mathbf{R}, \quad 1 \le k \le N-1.
\end{array}
\tag{10}
$$

e_v, $1 \le v \le 2N$ are real numbers defined by:

$$\epsilon_j = 1, \quad \epsilon_{N+j} = 0 \quad 1 \le j \le N. \tag{11}$$

3. EXPLICIT EXPRESSION OF RATIONAL SOLUTIONS OF OR-DER 7 DEPENDING ON 12 PARAMETERS

In the the following, we explicitly construct rational solutions to the KPI equation of order 7 depending on 12 parameters.

We cannot give the complete analytic expressions of the solutions to the KPI equation of order 7 with twelve parameters because of their lengths.

The rational solutions to the KPI equation can be written as v(x, y, t) = $-2\frac{|d_3(x,y,t)|^2}{d_1(x,y,t)^2}$

with d_3 and d_1 polynomials of degree 112 in x, y and t. The number of terms of the polynomials of the numerator d_3 and denominator d_1 of the solutions are shown in the table below (Table 1) when only one of the parameters a_i and b_i are set non equal to 0.

Table1. *Number of terms for the polynomials d_3 and d_1 of the solutions to the KPI equation in the case N = 7*

a_i	d_3	d_1	b_i	d_3	d_1
1	86 927	46 383	1	86 926	45 036
2	55 509	29 584	2	55 509	28 790
3	42 219	22 489	3	42 309	21 962
4	34 968	18 608	4	34 968	18 167
5	30 342	16 132	5	30 342	15 778
6	26 595	14 099	6	26 595	13 846

We give patterns of the modulus of the solutions in the plane (x, y) of coordinates in functions of parameters a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5, b_6 and time t.

The maximum of modulus of theses solutions is checked equal in this case N = 7 to $2(2N + 1)^2 = 2 \times 15^2 = 450$.

When all the parameters are equal to 0, we obtain the lump L_7 with a highest amplitude of the modulus equal to 450.

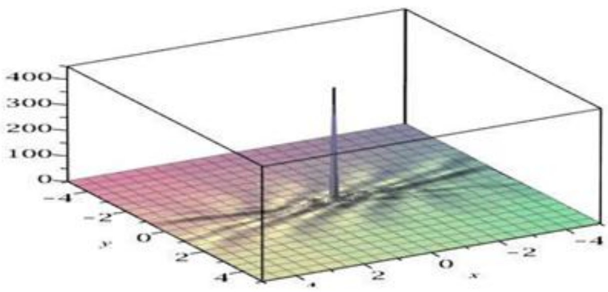

Figure. *Solution of order 7 to KPI, for t = 0 when all parameters equal to 0*

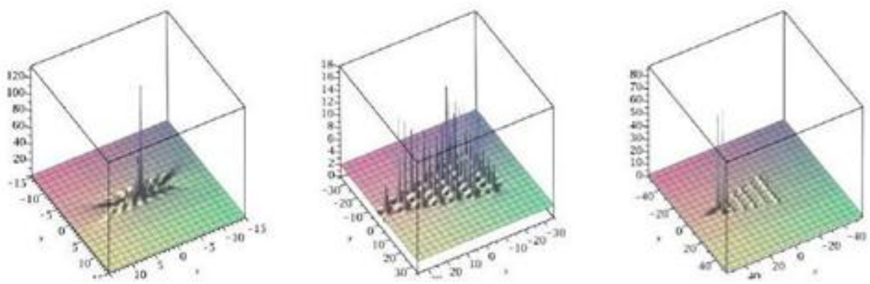

Figure1. *Solution of order 7 to KPI, for a_1 6= 0 and all other parameters equal to 0; on the left for t = 0 and a_1 = 1; in the center for t = 0 and $a_1 = 10^3$; on the right for t = 1 and $a_1 = 10^5$.*

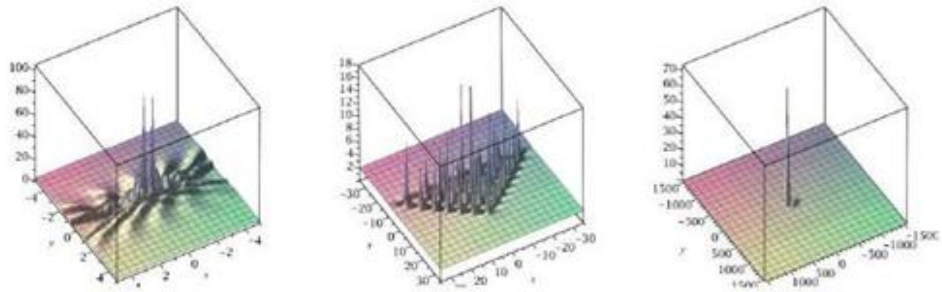

Figure2. *Solution of order 7 to KPI, for b_1 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $b_1 = 1$; in the center for $t = 0$ and $b_1 = 10^3$; on the right for $t = 10$ and $b_1 = 10^6$*

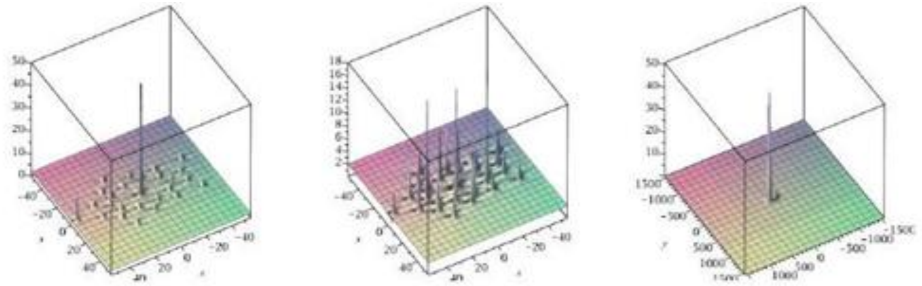

Figure3. *Solution of order 7 to KPI, for a_2 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $a_2 = 10^6$; in the center for $t = 0.1$ and $a_2 = 10^6$; on the right for $t = 10$ and $a_2 = 10^3$.*

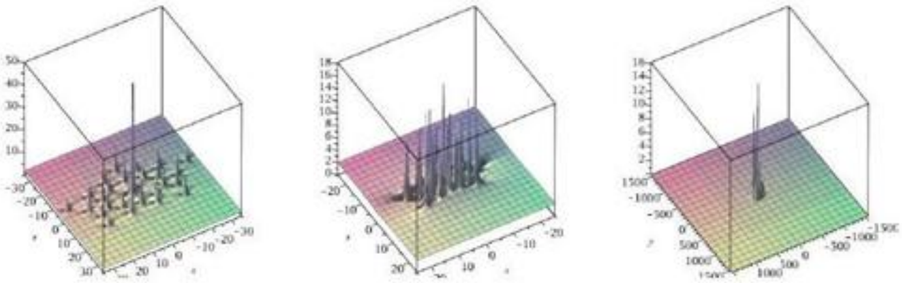

Figure4. *Solution of order 7 to KPI, for b_2 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $b_2 = 10^5$; in the center for $t = 0.1$ and $b_2 = 10^3$; on the right for $t = 10$ and $b_2 = 10^5$*

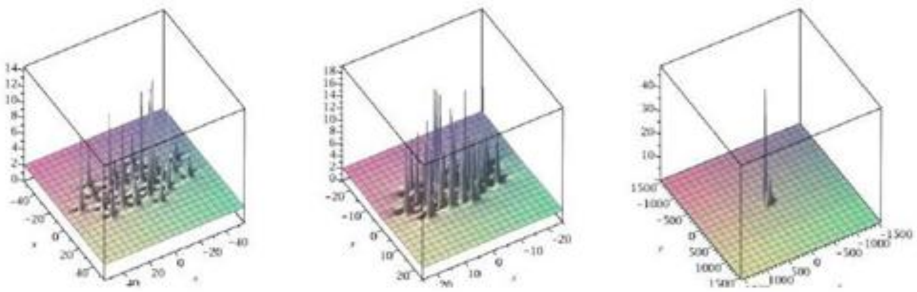

Figure5. *Solution of order 7 to KPI, for a_3 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $a_3 = 10^8$; in the center for $t = 0.1$ and $a_3 = 10^5$; on the right for $t = 10$ and $a_3 = 10^3$*

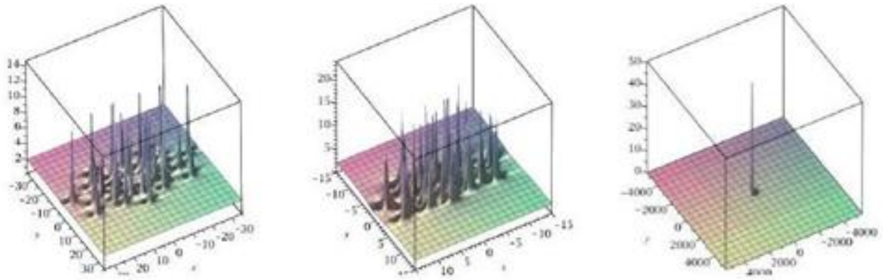

Figure6. *Solution of order 7 to KPI, for b_3 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $b_3 = 10^7$; in the center for $t = 0.1$ and $b_3 = 10^4$; on the right for $t = 10$ and $b_3 = 10^3$*

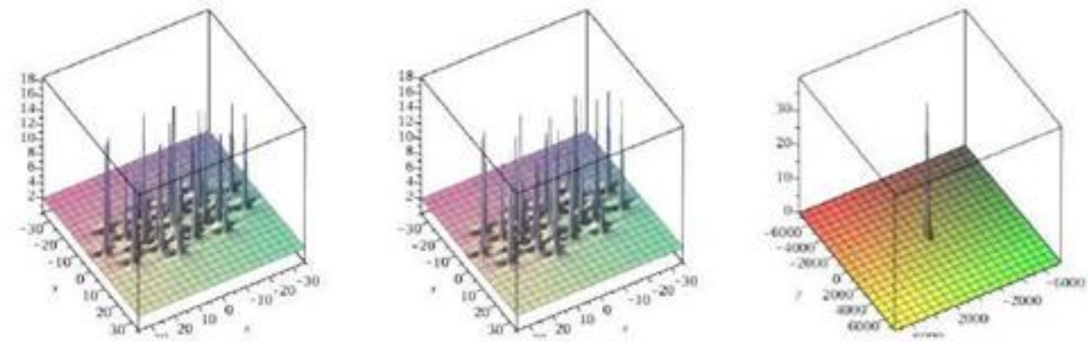

Figure7. *Solution of order 7 to KPI, for a_4 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $a_4 = 10^9$; in the center for $t = 0.1$ and $a_4 = 10^9$; on the right for $t = 10$ and $a_4 = 10^3$*

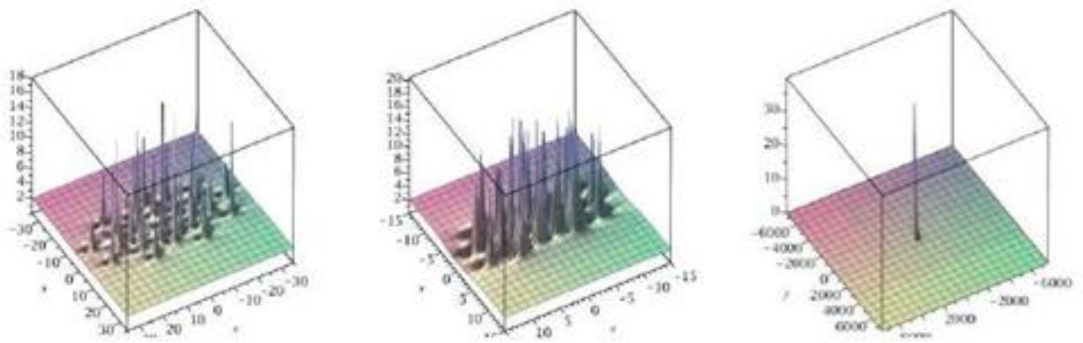

Figure8. *Solution of order 7 to KPI, for b_4 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $b_4 = 10^9$; in the center for $t = 0.1$ and $b_4 = 10^5$; on the right for $t = 10$ and $b_4 = 10^3$*

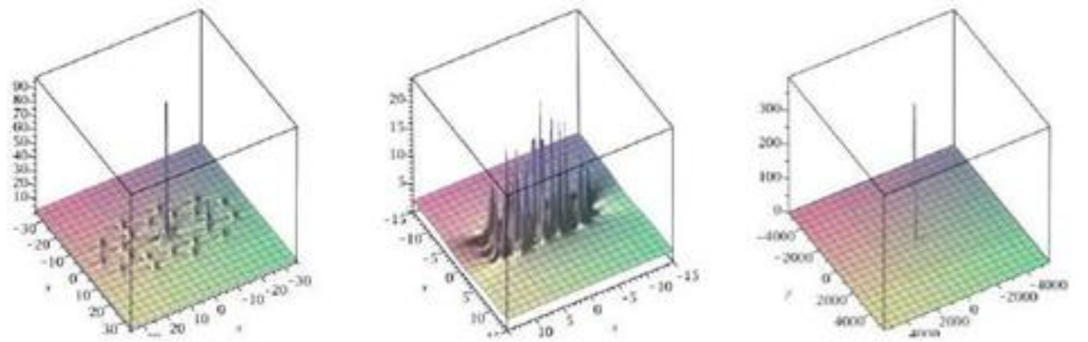

Figure9. *Solution of order 7 to KPI, for a_5 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $a_5 = 10^{11}$; in the center for $t = 0.1$ and $a_5 = 10^5$; on the right for $t = 20$ and $a_5 = 10^{11}$*

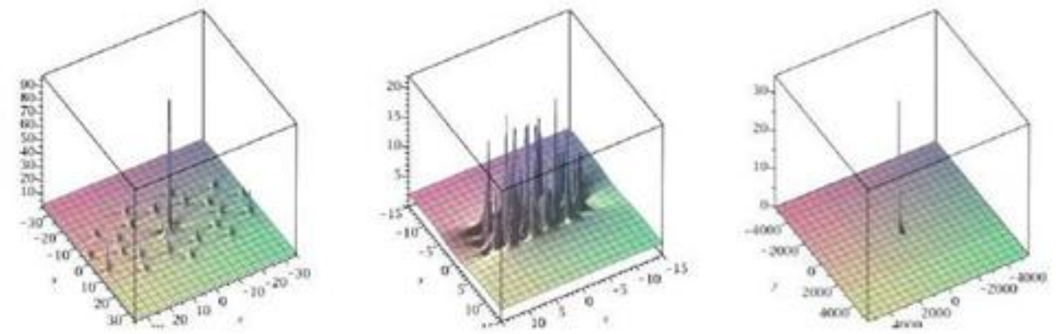

Figure10. *Solution of order 7 to KPI, for b_5 6= 0 and all other parameters equal to 0; on the left for $t = 0$ and $b_5 = 10^{12}$; in the center for $t = 0.1$ and $b_5 = 10^5$; on the right for $t = 50$ and $b_5 = 10^{11}$*

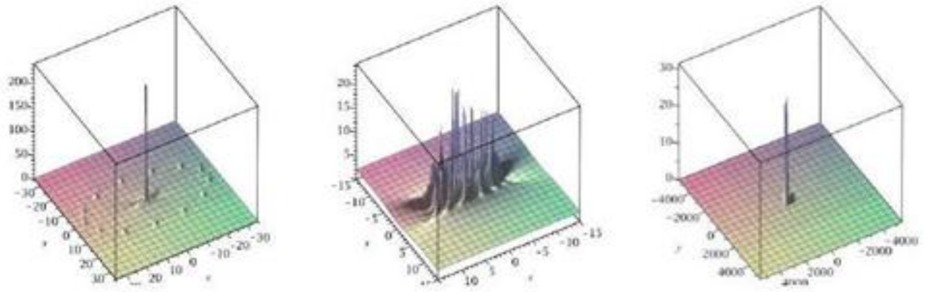

Figure11. *Solution of order 7 to KPI, for a_6 6= 0 and all other parameters equal to 0; on the left for t = 0 and $a_6 = 10^{14}$; in the center for t = 0.1 and $a_6 = 10^5$; on the right for t = 20 and $a_6 = 10^{11}$*

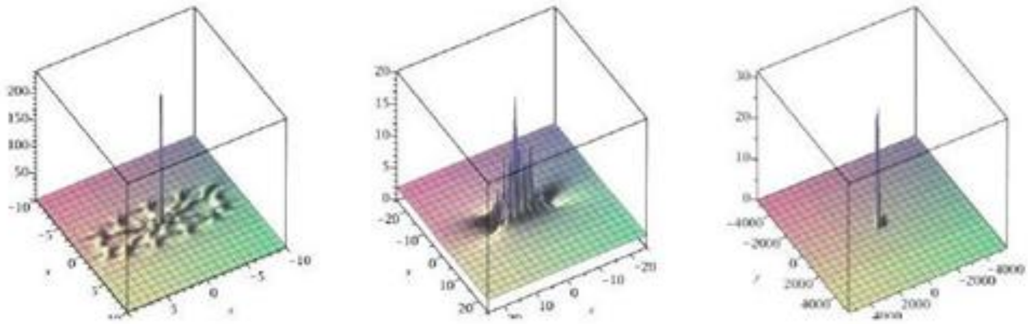

Figure12. *Solution of order 7 to KPI, for b_6 6= 0 and all other parameters equal to 0; on the left for t = 0 and $b_6 = 10^6$; in the center for t = 0.1 and $b_6 = 10^3$; on the right for t = 20 and $b_6 = 10^{11}$*

4. CONCLUSION

We construct 7-th order rational solutions to the KPI equation depending on 12 real parameters. These solutions can be expressed in terms of a ratio of two polynomials of degree 2N (N + 1) = 112 in x, y and t. The maximum of the modulus of these solutions is equal to $2(2N + 1)^2 = 450$; this solution which can be called lump L_7 is obtained when all parameters are equal to 0 at the instant t = 0. Here we have given a complete description of rational solutions of order 7 with 12 parameters by constructing explicit expressions of polynomials of these solutions.

We deduce the construction of the modulus of solutions in the (x, y) plane of coordinates; different structures appear. For a given t close to 0, when one parameter grows and the other ones are equal to 0 we obtain triangles, rings or concentric rings. There are six types of patterns. In the cases $a_1 \neq 0$ or $b_1 \neq 0$ we obtain triangles with a maximum of 28 peaks (figures 1 and2); for $a_2 \neq 0$ or $b_2 \neq 0$, we have 3 concentric rings with two of them with 10 peaks and an another with 5 peaks (figures 3 and 4). For $a_3 \neq 0$ or $b_3 \neq 0$, we obtain 4 concentric rings without central peak with 7 peaks on each of them (figures 5 and 6). For $a_4 \neq 0$ or $b_4 \neq 0$, we have 3 concentric rings with 9 peaks, with a peak in the center(figures 7 and 8). For $a_5 \neq 0$ or $b_5 \neq 0$, we obtain 2 concentric rings without central peak with 11 peaks on each of them (figures 9 and 10). For $a_6 \neq 0$ or $b_6 \neq 0$, we have only one ring with 13 peaks (figures 11 and 12). But, when t grows, all the structures disappear very quickly and the heights of the peaks decrease even more quickly.

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