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Basic Principles of the Field Theory: Connection of the Field-Theory Equations with the Equations of Mathematical Physics

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Abstract: *It is shown that the equations of mathematical physics possess invariant properties. And this discloses a connection between the field-theory equations and the equations of mathematical physics. This connection enables to understand the basic principles of field theory and the connection between physical fields and material media.*

The field-theory equations are connected with the equations of mathematical physics, which describe material media (material systems). The equations of conservation laws for energy, linear momentum, angular momentum, and mass, which describe such material systems as the thermodynamic, gas-dynamic and cosmic systems, as well as the systems of charged particles and others, are precisely such equations of mathematical physics.

1. Introduction

The present investigation bases on the properties of conservation laws. In this case, two specific features of the conservation laws, which were established by the author with the help of skew-symmetric forms, are taken into account.

1. *The conservation laws for physical fields (described by the field-theory equations) and the conservation laws for material media (described by the equations of mathematical physics) are different conservation laws.*

The conservation laws for physical fields are conservation laws that state the presence of conservative quantities or objects (structures). Such conservation laws, which can be named exact ones, are described by closed exterior skew-symmetric forms [1]. (The Noether theorem is an example). And the conservation laws for material media are conservation laws for energy, linear momentum, angular momentum, and mass, which are noncommutative ones. Such conservation laws are described by differential equations. (These conservation laws establish a balance between the change physical quantities of material media and external actions.)

2. *There exists a connection between the conservation laws for material media and those for physical fields. A peculiarity consists in the fact that this connection is realized discretely in the evolutionary process if there are any degrees of freedom.*

Such a process, which is described with the help of skew-symmetric differential forms, discloses the connection of the field-theory equations and the equations of mathematical physics, and this enables to understand the foundations of field theory.

The present investigation also bases on the properties of the mathematical physics equations that are hidden and reveal only under description of evolutionary processes. These properties were obtained when investigating the integrability of the mathematical physics equations that depends on the consistency of the conservation law equations.

Under such investigation the relations for functionals that characterize the material system state is obtained from the mathematical physics equations. It appears the functionals such as wave functions, entropy, the action functional, Pointing's vector, Einstein's tensor and so on are just such state functionals. It is known that the field-theory equations are equations for such functionals. And this substantiates a connection between the mathematical physics equations and the field-theory equations. [In this case the properties of physical quantities of material media were taken into account. Since the physical quantities (like temperature, energy, pressure or density) relates to a single material medium, a connection between them should exist. Such a connection is described by state functionals. The functionals pointed above are such state functionals.]

The results of this work were obtained by using skew-symmetric forms. In addition to exterior forms, the skew-symmetric forms, which are obtained from differential equations and, in distinction to exterior forms, are evolutionary ones and are defined on nonintegrable manifolds, were used.

2. Basis of the Field Theory: Closed Exterior Forms

Closed exterior forms possess unique properties, namely, they are invariants. Since a closed form is a differential (the total one $\theta^p = d\,\theta^{p-1}$ if the form is exact, or the interior one $\theta^p = d_\pi\,\theta^{p-1}$ on some pseudostructure π if the form is inexact) [2, 3], it is obvious that a closed form will turn out to be invariant under all transformations that conserve the differential. The nondegenerate transformations in mathematics and mathematical physics such as the unitary, tangent, canonical, gradient, and other nondegenerate transformations are examples of such transformations that conserve the differential. In this case the covariance of a dual form is directly connected with the invariance of an inexact exterior closed form.

Invariant properties of closed exterior forms and covariance properties of a dual form explicitly or implicitly manifest themselves essentially in all invariant formalisms of field theory, such as the Hamilton formalism, tensor calculus, group theory, Yang-Mills theory and others. The postulates of invariance or covariance are the basis of field theories [4].

The gauge transformations of field theories are transformation of closed exterior forms (transformation that conserve the differential). The field theory operators are operators connected with nondegenerate transformations of closed exterior differential forms [2].

A role of closed exterior forms in field theories, namely, in the theories that describe physical fields, is explained by the fact that they correspond to the conservation laws for physical fields.

From the closure conditions for exterior differential form $d\theta^p = 0$ [2] one can see that the closed exterior differential form is a conservative quantity (here θ^p is the exterior differential form of degree p (p- form)). This means that the closed exterior differential form can correspond to conservation law for physical fields, namely, to existence of a certain conservative physical quantity. If the exterior form is a closed inexact form, i.e. is closed only on some pseudostructure, the closure condition of exterior form is written as $d_\pi\,\theta^p = 0$, where the pseudostructure π obeys the closure condition for dual form $d_\pi {}^*\theta^p = 0$ (here ${}^*\theta^p$ is the dual form.) From conditions of closure for the dual form (pseudostructure) and closed inexact form (conservative quantity) one can see that the dual form and inexact form describe the structure on which the conservation law is fulfilled. The physical structures, which made up physical fields and correspond to conservation laws, are just such structures. This substantiate the fact that the properties of closed inexact forms and relevant dual forms must lie at the basis of theories that describe physical fields.

One can make sure that closed inexact exterior or dual forms are solutions to the field-theory equations.

Here it should be noted the following. [The expressions within the brackets will be disclosed below.]

Closed exterior forms and its corresponding dual form of the first degree (and, in addition, of zero degree) are solutions to the equations of Hamilton formalism.

Dirac's *bra*- and *cket*- vectors made up a closed exterior form of zero degree.

The electromagnetic field equations are based on closed exterior forms and its corresponding dual form of second degree (and, in addition, on forms of first and zero degree).

The gravitational field are based on closed exterior and dual forms of third degree (and, in addition, on forms of second, first and zero degrees). (Below it will be disclosed a specific features of the Einstein equations that have solutions of the forms of first degree.)

The identical relations of the field theory such as

- the Poincare invariant, which connects closed exterior forms of first degree;
- the relations $d\theta^2 = 0,\ d^*\theta^2 = 0$ are those for closed exterior forms of second degree obtained from the Maxwell equations;
- the Bianchi identity for gravitational field

are identical relations of closed exterior differential forms. That is, they are identical relations from which closed exterior forms are obtained.

[One can see that field theory equations are connected with closed exterior forms of a certain degree. This enables one to introduce a classification of physical fields in degrees of closed exterior forms. Such a classification shows that there exists an internal connection between field theories that describe physical fields of various types. It is evident that the degree of closed exterior forms is a parameter that integrates field theories into unified field theory. (A significance of exterior differential forms for field theories consists in the fact that they disclose the properties that are common for all field theories and physical fields irrespective to their specific type. This is a step to building a unified field theory.)]

Thus one can see that the existing field theories are based on the properties of closed exterior forms that correspond to the conservation laws for physical fields.

In this case, as it was noted, the closed exterior forms are found from the field-theory equations obtained from the conditions of invariance and covariance.

As it will be shown below, the closed inexact exterior forms, which describe conservation laws for physical fields, are also obtained from the mathematical physics equations, which describe material media. And this will be a substantiation of the fact that there exists a connection between the field-theory and the equations of mathematical physics. Such a connection will point out to the fact that the equations of mathematical physics made up the foundations of field theory.

3. Hidden Properties and Peculiarities of the Equations of Mathematical Physics. Evolutionary Relation. Generating Closed Inexact Exterior Forms

The field-theory equations are connected with the equations of mathematical physics, which describe material media (material systems).

The equations of mathematical physics which consist of the equations of the conservation laws for energy, linear momentum, angular momentum, and mass, are precisely such equations [5-9]. They describe such material systems as the thermodynamic, gas-dynamic and cosmic systems, as well as the systems of charged particles and others. (Examples of such equations and references to such equations are presented in Appendix 1.)

It occurs that such equations possess the properties that are hidden and reveal only under interaction of the conservation law equations between themselves. Such properties are revealed when investigating the integrability of the equations of mathematical physics, which depends on the consistency of equations in the set of equations.

3.1. Analysis of Consistency of the Conservation Law Equations. Evolutionary Relation for the State Functional

The consistency of the conservation law equations is realized under correlation of the conservation law equation between themselves. Let us analyze the correlation of the equations that describe the conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with the material system), and the second is an accompanying one (this frame of reference is connected with the manifold built by the trajectories of the material system elements). (The Euler and Lagrange frames of reference are examples of such frames).

In the inertial frame of reference the energy equation can be reduced to the form:

$$\frac{D\psi}{Dt} = A_1$$

where D/Dt is the total derivative with respect to time, A_1 is a quantity that depends on specific features of material system and on external energy actions onto the system, ψ is the state functional that specifies a material system. The action functional, entropy, wave function, Einstein's tensor, Pointing's vector and others can be regarded as examples of the functional ψ. (Thus, the equation for energy presented in terms of the action functional S has a similar form: $DS/Dt = L$, where $\psi = S$, $A_1 = L$ is the Lagrange function. In mechanics of continuous media the equation for energy of an ideal gas can be presented in the form [8]: $Ds/Dt = 0$, where s is the entropy.) [In the present case the properties of physical quantities of material medium are accounted. As it was already mentioned in introduction, the connection between the physical quantities of material medium (like temperature, energy, pressure or density) has to be exist since they relate to a single state functionals. Physical meaning of such functionals and the equations for functionals like entropy, action functional and Pointing's vector were considered in paper [10].]

In the accompanying frame of reference the total derivative with respect to time is transformed into the derivative along the trajectory. Now the equation of energy is written in the form

$$\frac{\partial\psi}{\partial\xi^1} = A_1 \qquad (1)$$

Here ξ^1 is the coordinate along the trajectory. In a similar manner, in the accompanying reference frame the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial\psi}{\partial\xi^\nu} = A_\nu, \quad \nu = 2\ldots \qquad (2)$$

where ξ^ν are the coordinates in the direction normal to the trajectory, A_ν are the quantities that depend on the specific features of material system and external force actions. [In the accompanying frame of reference the conservation law equations of gas-dynamics for energy and linear momentum see Appendix 2 (monograph [8], Chapter 6).]

Eqs. (1) and (2) can be convoluted into the relation

$$d\psi = A_\mu d\xi^\mu \quad (\mu = 1,\nu) \qquad (3)$$

where $d\psi$ is the differential expression $d\psi = (\partial\psi/\partial\xi^\mu)d\xi^\mu$.

Relation (3) can be written as

$$d\psi = \omega \qquad (4)$$

here $\omega = A_\mu d\xi^\mu$ is the skew-symmetric differential form of the first degree. (A summing over repeated indices is carried out.)

Relation (4) has been obtained from the equation of conservation laws for energy and linear momentum. In this relation the form ω is that of the first degree. If the equations of the conservation laws for angular momentum be added to the equations for energy and linear momentum, this form will be a form of the second degree. And in combination with the equation of the conservation law for mass this form will be a form of degree 3. In general case the evolutionary relation can be written as

$$d\psi = \omega^p \qquad (5)$$

where the form degree p takes the values $p = 0,1,2,3$. (The relation for $p = 0$ is an analog to that in the differential forms, and it was obtained from the interaction of energy and time.) [A concrete form of relation (4) and its properties in the case of the Euler and Navier-Stokes equations were considered in paper [11]. In this case the functional ψ is the entropy S. A concrete form of relation (5) for $p = 2$ were considered for electromagnetic field in paper http://arxiv.org/pdf/math-ph/0310050v1.pdf. In this case the functional ψ is Pointing's vector. The relation for Einstein's tensor is obtained when integrating the evolutionary relation for $p = 3$ (about this it will be told below).]

Since the conservation law equations are evolutionary ones, the relations obtained are also evolutionary relations, and the skew-symmetric forms ω and ω^p are evolutionary ones as well.

Significance of the evolutionary relation consists in the fact that it answers the question of whether or not the conservation law equations are consistent. As it will be shown below, the evolutionary processes in material systems and the processes of the physical fields generation depend on that. (This discloses the properties of field-theory equations.)

3.1.1. Properties of Evolutionary Relation

Evolutionary relation obtained from the conservation law equations possesses some peculiarity. This relation proves to be nonidentical and selfvarying.

3.1.2. Nonidentity of the Evolutionary Relation

Evolutionary relation proves to be a nonidentical relation since the differential form in the right-hand side of this relation is not a closed form, and, hence, this form cannot be a differential like the left-hand side. The evolutionary relation was obtained in the accompanying frame of reference, which is connected with the manifold built up by the trajectories of the material system elements. Such a manifold is a deforming nonintegrable one. The skew-symmetric form defined on nonintegrable manifold cannot be closed since the commutator of skew-symmetric form defined on such manifold includes an additional term, namely, the commutator of metric form, which is nonzero (because the metric form of nonintegrable manifold is not closed one). Some properties of skew-symmetric form, which basis are nonintegrable manifolds, are presented in [12] http://arxiv.org/abs/1007.4757 (see pp. 1-4, 21-24).

The skew-symmetric form $\omega = A_\mu d\xi^\mu$ in relation (4) is not a close form since its differential is nonzero. The differential $d\omega$ form $\omega = A_\mu d\xi^\mu$ can be written as $d\omega = K_{\alpha\beta} d\xi^\alpha d\xi^\beta$, where $K_{\alpha\beta} = A_{\beta;\alpha} - A_{\alpha;\beta}$ are the components of the commutator of the form ω, and $A_{\beta;\alpha}$, $A_{\alpha;\beta}$ are the covariant derivatives. If we express the covariant derivatives in terms of the connectedness (if it is possible), we obtain the following expression for the commutator components of the form ω (see [12]):

$$K_{\alpha\beta} = \left(\frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta}\right) + \left(\Gamma^\sigma{}_{\beta\alpha} - \Gamma^\sigma{}_{\alpha\beta}\right) A_\sigma$$

The coefficients A_μ of the form ω have been obtained either from the equation of the conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The first term in the commutator of the form ω made up by derivatives of such coefficients is nonzero. The expressions ($\Gamma^\sigma{}_{\beta\alpha} - \Gamma^\sigma{}_{\alpha\beta}$) entered into the second term are just components of commutator of metric form that specifies the manifold deformation and hence is nonzero (see [12]). Hence, it results that the differential of the form ω is nonzero. Thus, the form ω proves to be unclosed and cannot be a differential like the left-hand side.

Since the evolutionary form is unclosed and is not a differential, the evolutionary relation (4) turns out to be nonidentical.

In similar manner it can be shown that general relation (5) is also nonidentical. In this case the evolutionary forms of degree $(1-3)$ will be unclosed because the commutator of these evolutionary forms will contain nonvanishing commutators of the metric form of the first-, second- and third degrees specifying, respectively, torsion, rotation and curvature.

Hence, without a knowledge of a particular expression for evolutionary forms one can argue that for actual processes the evolutionary relation proves to be nonidentical. (It should be noted that the relation obtained will remain to be nonidentical regardless of the accuracy with what the conservation law equations were written. And this fact has a physical meaning.)

3.1.3. Self Variation of the Evolutionary Relation

The evolutionary relation possesses one more peculiarity, namely, this relation is a selfvarying relation.

The evolutionary nonidentical relation is a selfvarying one, because, firstly, it is a nonidentical, namely, it contains two objects one of which appears to be unmeasurable, and, secondly, it is an evolutionary relation, that is, the variation of any object of the relation in some process leads to a variation of another object; and, in turn, the variation of the latter leads to variation of the former. Since one of the objects is an unmeasurable quantity, the other cannot be compared with the first one, and hence, the process of mutual variation cannot stop.

3.2. Significance of Evolutionary Relation for the Equations of Mathematical Physics

The evolutionary relation has an unique significance for the equations of mathematical physics. This relation, firstly, discloses specific features of the solutions to the equations of mathematical physics, such as an existence of double solutions (which enable to describe the evolutionary processes in material medium and the processes of emergence of various structures). And, secondly, the evolutionary relation discloses a connection between the field-theory equations and the equations of mathematical physics for material media. (Such possibilities of evolutionary relation relate to a nontraditional mathematical apparatus of evolutionary forms, which contains such elements as the degenerate transformations and generating closed exterior forms.)

3.2.1. Double Solutions of the Equations of Mathematical Physics

The evolutionary relation was obtained when investigating the consistency of the conservation law equations. The nonidentity of the evolutionary relation points out to the fact that the conservation law equations appear to be inconsistent. This means that the initial set of equations of mathematical physics proves to be nonintegrable (it cannot be convoluted into identical relation for differentials and be integrated). This point to that the solutions to the mathematical physics equations are not functions (they will depend on the commutator of the form ω^p).

However, under additional conditions, as it follows from the evolutionary relation, the mathematical physics equations can have solutions which are functions. It is possible in the case *when from the evolutionary skew-symmetric form in the right-hand side of nonidentical evolutionary relation a closed skew-symmetric form, which is a differential, is realized.* In this case the identical relation is obtained from the nonidentical relation, and this will point out to a consistency of the conservation law equations and an integrability of the mathematical physics equations.

Here there is some delicate matter. From the evolutionary unclosed skew-symmetric form, which differential is nonzero, one can obtain a closed exterior form with a differential being equal to zero only under ***degenerate transformation***, namely, under a transformation that does not conserve differential. (The Legendre transformation is an example of such a transformation.)

Such degenerate transformations can take place under additional conditions, which are due degrees of freedom. *The vanishing of such functional expressions as determinants, Jacobians, Poisson's brackets, residues, and others corresponds to these additional conditions.* The conditions of degenerate transformation specify the integrable structures on which the solutions become discrete functions. These conditions can be realized under a change of nonidentical evolutionary relation, which, as it was noted, appears to be a selfvarying relation.

If the conditions of degenerate transformation are realized, from the unclosed evolutionary form ω^p (see evolutionary relation (5)) with non vanishing differential $d\omega^p \neq 0$, one can obtain a closed inexact (only on some pseudostructure) exterior form with vanishing (interior) differential.

That is, it is realized the transition

$$d\omega^p \neq 0 \to \text{(degenerate transformation)} \to d_\pi \omega^p = 0,\ d_\pi {}^*\omega^p = 0$$

The realization of the conditions $d_\pi \omega^p = 0$ and $d_\pi {}^*\omega^p = 0$ means that it is realized the closed dual form ${}^*\omega^p$, which describes some structure π (which is a pseudostructure with respect to its metric properties), and the closed exterior (inexact) form ω^p_π, which basis is a pseudostructure, is obtained.

On an pseudostructure from evolutionary relation (5) it follows the relation

$$d\psi_\pi = \omega_\pi^p \tag{6}$$

which occurs to be an identical one, since the form ω^p_π is a differential.

The identity of the relation (6) obtained from the evolutionary relation means that on the pseudostructure the equations of conservation laws (of the equations of mathematical physics) become consistent. This points out to that the equations of mathematical physics become locally integrable (only on pseudostructure). In this case the pseudostructure is an integrable structure. The solutions to the mathematical physics equations on integrable structures are generalized solutions, which are discrete functions, since they are realized only under additional conditions (on the integrable structures). [On integrable structures the desired quantities of the material system, such as the temperature, pressure, density, become functions of only independent variables and do not depend on the commutator (and on the path of integrating). Such functions may be found by means of integrating (on integrable structures) the equations of mathematical physics. (Since generalized solutions are defined only on realized integrable structures, they or their derivatives have discontinuities in the direction normal to integrable structure [13].) The solutions on characteristics or on potential surfaces are examples of such generalized solutions.]

Thus, *from the evolutionary relation it follows that the equations of mathematical physics have solutions of two types*:

(1) *solutions that are not functions*, i.e., they depend not only on independent variables, and

(2) *the generalized solutions, which are discrete functions*.

The specific feature is the fact that these solutions are defined on different spatial objects. The solutions of the first type are solutions defined on tangent nonintegrable manifold of original mathematical physics equations. And the solutions of the second type are generalized solutions defined on integrable structures that arise spontaneously under realization of additional conditions caused, for example, by degrees of freedom of the material medium described.

Such a peculiarity of the solutions to the equations of mathematical physics has a deep physical meaning, as it will be shown below, the double solutions describe the evolutionary processes proceeding in material medium, namely, the evolution of the material system state that is accompanied by emergence of observable formations and physical structures. This is also significant for the description of physical fields. As it will be shown below, the structure that make up physical fields are just such physical structures, i.e. structures that are generated by material media.)

3.3. Physical Meaning of Double Solutions to the Equations of Mathematical Physics. Description of the Material Medium State and an Advent of Observable Formations and Physical Structures

From the evolutionary relation it follows that this relation can describe the material medium state. This is due to the fact that it includes the state functional, which specifies the material system state. But here there is a peculiarity.

Although the evolutionary relation includes the state functional (which specifies the material medium state), but, since this relation is nonidentical one, from this relation one cannot get the differential of the state functional $d\psi$. This points out to the absence of the state function and means that the material medium is in the non-equilibrium state.

The non-equilibrium means that an internal force acts in material medium. It is evident that the internal force is described by the commutator of skew-symmetric form ω^p. (Everything that gives a contribution into the commutator of evolutionary form ω^p leads to emergence of internal forces that causes the non-equilibrium state of material medium (see [11]).) The solutions of the first type, which are not functions (since they depend on commutator of the form ω^p) describe such non-equilibrium state of material medium.

Another property of the nonidentical evolutionary relation, namely, its selfvariation, points out to the fact that the non-equilibrium state of material medium turns out to be selfvarying. State of material medium changes, but, in this case, it keeps to be non-equilibrium one during this process, because the evolutionary relation remains to be nonidentical during the process of selfvariation.

The realization of identical relation from evolutionary relation is indicative of the transition of material medium into the locally-equilibrium state. From identical relation one can find the differential of the state functional, and this points out to a presence of the state function and the transition of material medium from non-equilibrium state into equilibrium one. However, such a state of material medium turns out to be realized only locally due to the fact that differential of the state functional obtained is an differential interior (only on pseudostructure). And yet the total state of material medium remains to be non-equilibrium state because the evolutionary relation, which describes the material medium state, remains nonidentical one. (That is, there exists a duality. Nonidentical evolutionary relation goes on to act simultaneously with identical relation.)

[It can be noted that these results points out to the fact that the functionals of evolutionary relation are really state functionals. In this case they locally convert into the state function under realization of identical relation.]

The transition from non-equilibrium state to locally equilibrium state means that unmeasurable quantity, which is described by the evolutionary form commutator and act as internal force, converts into a measurable quantity of material medium. (However, not all unmeasurable quantity converts into measurable one since the process of transition into equilibrium state executes only locally. In this case the total state of material system keeps to be a nonequilibrium state.)

The transition of unmeasurable quantity into a measurable quantity of material medium reveals in emergence in material medium of some observed formations. Waves, vortices, fluctuations, turbulent pulsations and so on are examples of such formations. The intensity of such formations is controlled by a quantity accumulated by the evolutionary form commutator. (This discloses a mechanism of such processes like an origin of vortices and turbulence [11,14].)

Such emerged formations are described by generalized solutions to the equations of mathematical physics. The functions that correspond to generalized solutions, as it is known, are discrete functions, that have breaks (of functions itselves or its derivatives). The realization of integrable structures with such discontinuous functions and the transition from nonintegrable tangent manifold to integrable structure just describes the emergence of such formations. [It should be emphasized that the existence of double solutions and *degenerate transformation of the evolutionary skew-symmetric form* enables to describe the emergence of discrete formations. This cannot be described within the framework of another mathematical formalisms.]

The process of the discrete formation emergence relates to emergence of physical structures. This is described by the realization of the closed inexact exterior and dual forms.

3.3.1. Differential-Geometrical Structure

The closed dual form and associated closed inexact exterior form make up a differential-geometrical structure.

Such a differential-geometrical structure possesses a duality.

On the one hand, it describes the pseudostructure with generalized solution, namely, the integrable structure, and, on the other hand, it describes a pseudostructure with conservative quantity, i.e. a physical structure on which the exact conservation law is fulfilled.

The realization of integrable structures describes the transition of a material system from non-equilibrium state into a locally equilibrium state, which is accompanied by an emergence in material medium of observable formations.

On the other hand, the realization of pseudostructures with conservative quantity points out to emergence of physical structures, on which the exact conservation laws are fulfilled, that is, an emergence of physical structures from which physical fields can be formatted.

Such a duality of differential-geometrical structures discloses a connection between physical fields and material media. One can see that the transition of material media from non-equilibrium state to locally-equilibrium one is accompanied by the emergence of observable formations in material media and origination of physical structures that made up physical fields.

Physical structures and observed formations are a manifestation of the same phenomenon. (Light is an example of manifestation of such a duality, namely, as a massless particle (photon) and as a wave.) However, physical structures and observed formations are not identical objects. Whereas the wave is an observable formation, the element of wave front made up the physical structure in the process of its motion.

Thus, it has been shown that the equations of mathematical physics can describe evolutionary processes executed in material media and accompanied with emergence of observable formations (waves, vortices, turbulent pulsations and so on) and origin of physical structures.

This follows from the evolutionary relation for state functional that is obtained from the equations of mathematical physics and can generate closed exterior forms, which correspond to conservation laws for physical fields (exact conservations laws).

Such a property of the evolutionary relation discloses a connection between the field-theory equations and the equations of mathematical physics for material media.

4. CONNECTION THE FIELD-THEORY EQUATIONS WITH THE EQUATIONS OF MATHEMATICAL PHYSICS FOR MATERIAL MEDIA

The connection between the field-theory equations and the equations of mathematical physics for material media is caused by the correspondence between the field-theory equations, which solutions are closed exterior forms (relevant to conservation laws for physical fields), and the evolutionary relation that generates closed exterior forms.

4.1. Correspondence between the Field-Theory Equations and the Evolutionary Relation

The field-theory equations, which describe physical fields, are equations for functionals such as wave function, the action functional, Pointing's vector, Einstein's tensor, and others. The nonidentical evolutionary relations derived from the equations of mathematical physics, which describe material media, are relations for all these functionals.

To the correspondence between the field-theory equations and the evolutionary relation it also points out to the fact that, all equations of field theories, as well as the evolutionary relation, are nonidentical relations in differential forms or in the forms of their tensor or differential (i.e. expressed in terms of derivatives) analogs. For example,

- the Einstein equation is a relation in differential forms;
- the Dirac equation relates Dirac's *bra*- and *cket*- vectors, which made up a differential form of zero degree;
- the Maxwell equations have the form of tensor relations;
- the Schrodinger's equations have the form of relations expressed in terms of derivatives and their analogs.

[The field-theory equations are those whose solutions must be not functions but differentials (closed inexact exterior forms that must describe physical structures). Only the equations that have the form of relations (nonidentical) may have the solutions which are differentials rather then functions.]

From the field-theory equations, as well as from the nonidentical evolutionary relation, the identical relation, which contains the closed exterior form, is obtained. As one can see, from the field-theory equations it follows such identical relation as

- the Poincare invariant, which connects closed exterior forms of first degree;
- the relations $d\theta^2 = 0,\ d^*\theta^2 = 0$ are those for closed exterior forms of second degree obtained from Maxwell equations;
- the Bianchi identity for gravitational field.

The fact that closed exterior forms, which are obtained from the evolutionary relation, correspond to conservation laws for physical fields points out to a correspondence between the evolutionary relation and the field theory equations. From the evolutionary relation one obtains differential-geometrical structures constructed of dual and inexact exterior forms, which describe physical structures that made up physical fields.

Thus, one can see that there exists a correspondence between the field-theory equations, which describe physical fields, and the evolutionary relation obtained from the equations of mathematical physics for a material medium. Such a correspondence between the evolutionary relation and the field-theory equations points to a connection of the field-theory equations with the equations of mathematical physics for material media.

[Here it should be emphasized that there exist a certain departure of the field-theory equations from the evolutionary relation. The field-theory equations have been obtained from the invariance condition. This may be illustrated by the example of Einstein's equations. Thus, in the paper by Einstein [7] it was proved the invariance of equations for energy, momentum and continuity. In doing so it was proposed that the consistence of the conservation laws is identically fulfilled. This was used when finding the energy-momentum tensor in Einstein's equation.

However, as it was shown, the consistence of the conservation law equations (due to noncommutativity of conservation laws) is fulfilled only discretely. That is, the conservation law equations are not invariant identically (for example, due to nonpotentiality of boundary and initial conditions). In particular, the energy-momentum tensor if fulfilled discretely.

This means that Einstein's equation (as opposed to the evolutionary relation) fulfilled only discretely and cannot describe evolutionary processes. This equation can describe only realized gravitational fields, rather then the processes of realization of these fields. (Similarly, Schredinger's equation is fulfilled only discretely since the potential in the right-hand side of the equation is not fulfilled identically). Such a deviation of the field-theory equation from the evolutionary relation points out to the fact that the evolutionary relation, as opposed to the field-theory equations, possesses more wide potentialities. Namely, this relation describes evolutionary process.]

4.2. Hidden Properties of the Field-Theory

The connection of the field-theory equations with the equations mathematical physics for material media enables one to understand the basic principles of field theory and the properties of physical fields.

1. The functionals of field-theory equations (such as wave function, the action functional, the Pointing vector, Einstein's tensor, and others) are functionals that specify the state of relevant material medium (material system).
2. The closed exterior forms, which describe conservation laws for physical fields and lie at the basis of field theories, are generated by evolutionary forms obtained from the equations of mathematical physics for material media.
3. Nondegenerate transformations of field theories (transformations of closed exterior forms that conserve a differential) relate to the degenerate transformations of evolutionary forms obtained from the equations of mathematical physics for material media. The degenerate transformations

describe the process of emergence of physical structures, whereas the nondegenerate transformations describe transitions from one structure to another.

4. The symmetries of field theories are conditioned by the degrees of freedom of material media. In this case the interior symmetries of field theories relate to the conservation laws for physical fields, whereas the exterior symmetries of field theories, which are conditioned by the degrees of freedom of material media, relate to the equations of the conservation laws for material media.
5. The constants and characteristics of field theories are connected with characteristics of relevant material systems. (But this connection is indirect. This connection is realized in evolutionary process).
6. The emergence of physical structures occurs discretely under realization of the degrees of freedom of material systems. This explains the quantum character of field theories.
7. Type of physical structures generated by material media depends on

(1) the degree p of the evolutionary form in evolutionary relation connected with the number of interacting conservation laws;

(2) the degree k of the closed exterior form obtained, and

(3) the dimension n of the initial inertial space.

[This follows from the evolutionary relation under its integrating [2, 3]. By integrating (under realization of relevant degenerate transform) the nonidentical evolutionary relation with the forms of degree p the closed forms on the pseudostructure of sequential degrees $k = p$, $k = p-1,..., k = 0$ are formed (which indicates the creation of physical structures of a relevant type). When generating closed forms of sequential degrees $k = p$, $k = p-1,..., k = 0$ the pseudostructures of dimensions $(n+1-k)$: $1,...,n+1$ are obtained, where n is a dimension of original inertial space. As a result of transition to the exact closed form of zero degree the metric structure of the dimension $n+1$ is obtained.]

[Problem of integration of the nonidentical evolutionary relation discloses the properties of Einstein's equation and its connection with the space curvature, with the pseudo-Riemann manifold and the Riemann space. Einsteins' equations are obtained from evolutionary relation with the evolutionary skew-symmetric form of the third degree which differential is nonzero since the curvature is nonzero.

As it is known, when deriving the Einstein equation [4, 7, 15] it was supposed that the following conditions to be satisfied: the Bianchi identical is fulfilled, the connectedness coefficients are symmetric ones (the connectedness coefficients are the Christoffel symbols), and there exists a transformation under which the connectedness coefficient become zero. These conditions are those of realization of the degenerate transformations for the nonidentical evolutionary relations and transition to the identical relations while integrating [2,3]. Here it should be emphasized that these conditions are not identical. They are realized discretely under the degenerate transformations corresponding to any degrees of freedom of material system. In particular, the symmetry of the connectedness coefficients is not fulfilled identically. (It should be noted that in the evolutionary relation the connectednesses coefficients are nonsymmetric. Symmetric connectednesses coefficients are realized only discretely under degenerate transformation). (The attempts to apply nonsymmetric connectednesses coefficients were made by Einstein when trying to building a unified field theory. In the monograph by W. Pauli [6] (in the subsection "Remarks to the English edition" in the note 23, the point "Theories with nonsymmetric the connectednesses") this fact was noted.)]

8. It is evident that the field theory equations are connected with closed exterior forms of a certain degree. This points out to the fact that there exists an internal connection between field theories, which describe physical fields of various types. It is evident that the degree of closed exterior forms is a parameter that integrates field theories into unified field theory.
9. One can see the correspondence between the degree k of the closed forms realized and the type of interactions. Thus, $k=0$ corresponds to strong interaction, $k=1$ corresponds to weak

interaction, $k = 2$ corresponds to electromagnetic interaction, and $k = 3$ corresponds to gravitational interaction. (In the paper [2] it is presented the table of elementary particles, where physical fields and interactions in their dependence on the parameters p, k, n of evolutionary and closed exterior forms are demonstrated.)

10. The mechanism of origination the physical structures elucidates the mechanism of forming physical fields and manifolds [2]. (The dual forms, which are metric forms of the manifold, describe pseudostructures, from which pseudometric and metric manifolds are formatted). To every physical field it is assigned its own material medium (a material system). As examples of material systems it may be cosmic systems, systems of elementary particles and others.
11. [The evolutionary processes in material medium described by evolutionary relation help to elucidate the origination of dark energy and dark matter. Because of the noncommutativy of conservation laws (that does not allow the given nonpotential actions to convert into own quantities of material medium), the nonpotential inconsistent external actions upon material medium, lead to emergence of nonmeasurable quantity (described by the evolutionary form commutator) that acts as interior force.

As it was shown, under realization of any degrees of freedom of material system the nonmeasurable quantity partly converts into observable and measurable formations (that is, into own quantities of material medium). However, since this occurs only locally, only a part of nonmeasurable quantity converts into into own quantities of material medium. This means that a certain nonmeasurable quantity remains. The dark energy and dark matter are such nonmeasurable and nonobservable quantity (essence) that emerges due to various nonpotential actions and, because of the noncommuativity of conservation laws, cannot directly convert into own quantities of material medium.]

5. CONCLUSION

The fundamental result that clarifies the problems of field theories is the fact that the field theories, which are based on the conservation laws for physical fields, are connected with the equations of mathematical physics which consist of the equations of noncommutative conservation laws for material media (the balance conservation laws of energy, linear momentum, angular momentum, and mass as well as the analog of such laws for the time, which takes into account the noncommutativity of time and energy of material system). Such a connection, which is common to all field theories, discloses the general foundations of field theories. This connection has to be taken into account while building the general field theory.

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APPENDIX 1

The equations of mathematical physics describing material media. In the present paper, the equations of mathematical physics, which describe material media, are investigated.

Such equations are presented, for example, in monographs:

R.Courant, "Partial Differential Equations", New York.London, 1962 (Chapter 6, paragraph 3a),

W. Pauli "Theory of Relativity, Pergamon Press, 1958 (paragraphs 30,37).

The equations of gas-dynamics, which describe the flow of ideal gas, are the example of such equations.

In an inertial frame of reference such equations can be written in the following form [8].

The conservation law equations for energy (see 5.3.21)

$$\rho\frac{D}{Dt}(e+\tfrac{1}{2}u^2)=f_1$$

The conservation law equations for linear momentum (see 5.3.15):

$$\rho\frac{\partial u_j}{\partial t}+\rho u_k\frac{\partial u_j}{\partial x_k}=f_j$$

The conservation law equations for mass (see 5.3.12)

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_k}(\rho u_k)=0$$

In the accompanying frame of reference (this frame of reference is connected with the manifold made up by the trajectories of the material system elements) the conservation law equations for energy can be written as (see [8], Chapter 6):

$$\frac{\partial s}{\partial \xi^1}=\frac{1}{\rho}\frac{\partial}{\partial x_i}\left(-\frac{q_i}{T}\right)-\frac{q_i}{\rho T}\frac{\partial T}{\partial x_i}+\frac{\tau_{ki}}{\rho}\frac{\partial u_i}{\partial x_k}$$

(here s is *entropy*, ξ^1 is *the coordinate along the trajectory*.)

In the case of two-dimensional flow of ideal gas, the conservation law equations for linear momentum can be written in the following form (see [8], Chapter 6):

$$\frac{\partial s}{\partial \xi^\nu}=\frac{\partial h_0}{\partial \xi^\nu}+(u_1^2+u_2^2)^{1/2}\varsigma-F_\nu\frac{\partial U_\nu}{\partial t}$$

where $\varsigma=\partial u_2/\partial x-\partial u_1/\partial y$. Here ξ^ν is *the coordinate in the direction normal to the trajectory.*